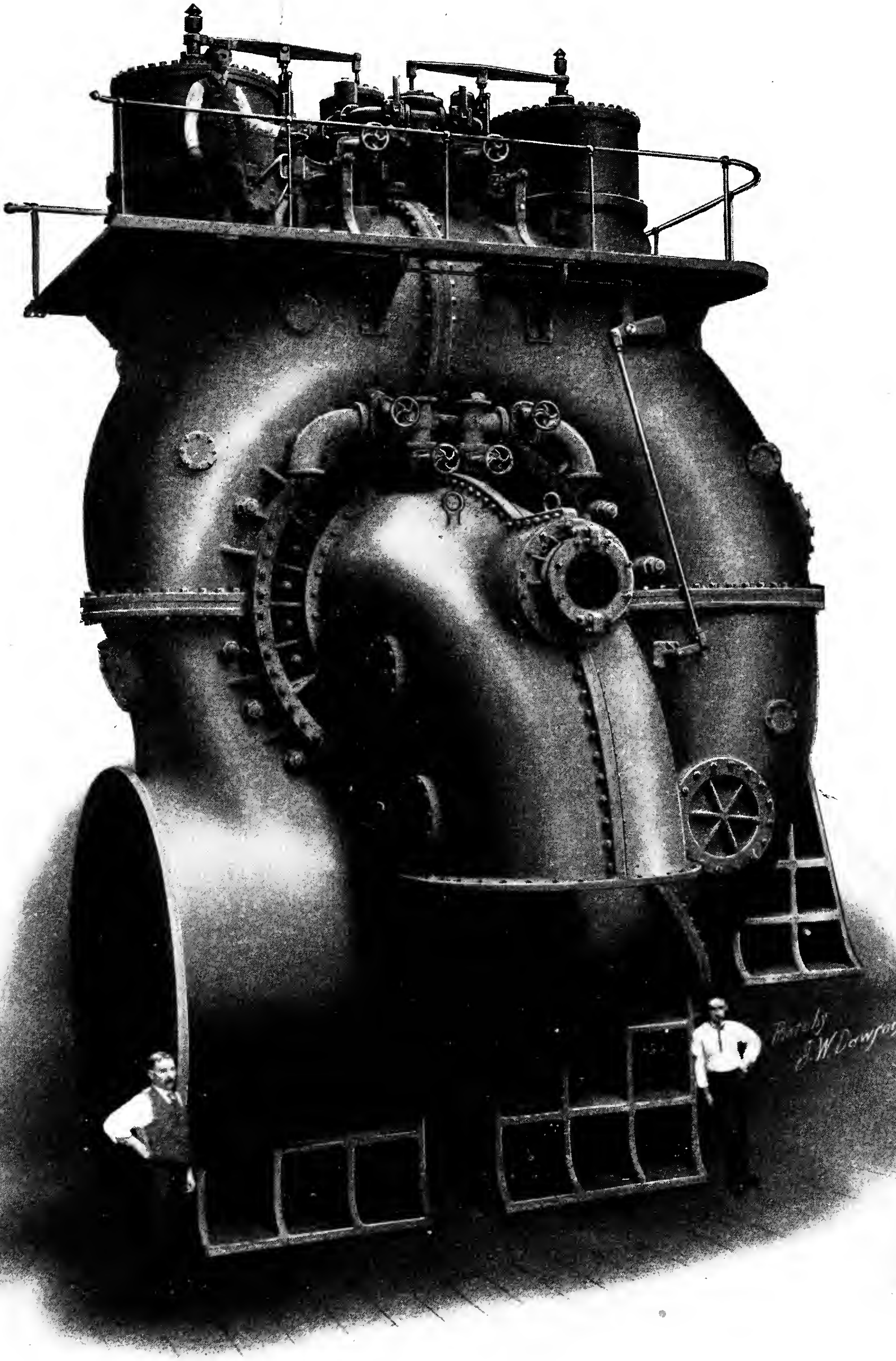

THE SHAWINIGAN 10,500 H.P. TURBINE.
Designed and built by the I. P. Morris Co., Phila., Pa.

HYDRAULIC MOTORS

WITH RELATED SUBJECTS

INCLUDING

CENTRIFUGAL PUMPS, PIPES, AND OPEN CHANNELS

DESIGNED AS

A TEXT-BOOK FOR ENGINEERING SCHOOLS

BY

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ADDENDA.

P. 4—In fourth line add: The weight of the wheels and buckets is a neutral force, since their center of gravity neither sinks nor rises.

P. 10—Third line. This value of P is also proved in this book. (See eq. (6), p. 66, with $\alpha = 180^\circ$.)

P. 133—The "Fig. 70." mentioned in first line will be found opposite p. 125.

ERRATUM.

P. 191—Eleventh line of § 117: for "increasing" read "decreasing."

Note.—For a short course, to include Hydraulic Motors proper, the following paragraphs may be selected, viz.: §§ 1–6, 14–59, 62–104a.

PREFACE.

By reason of the great increase that has taken place of recent years throughout the world in the utilization of water power, notably in connection with the electric transmission of energy, a special and growing prominence attaches to the subject of Hydraulic Motors in the curriculum of engineering schools.

In the preparation of the following pages, as forming a text-book on this important branch of hydraulics for the use of students of engineering, it has been borne in mind that to facilitate the acquirement of clear and sound ideas on the mechanics of the subject is the first essential of such a book; and, as greatly assisting to this end, ample numerical illustration has been provided in direct connection with the necessary algebraic treatment. At the same time, it is believed that sufficient descriptive matter has been introduced, relating to both past and present construction and design, to make the treatment a fairly practical one for its purpose, when regard is had to the limited time available for this subject in the ordinary course of study at an engineering school.

Some attention is also given to centrifugal pumps (so much improved of recent years) and other allied appliances; and to special problems, closely connected with the subject of water-power, involving pipes, weirs, and open channels. The experiments of Joukovsky on water-hammer are presented and the theory of this phenomenon is developed.

The student is supposed to be already well versed in the part of hydraulics dealing with stationary vessels and pipes, as set forth (for instance) in the writer's *Mechanics of Engineering* (in referring to which the abbreviation M. of E. is used).

It is hoped that the book may prove useful to practising engineers as well as students; in which connection attention is called to the diagrams of friction-heads in pipes and those for determining Kutter's coefficients for open channels. These have been especially prepared for the present work and will be found in the Appendix.

For the use of many of the illustrations appearing in the pages of this work the writer would acknowledge his great obligation to the following technical journals and engineering firms:

Engineering News (Figs. 9, 10, 37, 38, 39, 40, 89, and 90).
Cassier's Magazine (Figs. 52, 53, 54, and 55).
Kilburn, Lincoln and Co., Fall River, Mass.
I. P. Morris Co., Phila.
Risdon-Alcott Turbine Co., Mount Holly, N. J.
Pelton Water-wheel Co., San Francisco.
Abner Doble Co., San Francisco.
James Leffel Co., Springfield, Ohio.
Allis-Chalmers Co., Milwaukee, Wis.
Platt Iron Works Co., Dayton, Ohio (Victor Turbine).
Dayton Globe Iron Works, Dayton, Ohio (New American Turbine).
Lombard Governor Co., Ashland, Mass.
De Laval Steam Turbine Co., Trenton, N. J. (Centrifugal Pumps).
Lawrence Machine Co., Lawrence, Mass. (Centrifugal Pumps).
Irvin Van Wie, Syracuse, N. Y. (Centrifugal Pumps).
Henry R. Worthington, New York (Water-motor Pump).
Columbia Engineering Works, Portland, Oregon (Hydraulic Ram).
Goulds Manufac. Co., Senaca Falls, N. Y. (Hydraulic Ram).

Cornell University,
Ithaca, N. Y., Sept. 1905.

CONTENTS.

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Chapters on hydraulic motors may also be found in:

Cotterill's Applied Mechanics;
Merriman's Hydraulics;
Bovey's Hydraulics;
Rankine's Steam-engine;
Unwin's Hydromechanics (Encyc. Britann.).

NOTATION AND CONSTANTS.

The Greek letters α, β, δ, θ, ζ, λ, μ, and ϕ are used for angles [δ also for a ratio (pp. 25 and 32); ζ for a coefficient (pp. 105, 174, and 192); and μ as a coefficient in weir formulæ]; η for *efficiency;* and π for the ratio 3.1416.

γ (gamma) is the weight of a unit of volume of fresh water, viz. 62.3 lbs. per cub. ft. at 62° Fahr. (or 0.03604 lbs. per cub. inch); but 62.5 (or $1000 \div 16$) is quite accurate enough for ordinary hydraulic problems. (Sea water weighs 64 lbs. per cub. ft.)

ω (omega) is angular velocity of a rotating body (e.g., *radians per second;* in which case revs. per sec. would be $\omega \div 2\pi$).

A is Kutter's coefficient (p. 215).

b is the height of the (ideal) water-barometer. For a pressure of one standard atmosphere (14.70 lbs. per sq. inch or 2117 lbs. per sq. ft.) b is 34 lineal ft.; corresponding to a mercury column of 30 inches, nearly (29.95). In any actual case the value of b depends on the weather and the elevation above sea-level. For example, at 6000 ft. above sea-level it might be about 27 ft.

c, c_1, c_n, etc., are *relative* velocities of water, on pp. 37, 38, 57–61, and 73–187.

c is an *absolute* velocity (of water) on pp. 1–35 an 62–70.

v, v', v_1, etc., are linear velocities of points of a revolving body (turbine) on pp. 1–187. v and $c =$ mean velocity of water in pipe or channel on pp. 188–237.

w, w_1, w_n, etc., are *absolute* velocities of water passing through a motor; but = wetted perimeter, p. 229.

h, H, y, and z are used for vertical heights.

f is the coefficient of fluid friction, pp. 188–237.

l is a length.

F is the area of a cross section of the passageway of a turbine, or of pipe or channel.

u is "velocity of whirl" (p. 48).

V is "velocity of flow" (p. 172).

L is *power* (ft.-lbs. per sec., e.g.), or rate of work.

Q (rarely q) is the volume of water flowing per unit time in steady flow (e.g., cubic ft. per sec.; gallons per minute).

H.P. = *horse-power* (= ft.-lbs. per sec. power ÷ 550).

p is *unit pressure;* e.g., lbs. per sq. in. (but an acceleration on p. 40, and the height of a weir on p. 223).

P is a force (or total pressure) (lbs.); and R, or R', a resistance (i.e., a force) (lbs.). R is "hydraulic radius" on pp. 215, 216.

r, r_1, etc., are used for radii; d for diameter (also depth).

s is the slope of the water surface in an open channel, p. 214.

G is the total weight of a body.

g is the *acceleration of gravity.* In the temperate zones we may use, for all ordinary problems in hydraulics, the value 32.2 (for the English foot and second as units) (or 386.4 for the inch and second), as sufficiently accurate; the error involved being only a small fraction of one per cent. Near the equator $g = 32.09$ at sea-level, and 32.06 at 10,000 ft. elevation. It is 32.18 at London and 32.15 at Baltimore; 32.26 at the pole, sea-level.

One *U. S. gallon* of fresh water (see *Conversion Scales,* in Appendix) weighs 8.34 lbs. at ordinary temperatures and has a volume of 231 cub. in. (or 0.1336 cub. ft.). One cub. ft. contains 7.48 U. S. gallons. (N.B. This gallon measure is in common use in this country and must not be confused with the English, or Imperial, gallon, which contains 277.27 cub. in. An English gallon of fresh water weighs 10 lbs.)

HYDRAULIC MOTORS.

CHAPTER I.

General Considerations and Principal Types of Motors.

1. **Water-power.**—The descent of water from a higher to a lower level, through a properly designed machine, suitably regulated as to speed by the imposing of certain resisting forces to prevent acceleration of the motion of the machine, may be made the means of furnishing certain pressures or "working forces," acting at different parts of the machine, by whose action a *steady* or uniform motion of the machine may be kept up notwithstanding the presence of the resisting forces. In such a case the continuous overcoming of the resistances is said to be accomplished by **Water-power,** and the machine is called a **Hydraulic Motor.**

If the resultant pressure of the water on the machine or "motor" is P lbs., and its point of application travels uniformly at the rate of v ft. per second *in the direction* of the force P, then the power of the water exerted on the machine is the product Pv ft.-lbs. per second, (which divided by 550 gives Horse-power;) and if there is but one resistance, of R' lbs., applied to the motor, and its point of application is forced to travel backwards (backwards as regards the direction of pointing of the resistance R') at the rate of v' ft. per second, the power thus expended is $R'v'$ ft.-lbs. per second; and we have the equality

$$Pv=R'v', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

since the R' is supposed to have such a value that the motion of the machine is not accelerated. (See § 146, M. of E.)

2. Motors of the Gravity, Pressure, and Inertia Types.—The continuous maintenance of this working force, or pressure, P, of the water against the motor is due generally, in the last analysis, to *gravity*, i.e., to the weight of the water; but it is not necessarily due to the weight of the portions of water in actual contact with the motor; such is the fact, indeed, (or nearly so,) in the case of motors carrying detached bodies of water in buckets, and these may be called pure gravity motors; but in the case of pressure engines, with slowly moving pistons, the pressure is kept up by communication with a distant and large body of water; while with turbines, and with motors utilizing a "free jet" (i.e., a jet in the open air) of high velocity, the pressure is occasioned by causing the liquid to flow through channels or against surfaces of the motor in such a way that its absolute velocity is diminished by the constraint which the parts of the motor, if properly designed, exert upon its motion. This change of absolute velocity is usually accompanied by a *gradual* change of direction, to avoid waste of energy. These latter may be called **Inertia Motors.**

(In the case of an **Inertia** motor the water usually gains its initial absolute velocity, at entrance of the motor, through the *previous* action of gravity, though in some cases this velocity may be due to the action of a pump driven by steam or other power.) We may therefore distinguish between *Gravity Motors*, *Pressure Motors*, and *Inertia Motors* (or Kinetic Motors); though some belong to more than one of these categories, as will be seen.

3. Efficiency.—If a motor could be so designed (and *regulated*) as to use the full supply, Q cu. ft. per second, of a stream, and also the full "head," h (feet), or difference of level between the surface of the water in the "head-water" (or pond) and "tail-water" (or pool where the water flows away, below the motor), the maximum theoretical water-power would be equivalent to a working force equal to a weight of $Q\gamma$ lbs. (γ being the weight of one cubic foot of water) working through h ft., in

each second of time; i.e., equivalent to $Q\gamma h$ ft.-lbs. per second; but the useful power, $R'v'$, accomplished by a motor at its very best is always less than this, on account of various kinds of friction and because the water itself usually leaves the motor with a certain amount of velocity, thus carrying away, unutilized, a corresponding amount of kinetic energy, each second.

The ratio of the power usefully expended, viz., $R'v'$, to the full theoretical maximum, $Q\gamma h$, is called the *Efficiency* of the motor and will be denoted by the symbol η (pronounced ay-tah); that is,

$$\eta = \frac{R'v'}{Q\gamma h}. \quad \ldots \ldots \ldots \quad (2)$$

4. Example of a Gravity Motor.—A succession of buckets on an endless chain, confined in their motion to a vertical plane (the chain passing over two sprocket-wheels whose axles revolve in firm horizontal bearings) constitutes a nearly pure gravity motor. See Fig. 1. Each bucket as it moves down receives water at the point A and loses its contents at B. A resistance R' (tension in a rope, e.g., winding up on drum at C, being of sufficient value, we have a uniform velocity v of bucket, that of the rope being v'. It will be seen from the figure that the height h_1, from A to B, is a little less than that, h, from head-water surface H to tail-water surface at T. Since the motion of a bucket while holding water is *uniform* and *rectilinear*, the resultant pressure of the water within it upon the bucket is equal to the weight of its contents, which we may call G lbs.

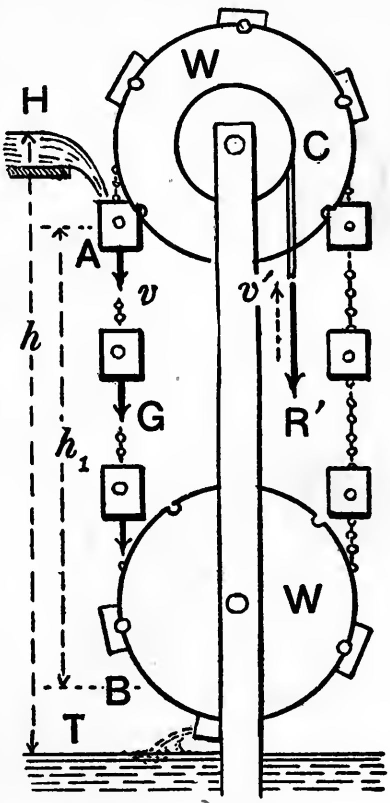

Fig. 1.

If we consider the buckets, sprocket-wheels, chain, and drum as a collection of rigid bodies forming a machine, and

apply the method of "Work and Energy" (see pp. 149–153, M. of E.), we note that there is a working force G acting on each of the n full buckets on the left; that R' is the only resistance (axle frictions are here neglected); and that the reactions at the bearings are neutral forces in this connection; and also that there is no change in the kinetic energy of the moving masses of the collection (by hypothesis) from second to second. Hence, considering the space of one second of time, we have

$$nGv = R'v'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let now t = time for a bucket to pass from A to B; then $v = h_1 \div t$ and

$$\therefore \; \frac{nGh_1}{t} = R'v'.$$

But nG lbs. of water $\div t$ = lbs. passing per second, = volume per second $\times \gamma$, i.e., $\frac{nG}{t} = Q\gamma$,

so that we have finally

$$Q\gamma h_1 = R'v'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Evidently, with greater perfection of design and operation the quantity $Q\gamma h_1$ could approach $Q\gamma h$ but could not exceed it; hence $Q\gamma h$ is called the full theoretical power of the "mill-site," and we have for the efficiency the ratio (as before defined)

$$\eta = \frac{R'v'}{Q\gamma h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Numerical Example.—With $Q = 2$ cub. ft. per sec. and $h = 20$ ft., we have, using the ft.-lb.-sec. system of units, $Q\gamma h = 2 \times 62.5 \times 20 = 2500$ ft.-lbs. per second, maximum theoretical power. Hence if the bucket-motor is so designed as to have an efficiency of 80 per cent and the velocity of cable at C is desired to be $v' = 2$ ft. per second, we may put $R'v' = 0.80 \times 2500$ and obtain $R' = 1000$ lbs. tension, as the resistance that could be overcome by the motor at that speed, in steady motion. If the velocity of the buckets themselves is kept at the value (say) $v = 3$ ft. per

second, the radius of the drum C must be made two thirds of that of the upper sprocket-wheel.

Strictly, the pressure on a bucket during filling at position A is a little greater than the weight of the water in it at any stage of the filling; again, both the filling and the emptying of any bucket are gradual. These facts are neglected at present, for simplicity, but will be considered later.

5. Buckets Moving in a Circular Path.—If the buckets are firmly attached to the rim of a single rigid wheel (revolving in a vertical plane) and thus constitute a vertical water-wheel, the resultant pressure on a bucket of the water in it is not equal to the weight of that water during uniform motion, but the effect as to power is the same; that is, we shall have $Q\gamma h_1 = R'v'$ as before; h_1 being the vertical distance from the point of filling to that of emptying.

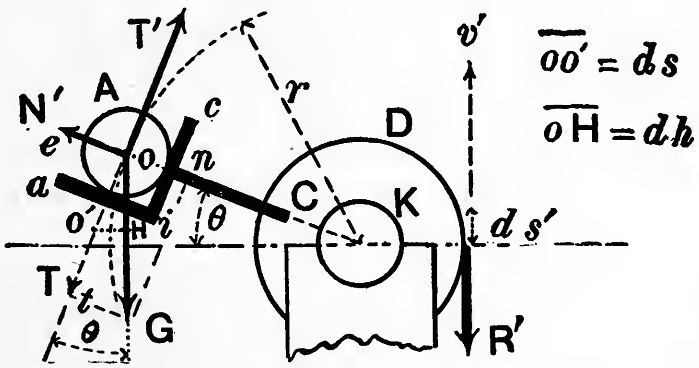

Fig. 2.

To prove this, in simple fashion, consider (Fig. 2) a heavy ball of weight G lbs., resting against a plate ai parallel to the radial arm nC, and upon another plate, ic, perpendicular to the same; both plates perpendicular to the vertical plane of the paper. The arm oC and plates are rigidly fastened to drum CD and axle K. There is a resistance R' acting at edge of drum (tension in a rope, say). The rigid body $ainCK$ is rotating uniformly, the ball with it, counter-clockwise, on axle K, in (vertical) plane of paper.

Let θ = the angle between the arm nC and the horizontal at this instant (or between the plate ic and the vertical). Let the reaction of plate ai against the ball be a force T' lbs.; that

of plate *in*, N' lbs. The only other force acting on the ball is that of the earth, i.e., its weight, G. The motion of the center of the ball being curvilinear, in the arc of a circle whose radius is r, and having a uniform velocity v, in that curve, we have [from p. 76, M. of E.]

$$\Sigma(\text{tang. compons.})=0, \quad \text{or} \quad G\cos\theta - T'=0;$$

and $$\Sigma(\text{norm. compons.})=(G\div g)v^2\div r;$$

i.e., $$G\sin\theta - N'=(G\div g)(v^2\div r).$$

Hence the value of the pressure T' against the ball is $G\cos\theta$, while that of N' is *not* $G\sin\theta$, but is $G\sin\theta-\dfrac{Gv^2}{gr}$.

However, when we apply the principle of work and energy to the rigid body *anD* for the very short time interval, dt, in which point o passes to o', describing a path of length ds, while a short length ds', of rope, winds up on the drum, dealing now with the equals and opposites of N' and T', we have, the motion being uniform, $T'.ds+N'\times\text{zero}=R'ds'$.

But $T'=G\cos\theta$ and $ds\cos\theta=\overline{OH}=dh$, = vertical descent of the center of gravity of the ball in time dt, and hence

$$Gdh=R'ds', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the same as would have been found in the foregoing case of the bucket-motor for the time dt (with nG in place of the present G); and therefore, for a complete second, we should have

$$Q\gamma h_1=R'v'; \quad \text{(see later, in the overshot wheel.)} \quad . \quad (2)$$

6. Simple Pressure Engine.—Fig. 3. Here we consider a single stroke, from left to right, of a piston of area F. sq. ft., under water pressure on both sides, from the tanks H (head-water) and T (tail-water), whose surfaces are h ft. apart, vertically. The motion is slow and uniform, acceleration being prevented by the action of a suitable resistance R' lbs. against the piston-rod (whose sectional area is small compared with that, F, of the piston). The unit-pressure on the left face of the piston is p_m (lbs. per sq. in.), a little *less* than the hydrostatic pressure due to the depth HE (plus the outside

atmospheric pressure p_a) on account of the loss of head at entrance E of communicating orifice, or port. Similarly, the unit-pressure at m', on the right-hand face, is $p_{m'}$, a little *greater* than that due to the vertical depth Tm' (plus atmosphere). If piezometric tubes A and B, open to the air, are provided in the sides of the cylinder, as shown, the heights, y and y', of the stationary water columns in them above the level mm' will, with atmospheric pressure added, measure the pressures

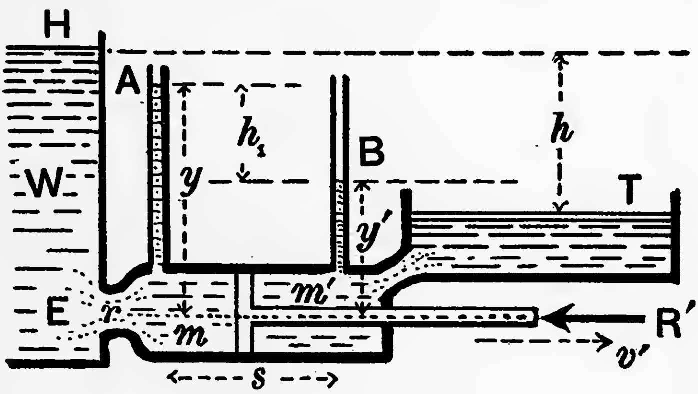

FIG. 3.

p_m and $p_{m'}$. The motion of the water is assumed to be a "steady flow," so that these water columns do not fluctuate in height. Hence we write

$$p_m = p_a + y\gamma, \quad \text{and} \quad p_{m'} = p_a + y'\gamma;$$

so that for steady motion the value of the resistance R' should be

$$R' = F[p_m - p_{m'}], \quad = F\gamma[y - y'] = F\gamma h_1.$$

Hence, the work done upon the resistance in one stroke being $R's$, we have $\qquad F\gamma s h_1 = R's. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

But, if n strokes are made in a unit of time, say one second, (provision being made, by means of valves and of air-vessels and by the employment of more than one cylinder and piston, for the maintenance of continuous operation and of a practically "steady flow,") we have

Work per second, i.e., the *power* $= nF\gamma s h_1, = R'(ns). \quad . \quad (4)$

Now $nFs =$ the volume of water used per unit of time, $= Q$,

and ns = the velocity, v', of the point of application of the resistance R' in the direction of the latter (in general, projected on the line of action of the resistance); whence the power, L, of the motor may be written

$$L, = Q\gamma h_1, = R'v'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

It is evident that h_1 can never quite equal h, though it may be made to approach it quite closely in the case of this kind of motor; that is, as before, the ideal maximum power is $Q\gamma h$, and the efficiency $= \frac{R'v'}{Q\gamma h}$.

We here note that in passing from position E to the point where it leaves the motor the water has not been subjected to any notable change in velocity, nor in vertical position; that is, that between E and m' there has been no change in kinetic, nor in potential, energy, but that there has occurred a great change in the internal fluid pressure; so that this kind of motor is sometimes described as acting by the surrender on the part of the water of some of the "pressure energy" possessed when in position m. But it should be remembered that these phrases are arbitrary and artificial, being employed simply for convenience. Some authors use the word potential energy as including pressure energy. Others would say that the potential energy contained in the water at H has been converted into the form of pressure energy at m, since no conversion into energy of motion (i.e., into kinetic energy) has taken place at that stage.

Numerical Example.—If a water-pressure engine is working steadily with a piston speed of $v' = 8$ in. per second, the diameter of piston being 11.72 in.; with value of $h = 70$ ft., and of $h_1 = 64$ ft.; we have for the power obtained (denote it by L)

$$L, = Q\gamma h_1, = Fv'\gamma h_1, = \frac{\pi}{4}\left[\frac{11.72}{12}\right]^2 \cdot \frac{2}{3} \times 62.5 \times 64,$$

$$= 2000 \text{ ft.-lbs. per second,} \quad \text{or} \quad 3.63 \text{ horse-power.}$$

The quantity of water used per second is $Q = Fv' = 0.5$ cu. ft. per second, and the thrust in the piston-rod R' is $L \div v'$ or

$2000 \div 0.666 = 3000$ lbs. (If we neglect the friction on edges of piston and in stuffing-box). The efficiency η is

$$\eta = \frac{R'v'}{Q\gamma h} = \frac{2000}{0.5 \times 62.5 \times 70} = 0.914,$$

or nearly 92 per cent. This might be obtained more easily by putting

$$\eta = h_1 \div h; \quad \text{or} \quad 64 \div 70; \quad = 0.914.$$

7. A Simple Inertia, or Kinetic, Motor.—It has already been proved in § 566 of the M. of E. (and will also be shown later in this work) that if, by provision of a proper resistance R, the speed of the cups of an impulse water-wheel, such as a Pelton or Doble wheel, be regulated to a value of about one half that of the water in the free "jet" (or jet in the open air) issuing from a nozzle and operating upon the cups in succession, a maximum power is obtained; that is, we have a maximum value for the product, Pv, of the (mean) tangential force (working force), P, of the jet against the cups of the wheel, by the linear velocity v of these cups, which is the distance through which the working force acts each second.

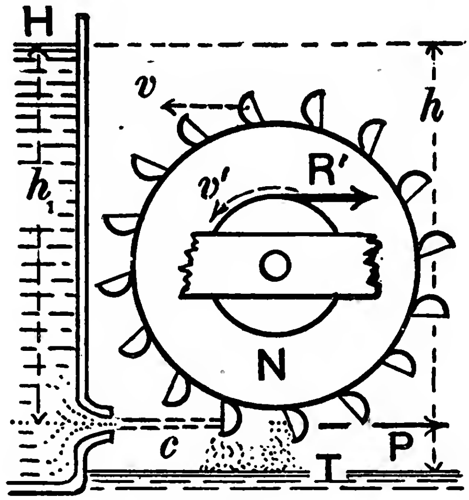

FIG. 4.

Fig. 4 shows such a wheel in steady operation, supplied with a free jet issuing from an orifice or nozzle in the side of a reservoir whose upper surface is h_1 ft. above the center of nozzle. The velocity, c, of the jet, since it is a free jet, is practically the same as if the wheel were not in position and has a value (see § 496, M. of E.) of $c = \phi\sqrt{2gh_1}$, where ϕ is the coefficient of velocity for the nozzle in question.

For uniform motion of the wheel, R' being the resistance applied to the rim of the smaller wheel (on same shaft) where the velocity is v', we must have, from the theory of work and

energy applied to the uniform motion of this rigid body, $Pv = R'v'$. But from p. 808, eq. (7), M. of E., we have, for a series of cups, the value of P, viz., $P = \frac{2Q\gamma(c-v)}{g}$, where $Q, = Fc$, is the volume of water passing per second from the nozzle. (F = the sectional area of the jet.)

Hence the power expended on R', $R'v'$, or exerted by P, is (after writing $\frac{c}{2}$ for v, for maximum power; see p. 808, M. of E.)

$$L = Pv = \frac{Q\gamma}{g} \cdot \frac{c^2}{2}, = R'v'. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Or, substituting from the equation $c = \phi\sqrt{2gh_1}$,

$$L, = R'v', = \phi^2 Q\gamma h_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

As the action of the water on the cups is more or less imperfect, the usual power ($R'v'$) obtained in practice is rarely more than 80 per cent. of this last expression. If this imperfection of action could be neglected and the value of ϕ taken as unity, with h_1 approximating to h (the total vertical distance between head- and tail-water surfaces), the theoretical ideal maximum power of the motor would be $Q\gamma h$, ft.-lbs. per sec., as in the other cases already instanced.

As before, the efficiency would be

$$\eta = \frac{R'v'}{Q\gamma h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Here we may say that the total power of the mill-site $Q\gamma h$ (ft.-lbs per sec.), i.e., $Q\gamma\frac{c^2}{2g}$, (if ϕ be unity,) has been converted into the kinetic form $\frac{Q\gamma}{g} \cdot \frac{c^2}{2}$ (or, mass per second $\times$ half-square of the velocity of jet) at the point where the water is about to act on the motor; so that this kind of motor utilizes the energy of the mill-site in the kinetic form. At the point of leaving the motor the water is at the same level as at entrance, and is under the same pressure (atmospheric pressure) as at

entrance, but has practically lost all its velocity (when cups have proper speed).

Numerical Example.—With a head, h_1, of 100 ft. and a value 0.95 for ϕ, we have for the velocity of the jet (free jet) $c=0.95\times\sqrt{2\times 32.2\times 100}$, or 76.23 ft. per second. If the mill-site furnishes $Q=2$ cu. ft. of water per second, the kinetic power (i.e., kinetic energy per second) of the jet just before impinging on the cups of the wheel is

$$\frac{Q\gamma}{g}\cdot\frac{c^2}{2}, =\frac{2\times 62.5}{32.2}\cdot\frac{(76.23)^2}{2};$$

i.e., 1128 ft.-lbs. per second. If the impulse wheel utilizing this jet has an efficiency of 80 per cent., the useful power obtained will be $L', =R'v', =0.80\times 1128=9024$ ft.-lbs. per second; for which result the speed of the cups must be maintained at the proper value, viz., $c\div 2$, or 38.1 ft. per second. To keep the speed of the cups from accelerating beyond this figure the value of the resistance R', if it is to act on a periphery of the wheel having (say) half the radius of that described by the center of the cups, will need to be

$R'=L'\div v'=9024\div 19.05=473$ lbs. (The value of P is 236 lbs.)

In the case of an impulse wheel the efficiency is usually referred to $Q\gamma h_1$ instead of $Q\gamma h$ (see Fig. 4).

8. Mixed Types of Motors.—It will be seen later that in the working of some kinds of motors (like the class termed "reaction-turbines") the water is not only under pressure in closed spaces at the entrance of the motor channels, but may have considerable velocity as well. In other words, the energy of the water at entrance is partly in the pressure form and partly in the kinetic. It will therefore be of interest and advantage to prove a general theorem of such a form as to bring into play all three of the quantities *pressure, velocity,* and *elevation* (above a convenient datum) of the point where the water enters the motor, or just before; and also similar quantities at the point of exit from the motor, or just downstream from such a point, as follows:

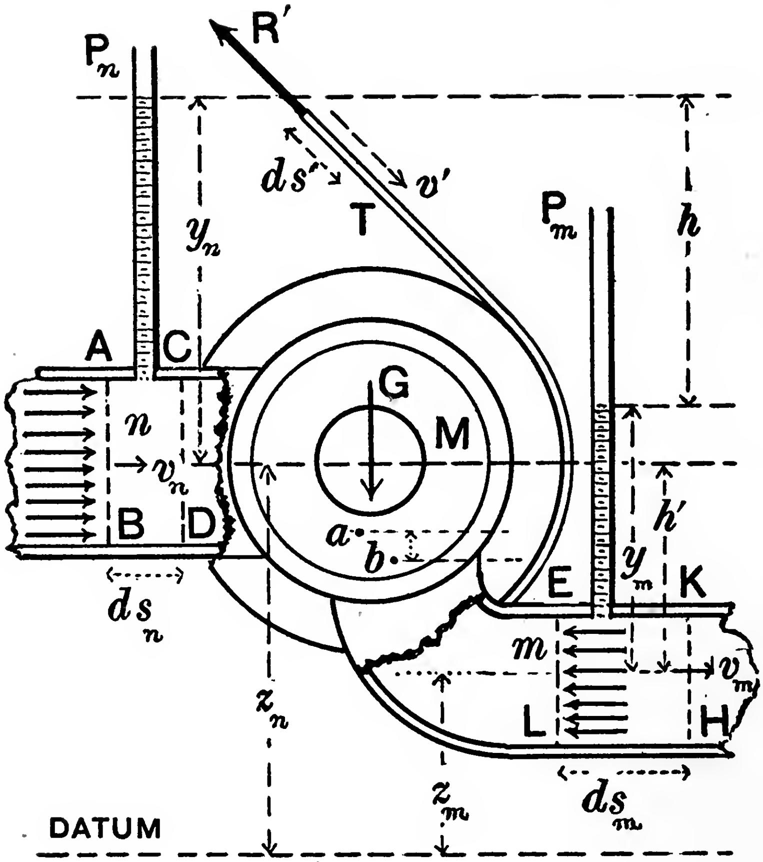

FIG. 5.

9. General Theorem for the Power Derived from any Hydraulic Motor in Steady Operation.—This will apply, whatever the nature of the motor may be (piston-motor, rotary motor, or what not) so long as its operation is smooth and steady, with uniform motion of the parts and a *steady flow* on the part of the water at rate of Q cu. ft. per sec. Fig. 5 shows a casing M, within which a water-motor is working. Water enters at n through a pipe AB, shown in longitudinal section, and leaves the motor at m through the pipe EL. All pipes are supposed full of water, as also all chambers, cells, or passageways of the motor, which is composed of rigid parts. Piezometers P_n and P_m being supposed inserted in the walls of the pipes at AC (up-stream pipe) and EK (down-stream pipe), the internal fluid pressure at point n, viz., P_n, will be indicated by the height, y_n, of the stationary water column (plus the atmospheric pressure, since the piezometer is an open one). That is, with p_a for atmospheric (unit) pressure, we have $p_n = p_a + y_n\gamma$; and likewise at the point m the internal fluid pressure is $p_m = p_a + y_m\gamma$. The mean velocity of the water in the cross-section of the pipe at n may be called v_n; and that at m, v_m. The height of n above the datum plane in figure will be called z_n; that of m, z_m. Elevation of n above $m = h'$; and the difference of elevation of the summits of the two piezometer columns, h.

The power of the motor is considered to be applied to the overcoming of a constant resistance, R' lbs., in the form of the tension in a rope or cable the velocity of any point of which is constant and is denoted by v'. That is, the cable is being wound upon a drum at a uniform rate. The value of R' is supposed to be such that the motion of the motor and of the water passing through is "*steady*," so that no part of the motor has any acceleration and the values of p_n, p_m, v_n, and v_m, and also that of Q (cubic feet per second, rate of flow of the water) remain constant. The sectional areas of the two pipes at n and m are F_n and F_m, respectively; whence we have $Q = F_n v_n$; and also $Q = F_m v_m$.

We are now to consider the assemblage, or collection, of

rigid bodies consisting of all the particles of water between the sectional plane AB of the inlet-pipe and that, EL, of the outlet-pipe, together with all the moving parts of the motor itself (including the cable up to the point where R' is applied (see Fig. 5). To the range of motion of this collection of rigid bodies which takes place during a short time, dt seconds, let us apply the general *Theorem of Work and Energy* as proved in § 142, p. 149, of M. of E. In this theorem the work done by, or upon, those forces only which are external to the bodies concerned need be considered, pressures between any two bodies of the collection being totally ignored unless of the nature of friction. Both internal and external frictions must be considered, each internal friction (i.e., friction between any two members of the collection) must be multiplied by the proper distance of *relative* travel. Any external force whose point of application moves at right angles to the line of the force is "neutral," i.e., does no work.

Items of Work.—During the small time, dt seconds, here considered, the pressure on the bounding plane AB, viz., $F_n p_n$ lbs., is a working force and works through the small distance $ds_n, = \overline{BD}$, the work so done being $F_n p_n ds_n$, while section AB is moving to a new position CD. Similarly, the resisting pressure $F_m p_m$ on the down-stream bounding plane (as if it were the face of a piston) EL, at m, is overcome through a corresponding small distance $ds_m, = \overline{LH}$; i.e., the work $F_m p_m ds_m$ is spent upon this resistance. The resistance R' is overcome through some small distance, ds' (amount of cable wound up in time dt). The work expended on external friction, such as axle, or shaft, friction, will be indicated by $\Sigma(R''ds'')$, while that of internal friction (of the water on itself or between the water and the moving blades or pistons of the motor) by $\Sigma(R'''ds''')$. During the motion of this collection of rigid bodies in time dt, the center of gravity of the portion of water now being considered, situated initially between AB and EL, the weight of which we may call G lbs., sinks from some position a to some lower position b. Denote the length of the vertical projection of this distance $a\,..\,b$ by dh (feet). This gravity-

force, G, does the work $G \cdot dh$. When the plane AB arrives at CD (and, correspondingly, plane EL reaches position KH) there is as much water, and in the same position, between CD and EL as there was before, and the weight of the lamina AD equals that of lamina EH (viz., $F_n ds_n \gamma = F_m ds_m \gamma$); hence the product of weight $F_n ds_n \gamma$ by the vertical height h' $(=z_n - z_m)$ is equal to that of weight G by dh (see § 32 for more detailed proof) and may replace it. Therefore, finally, the expression for the aggregate work done (positive and negative) in the time dt is

$$\overline{dW} = F_n p_n ds_n - F_m p_m ds_m + (F_n ds_n \gamma)(z_n - z_m) - R'ds' - \Sigma(R''ds'') - \Sigma(R'''ds'''). \quad (1)$$

It still remains to formulate the change that occurs during this time dt in the amount of kinetic energy possessed by the moving rigid bodies of the collection considered. Since the motion of all the parts of the motor itself is uniform, such change for them will be zero. As for the (rigid) particles of liquid concerned, consider all the particles of water between AB and EL to be divided into a vast number of contiguous groups, of equal volumes, each group having a volume equal to that of the lamina $ACDB$, this lamina being the first group of the series and having a mass $= dM, = F_n ds_n \gamma \div g$; these groups being so selected that in the short time dt the velocity of all the particles in any group shall have acquired a new value just equal to that which the particles of the group next ahead had at the beginning of the dt. It follows, therefore, from the definition of "steady flow" (see p. 648, M. of E.) that in subtracting the initial kinetic energy $\left(\frac{dMv^2}{2}\right)$ from the final, for each group of particles, and adding up these results for all the groups, from the first, AD, to the last, EH, (whose right-hand face is at EL at the beginning of the dt,) all the terms involved will *cancel out* except the initial kinetic energy of the first group (or lamina) and the final kinetic energy of the last group. That is to say, the result for the aggregate change in the kinetic energy of all the bodies of the collection, in time dt, is

$$d(\text{K.E.}),=\frac{F_m ds_m\gamma}{g}\cdot\frac{v_m^2}{2}-\frac{F_n ds_n\gamma}{g}\cdot\frac{v_n^2}{2}. \quad . \quad . \quad . \quad (2)$$

Equating the expressions for dW and d(K.E.), and replacing $F_m ds_m$ (and also its equal $F_n ds_n$) by $Q\cdot dt$ (the volume flowing in time dt) and then dividing through by dt, noting that $ds'\div dt=v'$, the velocity of a point in the cable, while v'' is the velocity of the rubbing parts for any friction such as R'', and v''' has a similar meaning (relative velocity) for any internal friction, R''', we have, finally,

$$Q\gamma\left[\left(\frac{v_n^2}{2g}+z_n+\frac{p_n}{\gamma}\right)-\left(\frac{v_m^2}{2g}+z_m+\frac{p_m}{\gamma}\right)\right]$$
$$=R'v'+\Sigma(R''v'')+\Sigma(R'''v'''). \quad (3)$$

(Each side of this equation is *ft.-lbs. per second; power.*)

$R'v'$ may be called the *useful power* of the motor and the other items, $\Sigma(R''v'')$ and $\Sigma(R'''v''')$, the *lost power*, or that wasted in friction. We may therefore say that the power of the motor, partly spent in the useful power, $R'v'$, and the remainder wasted in the work of friction (both of fluid friction and that between solids) is equal to the product of the weight $Q\gamma$ (lbs. of water used per second) by the difference between the sum of the three heads (viz., velocity-head, pressure-head, and potential-head or elevation above datum) at the point of entrance to the motor, and the sum of those at the point of exit therefrom.

Just as $\frac{Q\gamma}{g}\cdot\frac{v^2}{2}$ is called the kinetic energy of the mass $\frac{Q\gamma}{g}$ of water, as due to its velocity v, and $Q\gamma z$ its potential energy due to elevation above datum, similarly $Q\gamma\frac{p}{\gamma}$ may be called the "*pressure-energy,*" due to internal fluid pressure; (a mere name, however; useful when the *flow is steady;* this would not imply that a receiver full of stationary water under pressure possessed thereby more than a trifling amount of energy, due to its pressure.)

Hence eq. (3) might be reread as follows: The amount of energy (of the three kinds defined) lost during passage

through the motor by the weight (say lbs.) of water used per second, is equal to the power spent by the motor on the useful resistance and the various frictional resistances.

In actual practice, with a good motor run at proper speed, the useful power, $R'v'$, may be as much as 85 per cent. of the power given up by the water; i.e., may be 85 per cent. of the sum of the useful power and the power wasted in friction (this latter part reappears in the form of heat).

[N.B. Evidently, if no motor is placed in the line of pipe between n and m, $R'v'$ and $R''v''$ disappear and we have from eq. (3) Bernoulli's Theorem for steady flow in a stationary rigid pipe, the loss of head between n and m being represented by $\dfrac{\Sigma(R'''v''')}{Q\gamma}$.]

10. Another Form of Equation (3).—In Fig. 5 we may note the following relations (b being the height of the water barometer, or about 34 ft.):

$$\frac{p_n}{\gamma}=y_n+b, \quad \text{and} \quad \frac{p_m}{\gamma}=y_m+b;$$

also $$h'+y_n=y_m+h, \quad \text{and} \quad z_n-z_m=h'.$$

h denotes the vertical distance, or "drop," from the summit of the up-stream piezometer column, at A, to that in the lower, at E. Eq. (3) may now be written in the form

$$Q\gamma\left[h+\left(\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right)\right]=R'v'+\Sigma(R''v'')+\Sigma(R'''v'''). \quad . \quad (4)$$

Hence, if the entrance- and exit-pipes were *equal in sectional area*, thus making v_n equal to v_m, we should have

$$Q\gamma h=R'v'+\Sigma(R''v'')+\Sigma(R'''v'''). \quad . \quad . \quad . \quad (5)$$

11. Numerical Example of Foregoing.—In a test of a hydraulic motor it is found that when a value of R' (friction of a brake on pulley) of 240 lbs. is furnished for the motor to work against, on the rim of a pulley of $r=1$ ft. radius keyed upon the shaft of motor, the uniform speed to which the motor adjusts itself is $n=306$ revs. per minute, the consumption of water is $Q=1.2$ cu. ft. per second, while the pressure-gauge

readings at n and m respectively (see Fig. 5) are 56 lbs., and 6 lbs., per sq. in., above the atmosphere. The point n in the supply-pipe, which is 4 in. in diameter, is at an elevation of 4 ft. above the point m in the discharge- or "exit"-pipe, 6 in. in diameter.

Required the useful power, $R'v'$, and the efficiency of the motor, η, at this speed.

Solution.—Here we have (see Fig. 5), using ft., lb., and second,

$$h=\left[4'+\frac{56\times 144}{62.5}+b\right]-\left[\frac{6\times 144}{62.5}+b\right]=119.7 \text{ ft.}$$

Also, $\quad v_n=Q\div F_n=1.20\div\left[\frac{\pi}{4}\left(\frac{4}{12}\right)^2\right]=13.7$ ft. per sec.,

and $\quad v_m=Q\div F_m=1.20\div\left[\frac{\pi}{4}\left(\frac{6}{12}\right)^2\right]=6.1$ ft. per sec.;

whence $\quad \frac{v_n^2}{2g}=2.91$ ft., and $\frac{v_m^2}{2g}=0.58$ ft.

Hence

$$Q\gamma\left[h+\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right]=1.20\times 62.5[119.7+2.91-0.58]$$
$$=9290 \text{ ft.-lbs. per sec.}$$

= energy given up by the water each second in passing through the motor.

Now the useful power being $R'v'$, viz.,

$$R'v',=R'(2\pi rn),=240\times 2\pi\times 1\times 5.10=7690 \text{ ft.-lbs. per sec.,}$$

it follows that in this test, at speed of 306 revs. per minute, the motor developed an efficiency of 83 per cent.; since

$$\eta=\frac{R'v'}{Q\gamma\left[h+\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right]}=\frac{7690}{9290}=0.83.$$

The difference between the 9290 and the 7690, i.e., 1600 ft.-lbs. per second, is, of course, the value of the lost power (heat); amounting to some 17 per cent. of the power given up by the water. At other speeds, to secure which the value of R' would have to be changed to various other values, succes-

sively, the efficiency would be different; and it is usually an important object in the testing of a motor to ascertain at what speed it develops the greatest efficiency, this speed being the "*best speed*" for its operation. The quantity of water used per second, Q, may, or may not, be the same at different speeds; this depending on the kind of motor employed.

12. **Pump, instead of Motor.**—In this connection it will be of advantage to consider another kind of test. In Fig. 5 conceive a pump of some kind, say a centrifugal pump, the theory of which will be presented later, to be placed inside of the casing M and to be operated by the application of working force P lbs., applied tangentially to the periphery of a pulley (radius $=r$) keyed upon the shaft. If the rim of this pulley travels with a velocity v and the force P is the tension in an *unwinding* cable (or perhaps the tangential component of the pressure of a pinion-tooth against the tooth of a gear-wheel), the power applied in working the pump will be Pv ft.-lbs. per second, and water will be caused to pass in steady flow through the pump from point m, in what is now an inlet-pipe, to point n in the pipe AB (now a discharge-pipe); that is, from a point where the pressure-head, velocity-head, and potential-head are $\frac{p_m}{\gamma}$, $\frac{v_m^2}{2g}$, and z_m, respectively, to a point n where the sum of the corresponding heads is greater than at m. (v_n and v_m now point to left.)

If Q is the volume of water pumped per second, it is easily proved, by the same method as that just followed in § 9 (considering that in the present case P and $F_m p_m$ are working forces, and $F_n p_n$ and G resistances), that

$$Pv-\Sigma(R''v'')-\Sigma(R'''v''')$$
$$=Q\gamma\left[\left(\frac{v_n^2}{2g}+\frac{p_n}{\gamma}+z_n\right)-\left(\frac{v_m^2}{2g}+\frac{p_n}{\gamma}+z_n\right)\right];$$

or, more conveniently, that

$$Pv=Q\gamma\left[h+\left(\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right)\right]+\Sigma(R''v'')+\Sigma(R'''v''') . \quad . \quad (1)$$

Here, as before, $\Sigma(R'''v''')$ denotes the power lost in fluid

friction, whether in the passages of the pump or in portions of the stationary pipes between m and n; while $\Sigma(R''v'')$ is the power lost in friction between the solid parts.

Eq. (1) declares that, of the applied power Pv necessary from some external source (such as a steam-engine or water-wheel) to operate the pump at a certain uniform speed, the portions $\Sigma(R''v'')$ and $\Sigma(R'''v''')$ (ft.-lbs. per sec.) are lost, or wasted; while the remainder, i.e., $Q\gamma\left[h+\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right]$, is usefully employed in pumping water. The efficiency of the pump is the ratio, or fraction,

$$\eta=\frac{\text{useful power}}{\text{power applied}}=\frac{Q\gamma\left[h+\frac{v_n^2}{2g}-\frac{v_m^2}{2g}\right]}{Pv}, \quad . \quad . \quad . \quad (2)$$

or
$$\eta=\frac{Pv-[\Sigma(R''v'')+\Sigma(R'''v''')]}{Pv}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

13. Example of Test of Pump.—The following example represents very nearly the case of a test of a centrifugal pump used on a hydraulic dredge on the Mississippi River. (See pp. 136 to 167 of the Report of the Mississippi River Commission for 1903.) Although reference is now made to Fig. 5, it must be understood that the direction of flow of the water is from m toward n, and that in the place of a resistance R' we now have a working force P; the direction of motion of the cable (if we conceive that to be the manner of operating the pump-shaft) being the same as that of the force P.

From gauges inserted in the sides of the entrance-pipe (or "suction-pipe") at m and of the discharge-pipe at n, close to the pump-casing, and various other measuring appliances, the following data were obtained:

$p_m=3$ lbs. per sq. in. below the atmosphere;
$p_n=4.1$ " " " above " "

$v_m=13$ ft. per sec., hence $\frac{v_m^2}{2g}=2.7$ ft.;

$v_n=13.7$ " " " " $\frac{v_n^2}{2g}=2.9$ ft.

The delivery-point n was ($h' =$) 3 ft. higher than entry point m. $Q = 86.7$ cub. ft. per sec.

The steam-engine driving the pump was found to expend power in so doing at the rate (net) of 280 horse-power. Therefore $Pv = 280 \times 550 = 154,000$ ft.-lbs. per second.

It will be noted that the pressure at m was 3 lbs. per sq. in. *below* atmospheric pressure; in other words, the height y_m of Fig. 5 is *negative*. This value, 3 lbs. per sq. in., corresponds to a piezometric height of 6.95 ft. and hence the value of h (height from summit to summit of piezometer columns in Fig. 5) will be $9.5 + 3 + 6.95, = 19.45$ ft.

Hence the power expended in pumping water is

$$Q\gamma\left[h + \frac{v_n^2}{2g} - \frac{v_m^2}{2g}\right] = 86.7 \times 62.5[19.45 + 2.9 - 2.7]$$
$$= 105,400 \text{ ft.-lbs. per second;}$$

while the power exerted in running the pump is, as before, 154,000 ft.-lbs. per sec. Hence, for the efficiency of the pump in this trial, we find

$$\eta = \frac{105,400}{154,000} = 0.685; \quad \text{or} \quad 68\tfrac{1}{2} \text{ per cent.}$$

CHAPTER II.

Gravity Motors.

14. The Overshot Water-wheel.—This form of hydraulic motor, with others of the same type, though now nearly obsolete, will be given a few pages in the present work. Fig. 6 (from Weisbach's Mechanics) represents a wooden wheel of this class, revolving in a vertical plane on an axle in firm bearings. As seen from the figure it consists of two ring-shaped shroudings, or crowns, connected with the axle by radial arms, and a number of floats or buckets inserted between the crowns and forming, with them and a cylindrical boarding concentric with the axle, a series of cells. The water is supplied from a sluice or pen-trough near the top of the wheel, and is regulated by a gate, falling in a sheet, or jet, into the third or fourth bucket from the summit. These wheels have been constructed for falls of from 4 to 70 ft., sometimes receiving as much as $Q=50$ cub. ft. of water per second; and of from 3 to 50 or more horse-power. With high falls and a large supply of water it is better to use two or more small wheels rather than a single large one, whose necessarily great weight would be a disadvantage. The fall is measured from the surface in the supply channel, or *pen-trough*, to that of the tail-water. To lose the least head possible the wheel should hang just tangent to the tail-water; or, if the level of the latter is variable, high enough to avoid contact. In Fig. 6 H is the axle, B and C its gudgeons; DMF, $D'M'F'$, the crowns, or shroudings, made in 8 to 16 segments and from 3 to 5 in. thick, and fastened to each other by cross tie-bolts which

also pass through the arms. The cog-wheel E serves to transmit the power to the machinery.

More elaborate wheels of this class have been built of iron, with sheet-iron floats.

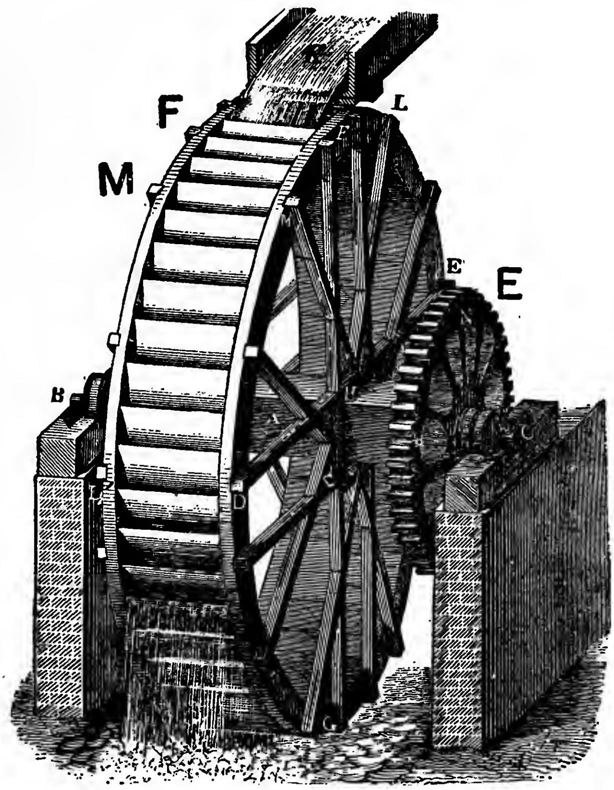

FIG. 6.

15. Theory of the Overshot Wheel.—In Fig. 6a we have a vertical section of a wheel of this kind, revolving counter-clockwise with uniform motion and overcoming the resistance R' at the rim of the gear-wheel A. The (uniform) velocity of the buckets should not exceed 5 ft. per second for small wheels, nor 10 ft. per sec. for high overshots. Otherwise the water would spill prematurely from the buckets and also carry away with it, unutilized, too great an amount of kinetic energy.

Each bucket is filled on passing some position E at a convenient angle θ with the vertical; the jet entering it having a velocity $c_1 = \sqrt{2gh_1}$, nearly; (say $=0.95\sqrt{2gh_1}$). The height

of wheel is nearly equal to h, the fall or vertical "head" from head-water, H, to tail-water, T.

If the various dimensions of the wheel, the quantity of water used per second (Q), the velocity of rotation of the wheel, and the resistance R' (lbs.) tangent to the rim of the gear-wheel A, are properly adjusted to each other, the motion remains uniform and the work per second done by the pressure of the water on the walls of the buckets or cells will (if we neglect axle friction) be equal to that spent per second (viz.,

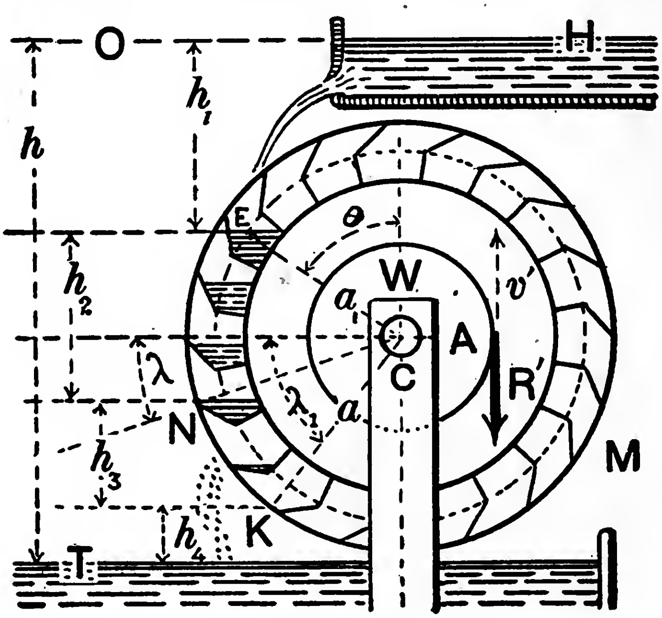

Fig. 6*a*.

$R'v'$ ft.-lbs. per sec.) on the resistance (v' being the velocity of the rim of wheel A). For example, R' may be the resisting pressure of the teeth of a pinion keyed on another shaft operating the machinery of a mill. The buckets should be of such shape and size, in connection with the proper speed, as to enable each cell to hold its contents as long as possible before reaching the lowest position. N shows the point where spilling begins, and K the position where the cell has completely emptied itself. During the filling of a cell under the jet the pressure against the cell walls is greater (for equal amounts of water) than it is later when the cell has passed the jet, since the water which first enters receives thereafter the impact

of the jet, without being driven out of the cell. The power due to this extra pressure is called the *power due to impact.* Its total amount is much less than that due to the steady action of the weight of the water after the cell has passed the jet, as will be seen.

16. Power Due to the Weight of the Water.—E may be taken as the point where the full action of the weight of the water begins, just after the mouth of the bucket has passed the jet. Let a_1 denote the radius of the "division circle" (dotted in figure) or circle half way out along the radial depth of a cell; a the radius of the outer edge of cell; θ, λ, and λ_1 the various angles marked in the figure. The whole fall, h, or vertical distance between the surfaces of the head- and tail-waters, H and T, may be considered to be made up of four parts, viz., h_1, serving to generate the velocity attained by the jet just before entering a cell, i.e., O to E, or $h_1=\frac{1.1c^2}{2g}$; a part, E to N, or $h_2,=a_1 \cos\theta+a \sin \lambda$; a part, N to K, or $h_3,=a \sin \lambda_1-a \sin \lambda$; the remainder being h_4.

The power due to the weight of the water in the buckets may now be written as the product of $Q\gamma$ lbs. per second by the height h_2 throughout which there is no spilling, plus the product of a certain fraction $(=\delta Q\gamma)$ of $Q\gamma$ by the height h_3 throughout which, on account of the progressive spilling, the average weight of water in action per sec. is $\delta Q\gamma$. (On the average, δ may be put $=0.50$.) We thus obtain, for the power due to the weight of the water,

$$L_g=Q\gamma[h_2+\delta h_3] \ldots \text{(ft.-lbs. per sec.)}. \quad . \quad . \quad . \quad (1)$$

17. Power Due to Impact.—As already stated, the pressure on the bottom of a cell, while water from the jet is entering, is greater than the weight of the amount of water so far entered, on account of the impact of the jet, so that the work per second done upon this part of the wheel is at a greater rate than if weight were the sole cause of the pressure on the cell.

This extra pressure, and corresponding power, due to impact will now be evaluated; it being borne in mind that the particles

of water impinging on the water already in the cell *do not fly out of the cell* after the impact, but join the water already in the cell and move on with it, and with the same velocity. (See Fig. 7.) The tangential component of the force of impact at the division circle, OA, where the impact may be considered to take place, may be computed thus: Let v_1 = the velocity of a point of a cell in the circumference of the division circle, and c_1 the absolute velocity of the particles of the jet where it intersects that circle. α_1 = included angle. During each small space of time Δt a small mass, Δm, of liquid has its velocity in the tangential direction OT changed from $c_1 \cos \alpha_1$ to v_1; i.e., its motion in that direction suffers a negative acceleration of $p = \dfrac{c_1 \cos \alpha_1 - v_1}{\Delta t}$; therefore the retarding force must be

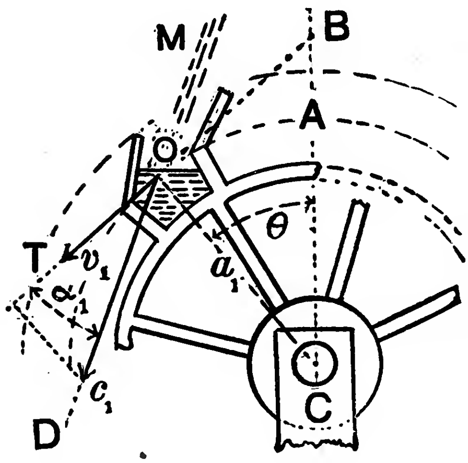

FIG. 7.

$$P_1 = \text{mass} \times \text{acc.} = \Delta m \,.\, p = \frac{\Delta m}{\Delta t}(c_1 \cos \alpha_1 - v_1),$$

which is also equal and opposite to the force in direction OT with which the mass Δm presses the bucket. But $\dfrac{\Delta m}{\Delta t} = \dfrac{Q_1 \gamma}{g}$ = the mass of water arriving per unit of time in the *bucket*. Hence this force or pressure can be written

$$P_1 = \frac{Q_1 \gamma}{g}[c_1 \cos \alpha_1 - v_1]; \text{ (lbs.)}$$

This is simply the continuous pressure, or working force, acting on the bucket, due to impact. The work done by it each second, i.e., the *power* steadily obtained from the impact, for each bucket in turn, is obtained by multiplying the force by the distance v_1 through which it works in each second in its own direction. Therefore, so long as a bucket receives

water, the work done upon it by impact is at the rate of $P_1 v_1$ per unit of time; i.e., the power due to impact

$$=[Q_1\gamma \div g][c_1 \cos \alpha_1 - v_1]v_1 \text{ ft.-lbs. per sec.} \quad . \quad . \quad (2)$$

for each bucket in turn. But the portion of jet intercepted between the edges of two consecutive buckets is free to do work on the forward of the two while other work is being done on the hinder one; hence, if Q = the volume of water passing the *pen-trough* H per unit of time, the rate of work, or power, in the long run, due to impact, is found by replacing the Q_1 of the last equation by Q; thus

$$L_1 = \frac{Q\gamma}{g}[c_1 \cos \alpha_1 - v_1]v_1. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If now we make v_1 variable, we find by Calculus that L_1 is a maximum when $v_1 = \frac{1}{2}c_1 \cos \alpha_1$, and therefore, by substitution,

$$L_{1\,\max.} = Q\gamma\frac{c_1^2}{2g} \cdot \tfrac{1}{2} \cos^2 \alpha_1, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which, even for $\alpha_1 = 0$, would only $=\frac{1}{2}\frac{Q\gamma}{g}\frac{c_1^2}{2}$, or only one half of the kinetic energy of the water supply before impact; and this is the maximum effect of impact.

Hence it is an object to use but a small portion of the total fall to impart entrance-velocity to the water. We also note that if the entrance-velocity is kept small, as above advised, and if the best effect of impact is obtained for $v_1 = \frac{1}{2}c_1$ (nearly), v_1 itself must be quite small. Hence a slow velocity tends to greater efficiency of the wheel, in this respect. There must be a limit, however, for a slow motion requires a greater width of wheel to accommodate the water, with a consequent increase of weight, the axle friction occasioned by which would, beyond a certain limit, consume more power than that gained by the slow motion; whence the limiting values of velocity mentioned in § 15.

18. Total Power of Overshot.—Adding the two items of power just obtained, viz., L_g and L_1, due to gravity and impact, respectively, we have, as the total power, L,

$$L = Q\gamma\left[\frac{(c_1 \cos \alpha_1 - v_1)v_1}{g} + h_2 + \delta h_3\right] \quad . \quad . \quad . \quad (5)$$

ft.-lbs. per sec. The efficiency is $\eta = L \div Q\gamma h$, if axle friction be neglected.

19. Numerical Example of Overshot.—Let the whole fall, h, be 32 ft., and let 30 ft. be adopted for the diameter of the wheel, with $Q = 10$ cub. ft. per second as the available water supply. Let the radius of the division circle be $a_1 = 14.5$ ft. and the angles θ, λ, λ_1, and α_1 be respectively equal to 20°, 46°, 70°, and 12°; (see Figs. 6*a* and 7.) Compute the power of the wheel, if 3 ft. be taken as h_1, and the value of δ as 0.5.

With foregoing data we have, for the entrance-velocity of jet, $c_1, = 0.95\sqrt{2gh_1}, = 0.95\sqrt{64.4} \times 3 = 13.1$ ft. per sec.; whence v_1 should be $13.1 \div 2, = 6.5$ ft. per second, for best effect of impact.

The fall with full buckets is then found to be

$$h_2, = a_1 \cos 20° + a \sin 46°, = 14.5 \times 0.94 + 15 \times 0.719,$$
$$= 13.62 + 10.78 = 24.40 \text{ ft.}$$

Also, $h_3 = a(\sin 70° - \sin 46°) = 15(0.940 - 0.719)$
$$= 15 \times 0.221 = 3.31 \text{ ft.}$$

We have, then, for the total power, from eq. (5) of § 18,

$$L = 10 \times 62.5\left[\frac{(13.1 \times 0.978 - 6.5)6.5}{32.2} + 24.40 + \frac{3.31}{2}\right]$$

$$= 10 \times 62.5[1.27 + 24.40 + 1.65] = 10 \times 62.5 \times 27.32$$
$$= 17{,}075 \text{ ft.-lbs. per sec.}, = 31.0 \text{ H.P.}$$

If there were no axle friction, nor resistance due to the atmosphere, this power would be equal to $R'v'$ (R' being the useful resistance, at edge of gear-wheel, and v' the velocity of that edge). If the radius of the gear-wheel is 3 ft., the value of v' is $\frac{3}{14.5}$ of v_1, or $v' = 1.34$ ft. per sec. From $R'v' = 17{,}075$, we have $R' = 17{,}075 \div 1.34 = 12{,}700$ lbs.

But the weight of the wheel and the water in the buckets and R' itself (unless the latter pointed upward, as it would do if on the other side of the shaft from its position in Fig. 6)

would occasion considerable axle friction. For example, if the total pressure on the main bearings of the shaft were 30,000 lbs., and the coefficient of axle friction were 0.10, the total friction would be R'', $(=0.10\times30{,}000,)=3000$ lbs. Supposing the diameter of each journal to be 6 in., we find that its circumference would rub against the bearing at a velocity of $(\frac{1}{4}\div14.5)6.5,=0.113$ ft. per second, $=v''$. Hence the product

$$R''v'',=3000\times0.113,=339 \text{ ft.-lbs. per second,}$$

would be the power lost through this cause. This friction being considered, therefore, we put

$$L,=17{,}075,=R'v'+R''v'', \quad \text{and obtain}$$
$$R'v'=17{,}075-339=16{,}736 \text{ ft.-lbs. per sec.;}$$

that is, with $v'=1.34$ ft. per sec., $R'=12{,}500$ lbs.

As to the efficiency of the overshot wheel in this example, the full theoretical power of the mill-site being $Q\gamma h=10\times62.5\times32$, or 20,000 ft.-lbs. per sec., we derive for the efficiency,

$$\eta=\frac{R'v'}{Q\gamma h}=\frac{16{,}736}{20{,}000}=0.836, \quad \text{or} \quad 83\tfrac{6}{10} \text{ per cent.}$$

It is seen from the above figures that the lowest point of the wheel hangs about 0.5 ft. above the surface of the tail-water.

19a. Special Overshots.—The largest overshot ever built is the Laxey wheel, 72 ft. in diameter, on the Isle of Man, England; developing some 150 horse-power and operating pumps for draining a lead-mine (see pp. 214 and 219 of Cassier's Magazine for July 1894). The largest wheel of this kind in the United States is at the Burden Iron Co.'s works, Troy, N. Y. Its diameter is 62 ft. and width 22 ft. and its weight 230 tons, 550 H.P. being developed. The great "sand-wheels" of the Calumet and Hecla Mining Co., at their stamp-mills in Lake Linden, Mich., are practically reversed overshot wheels, with buckets on the rim, by means of which, driven by suitable power, sand and water are elevated. The diameter of each is 54 ft., and width 11 ft. (Cassier's Mag., July 1894, pp. 217 and 218). These wheels, as also the Burden wheel above

mentioned, have rims supported by *tension spokes,* somewhat as bicycle wheels are constructed.

Space cannot be given in the present work for developing formulæ and rules for designing the form and number of buckets; to which end simple geometrical and mechanical principles apply, the main point being to have the cells only partly filled, that spilling may occur as late as possible. The parabolic path of the jet issuing from the head-basin, or pen-trough, must also be considered in arranging for the proper position of the sheet or jet entering the buckets. For details of this kind the reader is referred to Weisbach's "Hydraulic Motors," translated by Prof. Du Bois.

Values of efficiency as high as 80 per cent. have been reached by well-designed overshots, but their construction has been discontinued for many years.

20. Breast or Middleshot Wheels.—Wheels revolving in a vertical plane and having buckets or floats receiving the water at, or near, the level of the axle are called **middleshot** wheels; and if set in a flume closely fitting the water-holding arc, **Breast** wheels. The "apron" or surface of the flume fitting the wheel should not be more than $\frac{1}{2}$ to 1 in. from the circumference of the wheel, that but little water may escape. Instead of buckets, simple radial floats are generally used, sometimes slightly curved backwards near the circumference, to diminish resistance on rising from the tail-water.

A large number of floats is effective, not only because the loss of water between wheel and apron is smaller, but because, from the smaller interval between them, the impact head is smaller and the vertical distance through which the water acts by gravity is greater. Generally the outer distance between two consecutive floats is made $=d=$ the width of the shroudings, i.e., from 10 to 12 inches.

It is essential that middleshot wheels should be well "ventilated," that is, provision should be made for the passage of the air from the bucket-space toward the inside of the wheel; since the water on entering the wheel fills nearly the whole cross-section between the floats, thus preventing the ready

escape of the air outwardly. This is all the more necessary from the fact that the shrouding-space of these wheels is filled to one third or one half of its capacity. Breast wheels are in use on falls of 5 to 15 ft., and using from 5 to 80 cub. ft. per second.

The water may be introduced either by means of an overfall weir, or through a sluice-weir, with gates. Fig. 8 shows a vertical section of a breast wheel in which the former method has been adopted. If h_0 is the "head on the weir," or depth over the sill, while e is the width of the overfall (same as that of the wheel), and Q the volume of water used per second, we have (from p. 683, M. of E.)

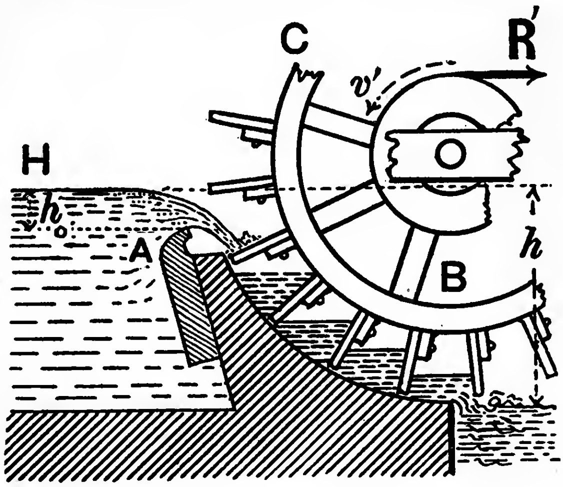

FIG. 8.

$$Q = \tfrac{2}{3}\mu e h_0 \sqrt{2gh_0}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

whence the required depth of the overfall (measured from the *surface of still water 3 or 4 feet back of the weir*) will be

$$h_0 = \left[\frac{\tfrac{3}{2}Q}{\mu e\sqrt{2g}}\right]^{\frac{2}{3}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The value of the coefficient μ varies from about 0.60 for a sharp-edged sill at upper edge of a vertical plate, to 0.80 or more for a rounded sill. In the wheel in Fig. 8 the sill, A, is adjustable, to suit different stages of water.

21. Power of Breast Wheels.—As in the case of the overshot, the work per unit of time is due partly to impact at entrance, but chiefly to the weight of the water after entrance. The whole fall h (see Fig. 8) may be divided into two parts, of which the upper, h_1, is the vertical distance from the surface of the head-water to the point of impact of the water on a float; while h_2 may denote the remaining lower portion. As

in the case of the overshot, let c_1 be the velocity $[c_1 = 0.95\sqrt{2gh_1}]$ of the water just before impinging on a bucket, at the "division circle," and v_1 the velocity of a point of the float in the "division circle" (see § 16). Also let $\delta Q\gamma$ denote the value of the effective weight (per second) of the water acting on the floats throughout the height h_2. The angle between c_1 and v_1 is α_1. We may therefore, by the same reasoning as in the case of the overshot, write the work done upon the wheel per second by the action of the water, both by impact and by weight,

$$L = \frac{Q\gamma}{g}\left[\frac{(c_1 \cos \alpha_1 - v_1)v_1}{g} + \delta h_2\right]. \quad . \quad . \quad . \quad . \quad (3)$$

As to the value of the ratio δ, Weisbach gives an example in which he makes application of his method of computation to a wheel where the distance between the apron and the edge of the floats is $\frac{1}{2}$ in. (§ 209, Vol. II), obtaining $\delta = 0.93$. In other cases where this distance is larger than $\frac{1}{2}$ in., as with wooden wheels, δ would be smaller, since the volume of water escaping between the apron and float-edges would be proportionally greater.

22. Modern Breast Wheels.—In Figs. 9 and 10 (on p. 33) are shown two varieties of breast wheel as manufactured by the firm of A. Wetzig, Wittenberg, Germany. That in Fig. 9 is constructed mainly of iron; the other of wood. The iron wheel is intended for flows of as much as 200 cub. ft. per sec. and for low heads of 2 to 6 ft.; while the wooden wheel is to be used for heads of 8 to 25 ft., with flows of 3 to 35 cub. ft. per second. A fuller description will be found in Engineering News of Nov. 27, 1902, p. 436. On the left in Fig. 9 is seen an "emergency gate," capable of falling quickly into position by the release of a rope.

In the first half of the nineteenth century breast wheels were very common in New England, but were gradually displaced by the turbine.

23. High Breast Wheels, or Back-pitch Wheels.—This name is given to wheels with buckets, like the overshot, but receiving the water between the level of the axle and the summit of the

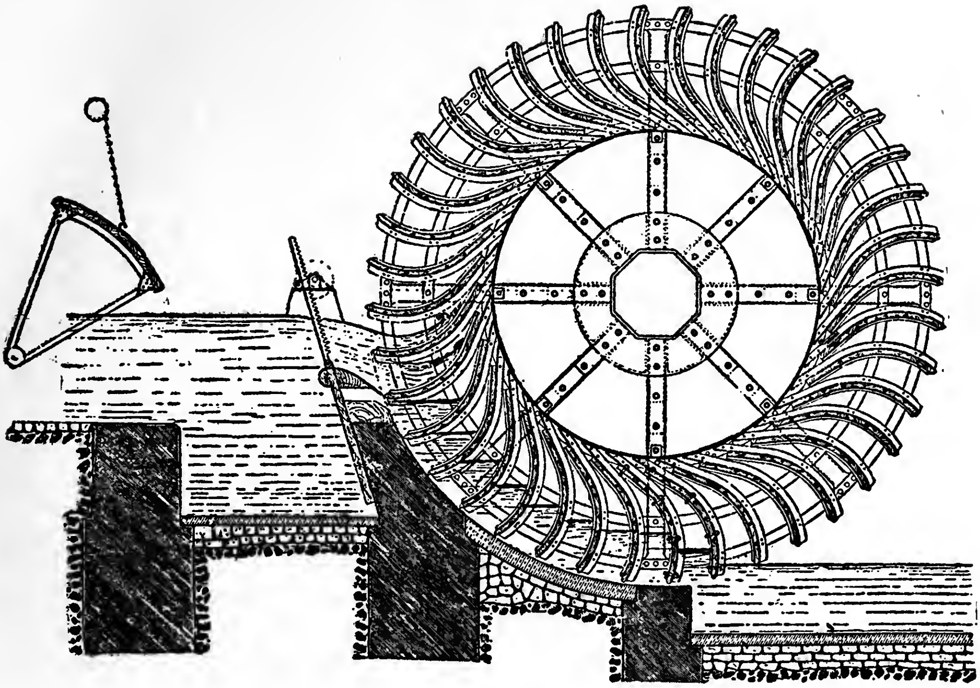

FIG. 9.

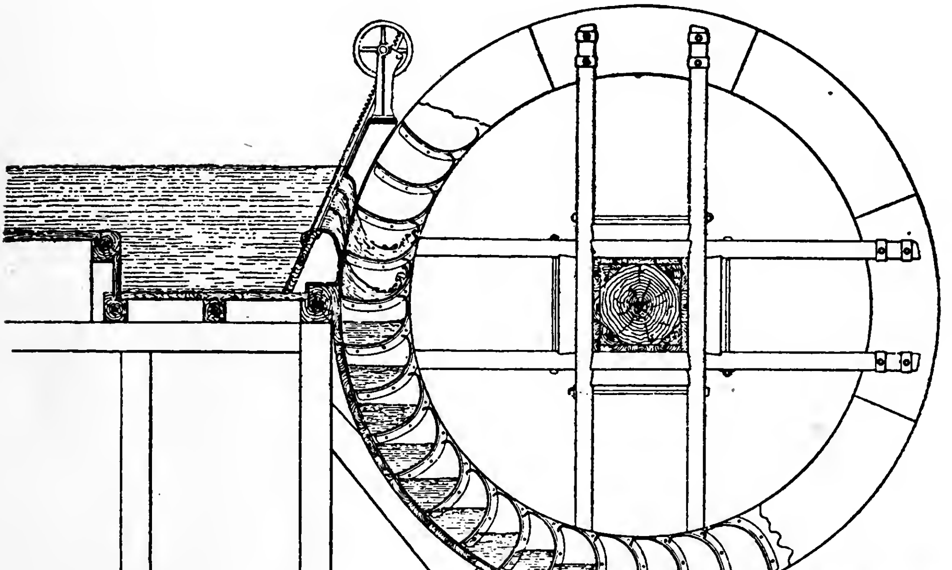

FIG. 10.

wheel. They are also set in a flume like ordinary breast wheels; and are peculiarly well adapted to situations where the surface-level of the head-water is liable to change, the gate being adjustable to different heads, and heights of orifice. To provide for the easy escape of air from the bucket as the water enters, "*ventilation*" is often resorted to, and is especially necessary in the case of these wheels. This was first proposed by Fairbairn, in one instance furnishing a saving of 3 per cent. of power, by actual experiment. See § 20.

24. **Efficiency of Breast Wheels.**—As a result of General

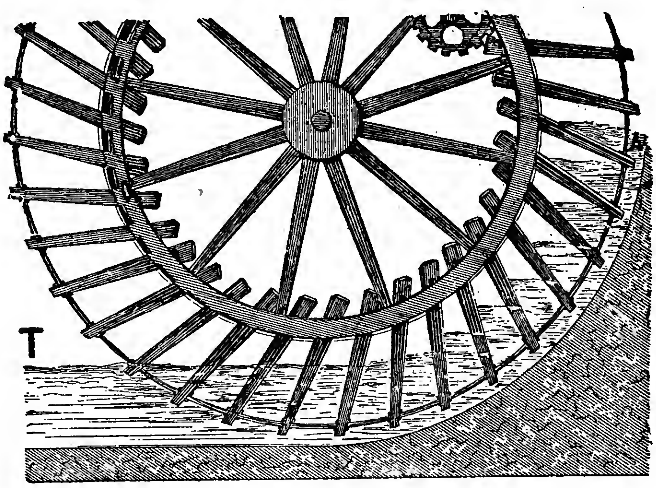

Fig. 11.

Morin's experiments with two breast wheels, both with well-fitting flumes, the efficiency, η, was found to be $=0.60$ and 0.70 respectively. In general η ranges from 0.65 to 0.75 for breast wheels; and for Wesserly wheels, as high breast wheels are sometimes called, from 0.65 to 0.72. Some overshots have been found to give efficiencies of 0.80 and above.

25. **The Sagebien Wheel.**—A peculiar variety of breast wheel, invented by Sagebien, is shown in Fig. 11. Its revolution must be very slow on account of the shape of the floats and their position. Hence much intermediate gearing is rendered necessary.

If the direction of motion of the Sagebien wheel is reversed, it requires power from without to drive it and becomes a *pump*,

the power spent upon it being used to raise water. Such a pump-wheel has been constructed and set up for purposes of irrigation on the Nile in Northern Egypt. (See the London "Engineer," Jan. 1886.)

26. Undershot Wheels.—These are almost entirely *inertia* motors, rejecting the water at about the same level as that at which it entered. The stream, with velocity $c=\sqrt{2gh}$ due to the head, issues horizontally from under a sluice-weir into the air and leaves the floats with about the same velocity v as that of the extremities of the floats, having impinged against them in its passage under the wheel. The floats are radial, or slightly curved from a radial position. Fig. 12 shows an ordinary construction. Evidently the whole power is due to *impact*.

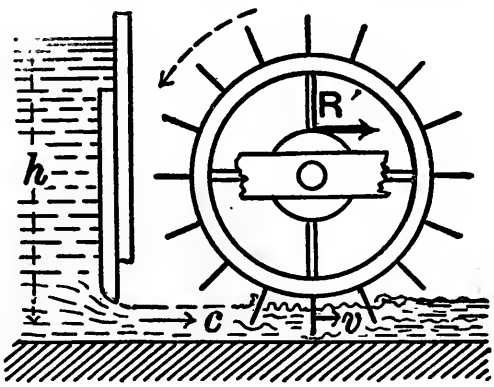

FIG 12.

27. Power of an Undershot.—Let Q' = the volume of water which *actually suffers impact* per unit of time. Let v = velocity of the middle of a float, and c that of the water before striking it. Then, as in § 17, we may write (remembering that α_1 is zero here) for the *total* power

$$L=\frac{Q'\gamma}{g}(c-v)v. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is a maximum for $v=\frac{1}{2}c$, but even then equals only half the initial kinetic energy of the water, i.e.,

$$L_{\text{max.}}=\frac{1}{2}\frac{Q'\gamma}{g}\cdot\frac{c^2}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Hence, even if $Q'=Q$ (the volume of water issuing from the sluice-opening per second), η for undershot wheels could never exceed 0.50. Roughly, Gerstner has computed from the experiments of the next paragraph that the power of a good

undershot is $L=0.61\frac{(c-v)v}{g}Q\gamma$, where Q is the volume of water passing the sluice-opening per time-unit; and therefore Q' must = about three fourths of Q.

As a best velocity for wheel circumference, Gerstner gives $v=0.4c$. In general it may be said that the efficiency of undershots ranges from 0.25 to 0.33 for the ordinary variety.

28. Experiments with Undershot Wheels, by Smeaton, Bossut, Morin, and others, have given somewhat varying results. Smeaton, with a small wheel 75 inches in circumference, found η no higher than 0.25, while Bossut, with slightly larger wheels, obtained a somewhat greater value. (See above.)

29. Current-wheels or Paddle-wheels. — These names are given to an undershot water-wheel, with comparatively few radial blades, hanging in an open current and supported on a pier; or, more advantageously, when the height of the water surface is variable, upon a floating dock or barge. They utilize less energy than the common variety of undershot just mentioned, not being enclosed in a flume and having fewer floats. Fig. 13 shows a simple construction.

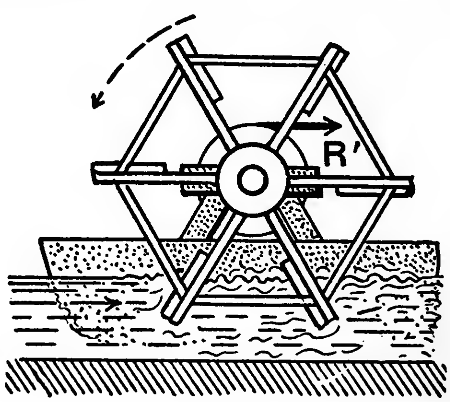

FIG. 13.

Current-wheels are in use for operating dredges on the river Rhine; and, in a crude form, have been constructed and used to some extent on the streams of the western part of the United States for irrigation and other purposes. For instance, one was constructed at Fayette Valley, Idaho, 28 ft. in diameter, and having 28 paddles, each 16 ft. long and 2.5 ft. wide. (See Engineering Record, Nov. 1904, p. 621.)

30. Poncelet Undershots. — In this peculiar and efficient wheel the floats are curved (see Fig. 14) and the crowns rather deep, the wheel being so designed, and run at such a speed, that the water enters without impact, mounts the curved side of

a float to a certain height and then descends, exerting a continuous pressure and losing its absolute velocity gradually; and leaving the float in a direction (relatively to the end of the float) opposite to that of entrance and at the same level. They are specially suitable for small falls, under 6 ft., utilizing about double the energy of an ordinary undershot. With greater falls they are excelled by breast wheels and are more difficult of construction. The wheel must fit the flume very accurately for the best results. Poncelet wheels have been built from 10 to 20 ft. in diameter with 32 to 48 floats of sheet iron or wood, iron being the better material.

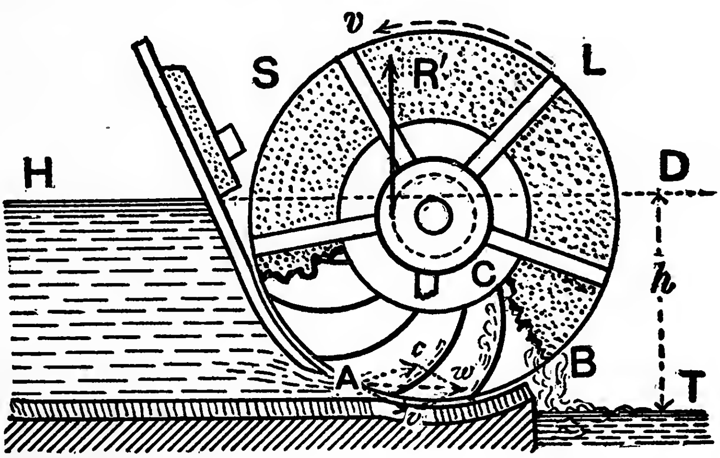

FIG. 14.

This variety of undershot owes its superiority in efficiency, when compared with the ordinary undershot, to the fact that the water is received upon the float at A without impact, and leaves the wheel at B with but little absolute velocity. At A the *absolute* velocity of the jet (that is, velocity relatively to the earth) is $w = \sqrt{2gh}$. The edge of the float has a velocity of $v =$ a little more than one-half c and in a different direction. If, therefore, a parallelogram of velocities be formed with w as diagonal and v as one side, the other side c is determined (see Fig. 14, at A) and the curve of the float should be made tangent to c, and not to w, since c is the velocity of the entering water relatively to the edge of the moving float (see p. 90, M. of E.), in order that the path of the water may suffer no sudden change of

direction. The water reaches a height C and then descends along the float, exerting continually a forward pressure against the float (the stream being open to the atmosphere on the other side). On reaching the exit-point, B, the relative velocity of the water is tangent to the lower extremity of the float-curve, as at entrance, and has about the same value c as at entrance, but is now directed *nearly backward* as regards the motion of the wheel. The result, therefore, of combining this relative velocity with the velocity v of the float-tip itself is to give an absolute velocity of exit w_n for the water which is small in value and nearly vertical in direction.

Since at both points A and B the water is under the same pressure (atmospheric) and these points are practically at the same level, the energy given up by the water per second is

$$\frac{Q\gamma}{g}\frac{w^2}{2}-\frac{Q\gamma}{g}\frac{w_n^2}{2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and since impact is largely avoided (by means already cited) a large portion of this power is transferred to the wheel, thus accounting for its superior performance. As high an efficiency as 68 per cent., with v regulated to a value of $v=0.58w$, has been obtained by test. On the whole, therefore, Poncelet wheels give about double the efficiency of ordinary undershots.

30a. Gearing of Overshots and High Breast Wheels.—In transmitting the power of these wheels, the axle may be largely relieved of the weight of the water in the buckets by so placing the pinion which gears with the cog-wheel or rack concentric with the axle of the wheel, that the tooth pressure between the two sets of teeth may be vertical and act in the vertical plane parallel to the axle and containing the center of gravity of the water in the buckets at any definite instant. This center of gravity may be found approximately by considering this quantity of water as forming a segment of a circular wire. (See p. 20, M. of E.)

CHAPTER III.

Preliminary Theorems, Fundamental to the Theory of Turbines and Centrifugal Pumps.

31. Remarks.—Lying at the basis of the theory of turbines and centrifugal pumps are the following theorems (A, B, and C), the presentation of which is necessary at this stage of the present work. The proofs of these theorems bring into play the fundamental principles of mechanics, and it must be particularly noted that without the existence of a "steady flow" Theorem C does not hold.

32. Theorem A.—Given a homogeneous mass *abcdefa* whose volume is V and center of gravity at C, Fig. 15; if a thin horizontal lamina AA' is removed from the upper part and placed so as to occupy the space BB' (also in the form of a horizontal lamina), then the center of gravity of the mass $a'b'c'd'defa'$

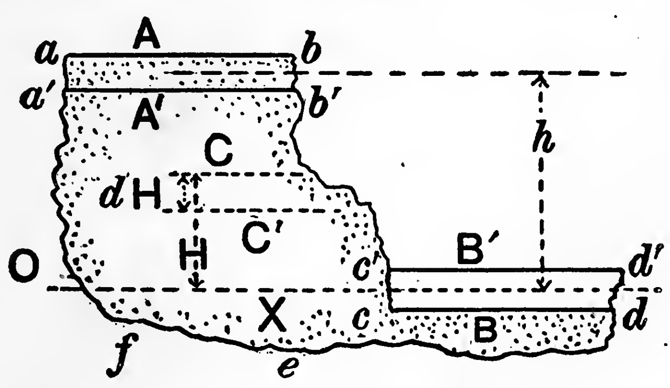

Fig. 15.

(whose volume is also equal to V) will occupy a position C' at some vertical distance dH lower than C. Let dV denote the volume of the horizontal lamina in question and h the

vertical distance between centers of gravity of AA' and BB'; then we are to prove that $V \,.\, dH = (dV) \,.\, h$.

Pass a horizontal reference plane OX through the center of gravity of lamina BB'. Let V' denote the volume of mass $a'b'cdefa'$ (mass common to both arrangements of the complete mass of volume V). Then from the properties of the "gravity" coordinates of a mass and of its component parts (see p. 19, M. of E.) for the original arrangement of the masses, denoting by H' the height of the center of gravity of V' above OX, we have

$$V \,.\, H = V' \,.\, H' + (dV) \,.\, h; \quad . \quad . \quad . \quad . \quad . \quad (1)$$

while in the second arrangement,

$$V(H - dH) = V'H' + (dV) \times \text{zero}. \quad . \quad . \quad . \quad . \quad (2)$$

Subtracting (2) from (1) we easily derive

$$V \,.\, dH = (dV) \,.\, h; \qquad \text{Q.E.D.} \quad . \quad . \quad . \quad . \quad (3)$$

33. Theorem B.—Let M be the mass of a small particle or "material point" (Fig. 16) which is describing a plane curve ABC under the action of one or more forces. Let AB be any element of the path, described in a time dt, while P is the resultant of all the forces acting on the particle at this point of its path. Denote the velocity of the particle in passing

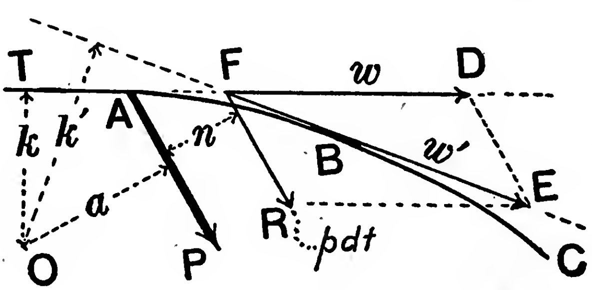

FIG. 16.

A by w, its velocity at B by w', (each being tangent to the curve at its proper point,) while p denotes the acceleration due to force P; this acceleration being (by Newton's law, p. 53, Mech. of Eng.) in the direction of the force. Of course, $p = P \div M$.

If this resultant force P were zero, the particle would keep along the straight line TD, tangent to curve at A and the velocity would remain constant, $=w$. But on account of the action of the force P we find that its path curves away from the tangent at A and that its velocity at B is of a different value, w'. If the velocity at A had been zero, and P had then acted, the particle would have moved in the line of P and its velocity at the end of dt seconds would have been $p \,.\, dt$. Hence, by the parallelogram of motions, it follows that the value of w' must be such as would be given by the diagonal of a parallelogram whose two sides are respectively equal (by scale), and parallel, to w and to $p \,.\, dt$. Hence note the intersection, F, of the two tangent lines (at A and B). A parallelogram whose side FD lies along the tangent drawn at A while the other side is FR parallel to P ($\overline{FD}$ being equal to w and $\overline{FR}$ to $p \,.\, dt$), must have for its diagonal FE, representing w' in amount and direction.

Now the parallelogram FE has the same geometrical properties as if it were a parallelogram of forces, that is, the "moment" of the resultant (diagonal) about any point is equal to the (algebraic) sum of those of its two components (sides) about the same point. Hence, if from any point, O, in the plane, perpendiculars are dropped upon the tangent line at A, the tangent line at B, and the line FR, the lengths of these perpendiculars being called k, k', and $a+n$ (n being the perpendicular distance of P at A from FR, while a is the length of the perpendicular dropped from O upon the line of the force P at A), we may write $w'k'=wk+p \,.\, dt(a+n)$, which may be written

$$w'k'-wk=p \,.\, dt(a+n), \quad \text{or} \quad \frac{1}{p}\cdot\frac{d(wk)}{dt}=a+n, \quad . \quad (4)$$

since $(w'k'-wk)$ is the increment, $d(wk)$, which the product (wk) receives as a consequence of the time t taking an increment dt. If now dt be made equal to zero, the distance n becomes zero, while $d(wk) \div dt$ is simply the "derivative" or first differential coefficient of the *product* (wk) with respect to the time t; so that, with $p=P \div M$, we may write

$$M \cdot \left[\frac{d(wk)}{dt}\right] = Pa.$$

That is, the moment (Pa) of the force about any point, = mass × the *time rate* of variation of the product wk about the same point.

(One of Kepler's laws for the planets may be proved by the aid of this relation.)

For subsequent use, this will be written in the form

$$M(w'k' - wk) = Pa \cdot dt. \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The quantity Mwk, or $Mw'k'$, is called *angular momentum*, and hence $M(w'k' - wk)$ may be called the *change* of angular momentum occurring in the small time dt.

34. **Theorem C.—Power of a turbine in steady motion = angular velocity × change of angular momentum experienced by the mass of water flowing per unit of time, in its passage through the turbine.**—A turbine channel is essentially one of a number of short curved pipes or passageways, forming a single rigid body (the turbine, or "runner"), their extremities lying in two circles concentric with the axis of rotation (vertical axis, here). This set of channels or "pipes" (see Fig. 19) revolves with *uniform* angular velocity (a sufficient resistance being offered to the wheel to prevent acceleration, so that the motion is "steady") and water is continually passing through them and is *under pressure;* the channels, therefore, being *always full.* The water passes into these channels from the mouths of other and *fixed* (stationary) passageways the walls of which are called *guides.* Fig. 18 shows an assemblage of guides which is supported in the interior of the ring (containing the wheel-passages) of Fig. 19. See also Fig. 56 on p. 112.

Each vertical partition (or "blade," "float," or "vane") between the wheel-channels experiences more pressure from the water on its concave than on its convex side, and the sum of the moments of all these excess forces, about the wheel axis, called $\Sigma(Pa)$, may be regarded as the moment of a single resultant couple representing the action of the moving water on the wheel, which couple maintains the uniform motion of the wheel against a proper resistance. Suppose each force

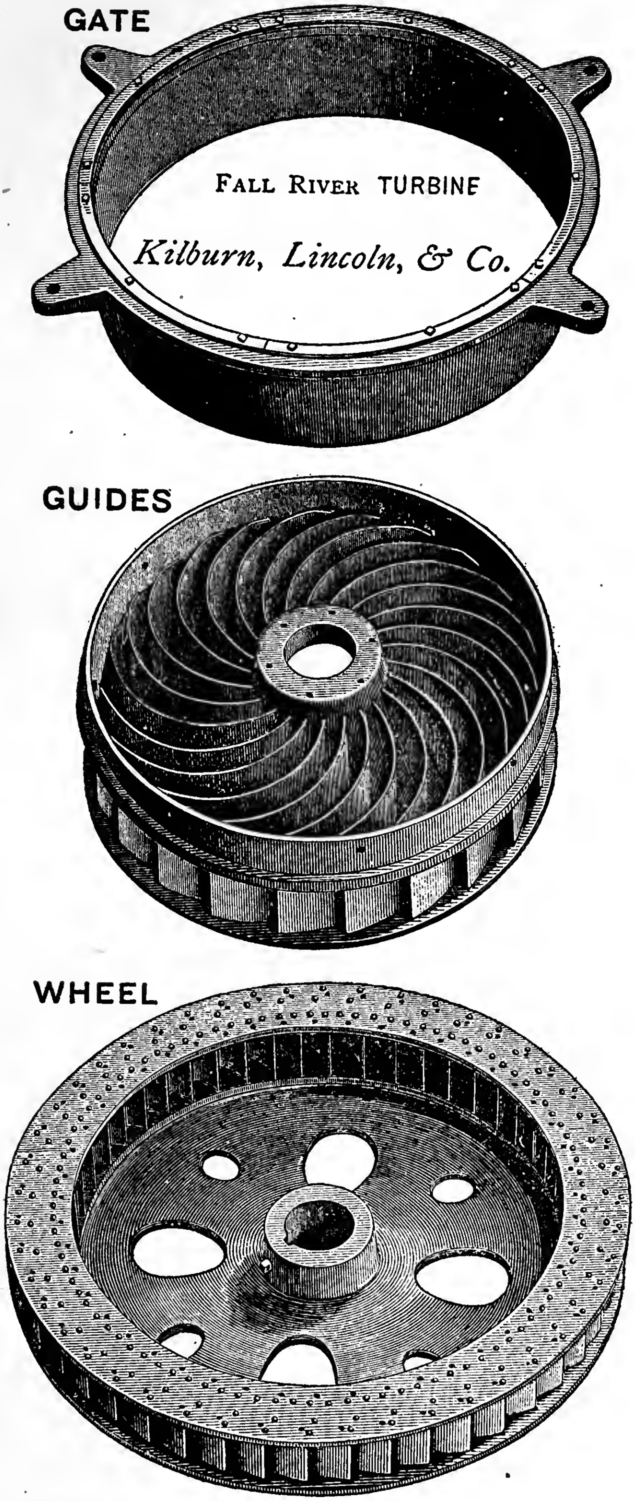

FIG. 17 (Gate,) 18 (Guides), and 19 (Wheel or Turbine).

of this couple to have a value P_0 lbs., with an arm of a_0 ft.; then $\Sigma(Pa) = P_0 \cdot a_0$. In a unit of time each of the two forces P_0 works through a distance $\omega a_0 \div 2$, where ω is the uniform angular velocity of the wheel. Hence the work per second, or power exerted, is $2P_0\omega a_0 \div 2 = \omega\Sigma(Pa)$ ft.-lbs per second, or L; that is,

$$L, = \textit{power of water on wheel,} = \omega P_0 a_0 = \omega\Sigma(Pa). \quad . \quad (6)$$

ft.-lbs. per sec.

Now conceive the water which at any instant lies in the turbine-passages to be subdivided into a great number of vertical rings, concentric with the wheel, of *equal volumes*, and of such small thickness that at the end of any small time, dt, each ring fills the exact space occupied by its (forward) neighbor at the beginning of the dt. (See Fig. 21.) The dotted line MN in Fig. 20 shows the absolute path (that is, the path relatively to the earth) and the initial and final velocities, w_1 and w_n, of a particle of water, as the ring to which it belongs passes *completely* through the wheel.

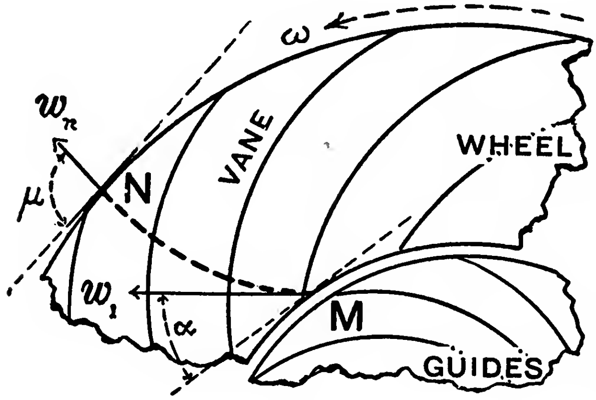

FIG. 20.

(The curvature of this path and the diminution of velocity are to be particularly noted.)

Fig. 21 shows the ideal division into rings (of water) for a segment of the turbine. In the small time dt in which any one ring passes (outwardly) into the next consecutive position, a portion, A (of the ring), included between any two neighboring partitions, or "vanes," passes into a position A' in the next ring-space, and in this new position, on account of the flow being "*steady*," has an absolute velocity w', equal to that, w'', which the portion B had at the beginning of the dt; while the length of the perpendicular, k', dropped on w' from the wheel

axis, is the same in value as for B at the beginning of the dt, since the positions A' and B are in the same ring-space. In other words, the "moment" of the absolute velocity for A' (i.e., the $w'k'$ of Fig. 16) about tne *axis of the wheel*, at the end of the dt, is the same in value as that for B at the beginning of the dt.

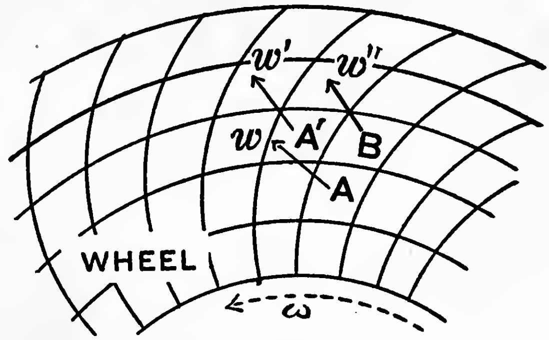

Fig. 21.

Now consider by itself (i.e., as a "free body") (see Fig. 22) the prism 1, at the entrance of any one of the wheel channels. P_1' and P_1'' are the pressures of the partitions against it; let P_1 represent their resultant; (it is, of course, the equal and opposite of the resultant pressure of the prism against the wheel at this instant.) Since the pressures of the neighboring prisms

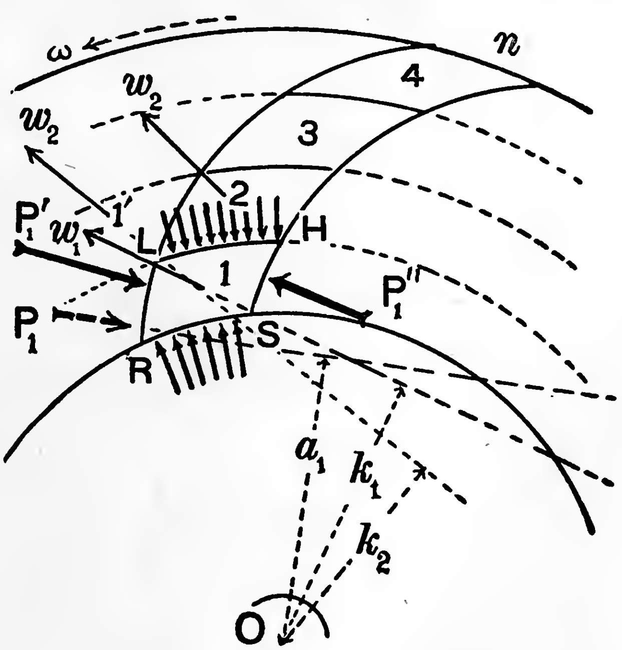

Fig. 22.

against 1 have lines of action containing O, the wheel axis, those pressures have zero moments about O, and the moment about O of P_1 (i.e., the moment P_1a_1) is therefore equal to

the moment about O of the resultant of P_1', P_1'', and the pressures on LH and RS. During the time dt, prism 1 moves to position 1′ in the next ring-space, w_1 changes to w_2, k_1 to k_2. Hence from eq. (5), with dM as mass of the elementary prism,

$$dM(w_1k_1 - w_2k_2) = P_1a_1dt.$$

Similarly for the other prisms in this channel between RS and n, as they, simultaneously with 1, in time dt, move into their consecutive positions, we may write (remembering that all the dM's are equal)

$$dM(w_2k_2 - w_3k_3) = P_2a_2dt; \quad dM(w_3k_3 - w_4k_4) = P_3a_3dt;$$

and so on, up to

$$dM(w_{n-1}k_{n-1} - w_n \, . \, k_n) = P_{n-1}a_{n-1}dt.$$

Adding these equations, member to member, we obtain

$$\Sigma(Pa) \text{ for one channel} = \frac{dM}{dt}(w_1k_1 - w_nk_n).$$

Hence, if the wheel has m channels,

$$\Sigma(Pa) \text{ for the whole wheel} = \frac{mdM}{dt}(w_1k_1 - w_nk_n).$$

Now $m \, . \, dM$ is the mass of water which leaves the wheel in time dt; hence if Q is the volume of water passing per unit of time, and γ is the weight of a unit volume of water, it follows that $mdM = (Q\gamma \div g) \, . \, dt$; therefore

$$\Sigma(Pa) \text{ for whole wheel} = \frac{Q\gamma}{g}(w_1k_1 - w_nk_n), \quad . \quad . \quad (7)$$

which is the moment of the couple to which the action of the water on the wheel, in this steady motion, is equivalent. *Hence the power of the wheel* at this speed [that is, the work per second done by this working couple] is, by eq. (6),

$$L = \omega\Sigma(Pa) = \omega\frac{Q\gamma}{g}(w_1k_1 - w_nk_n) \quad . \quad . \quad . \quad . \quad (8)$$

ft.-lbs. per sec.

The velocity w_n = the absolute velocity of the water at the exit-point of a channel (see Fig. 20).

The right-hand member of eq. (8) is seen to consist of the product of the (uniform) angular velocity, ω, by the difference between the quantities $\frac{Q\gamma}{g}w_1k_1$ and $\frac{Q\gamma}{g}w_nk_n$, or the change in the "*angular momentum*" of the mass, $\frac{Q\gamma}{g}$, of water flowing in a unit of time. (Q.E.D.)

Eq. (8) may be thrown into a more convenient form, thus, Fig. 23. By means of a rectangle, the velocity w_1 of the water at the entrance, M, of a wheel-channel can be resolved into two components, one, u_1, *tangent* to the inner circle of the wheel-ring of radius r_1, and the other along the radius drawn to that point, V_1.

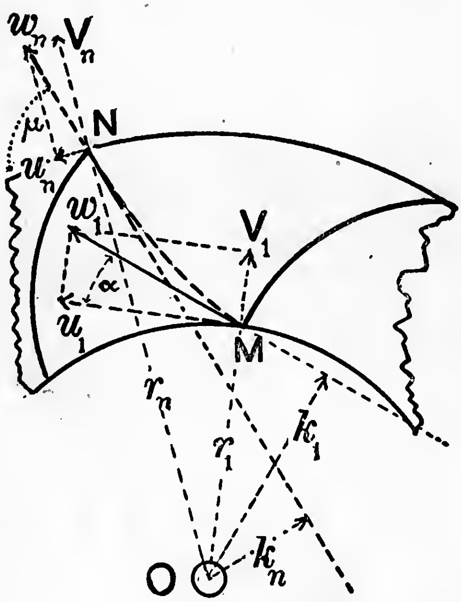

Fig. 23.

Similarly, at the exit-point, N, of a wheel-channel, the absolute velocity w_n can be decomposed into a *tangential* component u_n and a *radial* component V_n, at right angles to each other. If α and μ denote the angles that the absolute velocities make with their respective tangents (Fig. 23), we have

$$u_1 = w_1 \cos\alpha, \quad \text{and} \quad u_n = w_n \cos\mu.$$

Evidently, from the similar triangles involved, we have

$$k_1 : r_1 :: u_1 : w_1, \quad \text{and} \quad k_n : r_n :: u_n : w_n;$$

and hence eq. (8) may be written in the form

$$Power = L = \frac{\omega Q\gamma}{g}(u_1r_1 - u_nr_n). \quad . \quad . \quad . \quad . \quad (9)$$

Hence the moment of the couple would be

$$P_0a_0 = \Sigma(Pa) = \frac{L}{\omega} = \frac{Q\gamma}{g}[u_1r_1 - u_nr_n]. \quad . \quad . \quad . \quad (9a)$$

Again, since the product, angular velocity×radius, gives the linear velocity of the outer end of that radius, ωr_1 may be replaced by v_1, the velocity of the inner edge (or entrance-point) of the wheel-ring, and ωr_n by v_n, the velocity of the outer edge (or exit-point) of the wheel-ring; whence we may also write

$$\textit{Power of water on turbine} = L = \frac{Q\gamma}{g}(u_1 v_1 - u_n v_n) \quad . \quad (10)$$

ft.-lbs. per sec.

These tangential velocity-components of the water, u_1 and u_n, are sometimes called the *velocities of whirl* of the water; at entrance and exit, respectively.

This equation is remarkable in not involving the internal fluid pressures at entrance and exit.

35. Turbine Pump.—In the foregoing it has been supposed that the rigid body containing the set of rotating channels or pipes forms a "turbine," the action of the water on which is equivalent to a couple so directed as to tend to accelerate the rotary motion of the turbine; which acceleration is supposed to be prevented by application to the turbine of a system of resisting forces constituting a couple having a moment equal and opposite to that of the first, i.e., opposite in direction to the rotary motion of the rigid body. If the moment of this first equivalent couple is *negative* in any particular instance, it simply shows that the action of that couple tends to *retard* the motion of the rigid body, or set of channels; for the maintenance of whose uniform motion, therefore, the second, equilibrating, couple to be applied to the rigid body must have a moment coinciding in direction with that of the rotary motion of the rigid body itself, which in this case acts as a "centrifugal pump" (to be treated in a later chapter; see § 105).

36. Other Kinds of Turbines.—Although the turbine considered in the present discussion is for simplicity one in which the general course of the water is radially outward, in planes at right angles to the shaft of the turbine, the same kind of treatment may be applied whatever the nature of the turbine in question and corresponding path of the water (e.g., radial inward flow; radial and downward flow; or one in which the

path of a particle lies in a cylindrical surface parallel to the shaft; see later chapters). For any variety of turbine eq. (10) holds; v_1 and v_n being the respective velocities of the entrance- and exit-rims of the wheel, and u_1 and u_n the *projections*, upon the tangent lines of those rims, of the *absolute* velocities w_1 and w_n of the water, at entrance and exit respectively.

37. Numerical Example.—A turbine uses $Q=50$ cub. ft. of water per second in steady operation. The absolute velocity of the water at entrance is $w_1=50$ ft. per second at an angle of $\alpha=20°$ with the tangent to wheel-rim; and that at exit is $w_n=10$ ft. per second at an angle of $\mu=110°$. The speed of the wheel is 120 revs. per minute, the two radii being $r_1=1.5$ ft. and $r_n=2$ ft. Compute the power derived by wheel from the water under these conditions.

Solution.—From these data we have, for the "velocities of whirl,"

$$u_1=50 \cos 20°=50\times 0.940=47.0 \text{ ft. per sec.,}$$

and

$$u_n=10 \cos 110°=10\times(-0.342)=-3.42 \text{ ft. per sec.}$$

Since the angular velocity $=2\pi\frac{120}{60}=12.56$ radians per sec., $=\omega$, we have, for the velocity of the wheel-rim at entrance, $v_1=\omega r_1=18.84$ ft. per sec.; and, for that of outer wheel-rim, $v_n=\omega r_n=25.12$ ft. per sec. Hence the power derived is, from eq. (10),

$$L=\frac{50\times 62.5}{32.2}[47.0\times 18.84-(-3.42)(25.12)]=97.1[885.0+85.9]$$

$$=94{,}300 \text{ ft.-lbs. per sec.,} \quad \text{or} \quad 171.3 \text{ H.P.}$$

We also find that the moment of the couple to which the action of the water on the turbine is equivalent is

$$\Sigma(Pa), =L\div\omega=94{,}300\div 12.56=7510 \text{ ft.-lbs.}$$

38. Turbines. Fundamental Formula for Power.—In Fig. 24 is shown a vertical section (pulley in perspective, however) mainly diagrammatic, of a (radial outward-flow) turbine, with shaft vertical, in steady operation. A is the upper level or head-water, B the lower level or tail-water, the difference of

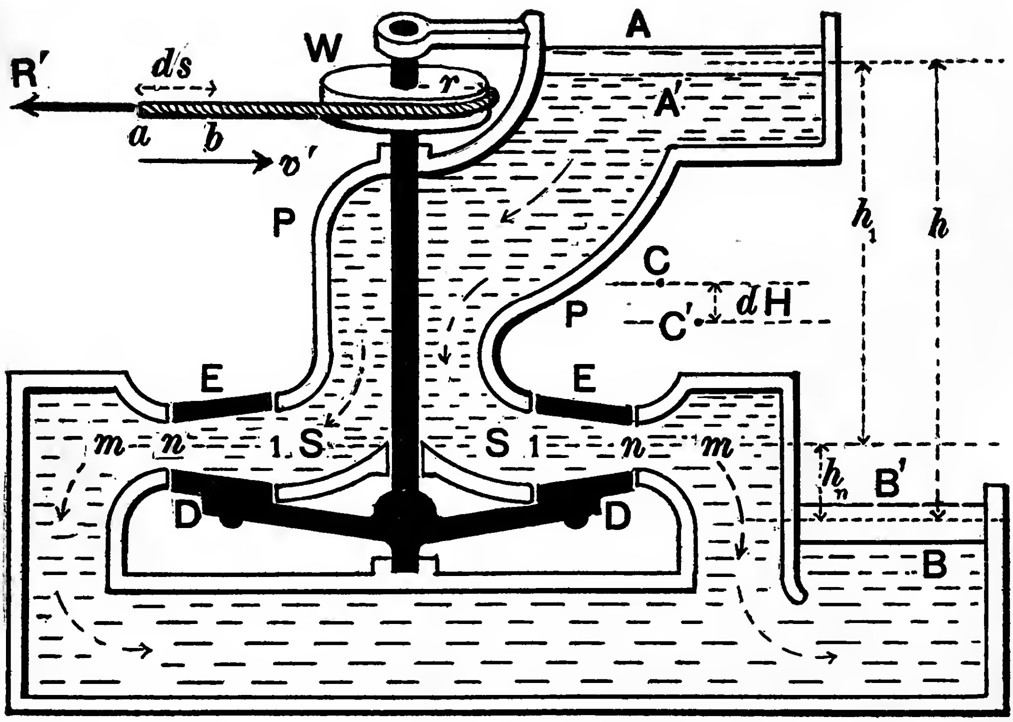

FIG. 24.

elevation, h, of their surfaces being the "head" of the mill site. The thick heavy lines indicate the turbine and its shafts, points 1 being at the entrance and n at the exit-point of a wheel-channel. In this case the heights of the wheel-channels at 1 and n are not the same. The guides are in the space S, S; the water being conducted to them through the rigid "pen-stock" $P P$.

Points n, where the water leaves the wheel, are in this figure (and quite often in practice) at a higher level than the surface B of the tail-water; the stationary tubes or vessel which the water enters on leaving the wheel at n being called the "draft-tube," or "suction-tube." The height h_n is rarely taken at more than 20 ft., in order that the water in the draft-tube may be under sufficient pressure to keep it full at the highest point n. In an ideal design for the best effect (but rarely met with)* the entrance of the draft-tube should be made with a very gradually enlarging section n to m, in order to reduce to a minimum the loss of head at this point in the progress of the water (more gradual than in this figure).

But little leakage is supposed to take place at 1 or n between the edges of the moving crown-plates (or shells) of the wheel (E, D) and the stationary edges of the guides, or of draft-tubes. As already shown in Fig. 19, the curved passage-ways or channels of the turbine lie in a ring, being separated from each other by vertical *curved* blades or vanes, and are closed in at top and bottom by the "crown-plates," or shells, E and D, which are rings, more or less flat, providing a floor and a roof for each passageway.

In this figure the power of the wheel is employed in winding up a cable on a drum W keyed upon the shaft of the wheel, the tension in the cable being R lbs. (Radius of drum $=r$.)

We shall now assume that the flow of water from A to B through the fixed pen-trough, moving wheel, and fixed draft-tube is *steady*, and that this flow takes place with full passage-ways (any air previously contained therein having been ex-

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* See Engineering News, Dec. 1903, p. 569.

pelled), the angular velocity of the wheel being *uniform*, its acceleration being prevented by a proper value of R', the tension in the cable.

With proper design of the wheel-passages, etc., the action of the water on the wheel is equivalent to a "couple" acting in a plane at right angles to the shaft and having a certain moment P_0a_0 ft.-lbs. The value of this moment depends on the speed at which the wheel is permitted to run. By "*steady motion*," then, both of water and wheel, we imply that the resistance provided (R' lbs.) is such that the moment $R'r=P_0a_0$, where P_0a_0 has the special value corresponding to the particular uniform speed of wheel; so that no acceleration takes place.

We also assume that the surfaces A and B are so large that the water in these surfaces has no appreciable velocity during the flow; and that all quantities concerned in the design are properly adjusted to each other to secure the most advantageous result for the permissible consumption of water (or rate of flow) Q cub. ft. per second. In other words, friction is reduced to a minimum. This latter is accomplished by such design (details given later) that all elbows, sudden enlargements of section, eddyings, etc., in the flow of the water, giving rise to fluid friction and consequent "loss of head" are avoided (as much as possible).

Such being the assumptions made for the wheel in Fig. 24, let us apply to it and the moving water the Principle of Work and (Kinetic) Energy (see p. 149, Mech. of Eng.). This holds good for any collection of *rigid* bodies moving among each other. The assemblage of rigid bodies to which we are now to apply it consists: *first*, of the wheel itself, with shaft and drum and the portion of cable shown in figure; the other rigid bodies being all the particles of water in the whole body of that liquid in the two ponds and all the internal spaces of the wheel, penstock, and draft-tube. (Water being practically incompressible, each particle of it is a "rigid body.")

The extent of motion that is to be considered is that taking place in a single element of time, dt, seconds. Since Q cub. ft.

per second is the rate of flow, the quantity flowing in a time dt will be $Q \,.\, dt$ cub. ft. In this short time, dt, surface A sinks to A', while surface B rises to B'; the volume of the horizontal lamina of water AA' being equal to that of the lamina BB', each being $=Q \,.\, dt$. The total atmospheric pressure acting on the surface A is an external force P_A, acting on our system of rigid bodies, and is a working force doing the work $P_A \times \overline{AA'}$, while that acting on B, P_B, is an external resistance, upon which is expended the work $P_B \times \overline{BB'}$. Now these products are equal and cancel each other in the summation of items of work (easily proved by the student).

In dt seconds the center of gravity C of the whole body of water (whose weight we denote by G lbs.) sinks through a small vertical distance dH. Hence the work done by this working force is $G \,.\, dH$ ft.-lbs; which, however, from § 32, eq. (3), can be replaced by $Q \,.\, dt\gamma h$. Also, in this time dt, the resistance R' in the cable is overcome a small distance ab, or ds, and the work done upon it is $R' \,.\, ds$. Disregarding friction for the present, we note that all the other external forces acting on the wheel and the water particles (that is, the pressures from the walls of penstock and draft-tube and the weight of the wheel itself) are "*neutral*" (that is, either they act at right angles to the path of the point of application or the point of application does not move at all); and that all the mutual pressures are normal to the rubbing surfaces and hence can be omitted from consideration. (See p. 149, Mech. of Eng.)

Next, as to the gain or loss of kinetic energy possessed by each of the rigid bodies of the collection considered, occurring between the beginning and the end of this small time dt; we proceed thus:

Conceive the whole body of flowing water to consist of a vast number of very small *groups* of particles, these groups containing equal masses of liquid and so situated that in dt seconds any group moves into the position just vacated by the adjacent group next ahead of it. (This means that the volume of each group is Qdt. The lamina at A is Group No. 1, while that at B is the last group of the series.) As a

consequence of the flow being "*steady*" it follows (by definition of steady flow) that the particles of any group acquire at the end of dt seconds a velocity equal to that which the particles of the next group ahead possessed at the beginning of the time dt. Now in the application of the principle of Work and Energy we are called upon to subtract the initial amount of Kinetic Energy of each mass from the final; but from the circumstances noted above it is seen that the initial kinetic energy of each group of particles is equal to the final kinetic energy of the group just behind it, so that when the subtractions indicated are all written out and added together a *total cancellation* or zero, is the net result of the aggregate change of kinetic energy of all the particles of water. As already postulated, the velocity of the group, or lamina, at A (and also at B) is insensible. Also, since the velocity of the rotating wheel is uniform, there is no change of kinetic energy on the part of that body, in the time dt.

Hence the net result of the whole operation of applying the method of Work and Energy to the rigid bodies mentioned, for the duration dt seconds, is simply

$$G \,.\, dH = R' \,.\, ds, \quad \ldots\ldots \quad (1)$$

i.e.,

$$Q\gamma dt \,.\, h = R' \,.\, ds. \quad \ldots\ldots \quad (2)$$

But (2) may be written

$$Q\gamma h = R \,.\, \frac{ds}{dt}; \quad \ldots\ldots \quad (3)$$

and again, since $ds \div dt$ is the uniform velocity of a point in the cable, or in the rim of the drum W, (call it v',)

$$Q\gamma h = R'v'. \quad \ldots\ldots \quad (4)$$

Now $R'v'$ is *lbs.*$\times$*ft. per sec.*, or *ft.-lbs. per sec.*, i.e., the *power* or rate at which work is expended on the resistance R'(lbs.); to the steady and continual overcoming of which the whole power $Q\gamma h$, due to the water supply Q and the head h, is applied. That is, in this ideal case of a water motor of perfect design and of perfect adjustment in operation, with no friction of any kind, the efficiency $= \dfrac{R'v'}{Q\gamma h} = 1.00$, or 100 per cent.; since

in general the efficiency = ratio of the part of the power that is usefully applied, to the whole theoretical power ($Q\gamma h$) of the mill-site.

39. Example.—If the full water-supply is $Q=48$ cub. ft. per second, and $h=100$ ft., we have $Q\gamma=48\times62.5=3000$ lbs. per second; so that $Q\gamma h=300{,}000$ ft.-lbs. per second is the full theoretical power of the mill-site (if 48 cub. ft. per second is the maximum available rate of supply). With a 100 per cent. motor to utilize this power we should have $R'v'=300{,}000$ ft.-lbs. per sec. The wheel being run at its best (most advantageous) speed of (say) 120 revs. per minute, while the radius of the drum is $r=1.5$ ft., the velocity of a point in the cable would be $v'=2\pi\times1.5\times120\div60=18.85$ ft. per second, and we have $R'\times18.85=300{,}000$; i.e., R' is 15,916 lbs., = the tension that can be overcome in the cable at the specified linear velocity of cable (18.85 ft. per sec.).

Even with the best designs, however, the useful power obtained would rarely be more than 85 per cent. of $Q\gamma h$ on account of fluid friction and the friction at the axle of the shaft; in such a case, therefore, we should have

$$R'v'=0.85\times300{,}000; \quad [=463 \text{ H.P.}],$$

and with v' the same as before we find that R' is only 13,528 lbs. (tension, or "load").

In case the power is taken off, not by a cable, but by a cog-wheel gearing with a pinion keyed on the shaft of the turbine, R' would represent the tangential component of the pressure between two engaging teeth, and v' the linear velocity of the pitch-circle of the pinion (or cog-wheel).

40. Turbine with Friction.—Referring again to Fig. 24, we note that if both fluid friction and axle friction are to be considered, the outcome will be as follows:

Let h' denote the "loss of head" that occurs between the surface A and the point 1 where the water enters the wheel; h'' the loss of head occurring in a wheel-channel (that is, between point 1 and point n); and again let h''' denote that occurring in the draft-tube (that is, between point n and the sur-

face B of the tail-water). If these three heads be deducted from the head h, the product $Q\gamma(h-h'-h''-h''')$ will express the power of the mill-site after fluid friction has been allowed for.

As to axle friction, if that be represented by R'' lbs., and the uniform velocity of the circumference of the axle is v'' ft. per sec. then the power lost in axle friction is $R''v''$ ft.-lbs. per sec.; and finally

$$Q\gamma(h-h'-h''-h''')=R'v'+R''v'', \quad . \quad . \quad . \qquad (4a)$$

as applicable to a turbine when friction is considered. If the wheel runs immersed in water, another term, R_0v_0, might be added on the right to represent the power lost in fluid friction on the wheel-casing (i.e., on the outside surface of wheel).

In fact, eq. (4a) might be written in the form

$$Q\gamma h=R'v'+R''v''+\Sigma(R'''v''');$$

(see § 9, eq. (3)) the detail of the term $\Sigma(R'''v''')$ being

$$\Sigma(R'''v''')=Q\gamma[h'+h''+h''']+R_0v_0.$$

41. Bernoulli's Theorem for a (Uniformly) Rotating Channel (Steady Flow of Water Therein).—See Fig. 24. Since the steady flow of water from A to point 1 occurs in a *stationary* (rigid) pipe or casing, we may apply Bernoulli's theorem for such flow, denoting by p_1 the internal fluid pressure at point 1, by b the height of the water barometer, by γ the heaviness of water, and by w_1 the *absolute* velocity of the water at 1; whence

$$\frac{p_1}{\gamma}+\frac{w_1^2}{2g}=b+h_1 \text{ (without friction)}. \quad . \quad . \quad . \qquad (5)$$

Considering friction, we have

$$\frac{p_1}{\gamma}+\frac{w_1^2}{2g}=b+h_1-h', \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5a)$$

where h' is the loss of head between A and point 1.

We also note that between points n and B the steady flow takes place in a *stationary* (rigid) pipe, to which if Bernoulli's Theorem is applied (first, without friction), we have

$$0+b=\frac{w_n^2}{2g}+\frac{p_n}{\gamma}+h_n; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

w_n being the *absolute* velocity of the water as it leaves the wheel at n, and p_n the internal fluid pressure at n; h_n is the height of n above tail-water surface at B.

Usually there is considerable loss of head between n and B, due to failure to make the change of section between n

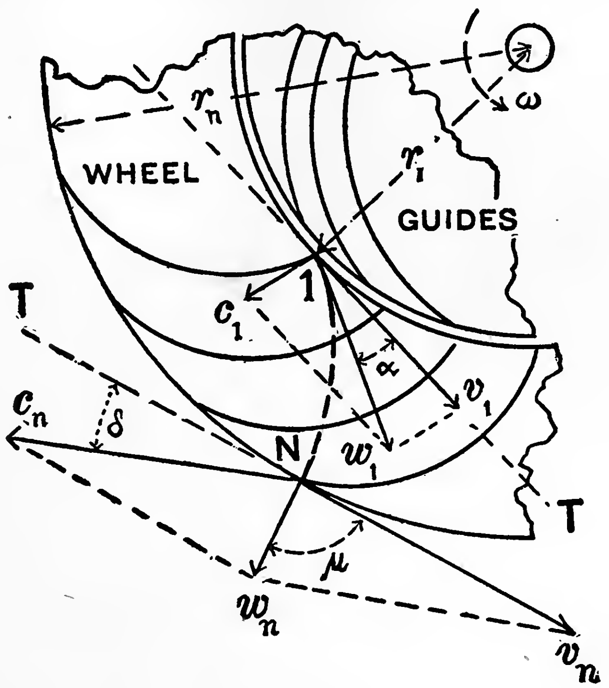

FIG. 25.

and m very gradual (Fig. 24). Calling this loss of head h''' [as before in eq. (4a)] and applying Bernoulli's Theorem *with friction*, (n to B,) we have

$$0+b=\frac{w_n^2}{2g}+\frac{p_n}{\gamma}+h_n-h''' . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

Now consider, in Fig. 25, the absolute path of a particle of water through the wheel; point 1 is entrance where the absolute velocity is w_1 and internal pressure p_1; while at exit, N, w_n is the absolute velocity and p_n is the internal fluid pressure.

Since the wheel-channel is in motion, w_1 is not the velocity of the water at 1 relatively to that point of the channel. This relative velocity must be found by drawing a parallelogram (see p. 89, Mech. of Eng.) of which the diagonal is made equal to w_1 and one side $= v_1$, the velocity of that point of the wheel (inner rim), the angle between these being called α.

The other side, c_1, being thus constructed, is the velocity of the water at 1 *relatively* to that point of the wheel ("relative velocity" c_1); and the tangent of the wheel-blade is made to coincide with this, c_1, in order that the water may follow the blade at once without having to make a sudden turn or elbow (at 1).

Similarly, at the exit, or point N, the absolute velocity w_n of the water particle is the diagonal of a parallelogram of which one side is v_n, the velocity of the outer rim of the wheel (which is $> v_1$ in ratio of the radii r_n and r_1), while the other side is c_n, the velocity of the water particle relatively to the point n of the wheel-channel ("relative velocity at exit"). Of course c_n is tangent to the extremity of the blade or vane at point N. Let δ and μ denote the angles marked in Fig. 25 at point N. Then, from trigonometry,

$$c_1^2 = w_1^2 + v_1^2 - 2v_1 w_1 \cos \alpha, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$c_n^2 = w_n^2 + v_n^2 - 2v_n w_n \cos \mu; \quad . \quad . \quad . \quad . \quad . \quad (8)$$

hence, by subtraction,

$$c_n^2 - c_1^2 = (w_n^2 - w_1^2) + (v_n^2 - v_1^2) - 2(v_n w_n \cos \mu - v_1 w_1 \cos \alpha). \quad (9)$$

Now from eqs. (5) and (6) we easily find, by subtraction,

$$\frac{p_1}{\gamma} - \frac{p_n}{\gamma} = \frac{w_n^2 - w_1^2}{2g} + h_1 + h_n, \quad . \quad . \quad . \quad . \quad (9')$$

in which $h_1 + h_n$ may be replaced by h. Between eqs. (9) and (9′) $w_n^2 - w_1^2$ is eliminated, whence, noting that $w_1 \cdot \cos \alpha = u_1$ and $w_n \cdot \cos \mu = u_n$ (Fig. 23), we have

$$\frac{c_n^2 - c_1^2}{2g} = \frac{p_1 - p_n}{\gamma} + \frac{v_n^2 - v_1^2}{2g} + \frac{u_1 v_1 - u_n v_n}{g} - h. \quad . \quad . \quad (10)$$

But from eq. (10) of § 34 the work done (per second) by the couple to which the water's action on wheel is equivalent is

$$L=\frac{Q\gamma}{g}(u_1v_1-u_nv_n), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

which in this case (without friction) must $=R'v'$ (see Fig. 24). We also have $Q\gamma h=R'v'$ (see eq. (4) of § 38); whence it follows that

$$h=\left[\frac{(u_1v_1-u_nv_n)}{g}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This being substituted in eq. (10) there results

$$\frac{c_n^2}{2g}+\frac{p_n}{\gamma}=\frac{c_1^2}{2g}+\frac{p_1}{\gamma}+\frac{(v_n^2-v_1^2)}{2g}, \quad . \quad . \quad . \quad . \quad (13)$$

which is known as *Bernoulli's theorem* (without friction) *for steady flow of water in a (uniformly) rotating casing* (rotating around a vertical axis). It is noticeable that it does not contain the absolute velocities of the water at entrance and exit of a channel, but only the *relative* velocities of the water, the fluid pressures, and the velocities v_1 and v_n of the two wheel-rims themselves. The term $\left(\frac{v_n^2-v_1^2}{2g}\right)$ is sometimes called the *centrifugal head.*

42. Bernoulli's Theorem (Rotating Casing) when Friction is Considered.—If eqs. (5*a*) and (6*a*) be combined we find

$$\frac{p_1}{\gamma}-\frac{p_n}{\gamma}=\frac{w_n^2-w_1^2}{2g}+h_1+h_n-h'-h''', \quad . \quad . \quad . \quad (9'a)$$

in which h_1+h_n may be replaced by h.

This may now be combined with (9) to eliminate $(w_n^2-w_1^2)$, remembering that $w_1 \cos\alpha=u_1$ and $w_n \cos\mu=u_n$, whence we have

$$\frac{c_n^2-c_1^2}{2g}=\frac{p_1}{\gamma}-\frac{p_n}{\gamma}+\frac{v_n^2-v_1^2}{2g}+\frac{u_1v_1-u_nv_n}{g}-h+h'+h'''. \quad (10a)$$

Now in the derivation of eq. (10), § 34, for the power due to the action of the "equivalent couple" of water on wheel, the forces dealt with, of water prisms on the wheel-blades, were the *actual* forces, including frictional components, if any. Hence that equation will *still stand* as to its form, now that we are considering friction. The equation is

$$L = \frac{Q\gamma}{g}[u_1 v_1 - u_n v_n]. \quad . \quad . \quad . \quad . \quad . \quad (11a)$$

We also have eq. (4*a*) of § 40, viz.,

$$Q\gamma(h - h' - h'' - h''') = R'v' + R''v'',$$

for the case where friction is considered, and note that the power given by eq. (11*a*) above is expended on R' and R'', that is,

$$R'v' + R''v'' = \frac{Q\gamma}{g}(u_1 v_1 - u_n v_n);$$

hence

$$h = h' + h'' + h''' + ([u_1 v_1 - u_n v_n] \div g); \quad . \quad . \quad . \quad (12a)$$

and this value of h, placed in eq. (10*a*) above, gives

$$\frac{c_n^2}{2g} + \frac{p_n}{\gamma} = \frac{c_1^2}{2g} + \frac{p_1}{\gamma} + \frac{v_n^2 - v_1^2}{2g} - h'', \quad . \quad . \quad . \quad (13a)$$

which is **Bernoulli's theorem for steady flow in a rotating casing when friction is considered.** It is seen that the quantity h'' is what was called the "loss of head occurring in the wheel-channel," so that (13*a*) differs from (13) only in the introduction of this loss of head.

Here we note again that the absolute velocities of the water, at points 1 and n, entrance and exit of a wheel-channel, do not appear in this theorem, but simply the *relative* velocities, the "pressure-heads," the "centrifugal head," and the loss of head h''.

42a. Turbine Pump.—If it is required in Fig. 24 that the direction of the flow of water be reversed; that is, that a steady flow of water is to be maintained from the lower level B to the upper level A, with steady operation of a properly designed rotating "pump" (as it now becomes), or reversed turbine, having properly curved channels, but with inlet 1 communicating with B and outlet n with A*; it is evident that all the relations of the previous paragraph still hold good with these differences: Instead of a resisting force R' we must have a working force P, and the cable must unwind from the drum instead of being wound up. The working force will have to be furnished by some external source of power, and if the velocity of the cable be now

called v, we have for the case where no loss of head occurs in any part between B and A (and no axle friction of pump)

$$Pv = Q\gamma h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

whereas, if losses of head, etc., occur (using same notation as in §§ 38 to 42),

$$Pv = Q\gamma[h + h' + h'' + h'''] + R''v'', \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

instead of eq. (4a).

Also, Bernoulli's Theorem for steady flow in a rotating pipe revolving uniformly in a horizontal plane remains the same as (13a), viz.,

$$\frac{c_n^2}{2g} + \frac{p_n}{\gamma} = \frac{c_1^2}{2g} + \frac{p_1}{\gamma} + \frac{v_n^2 - v_1^2}{2g} - h'', \quad . \quad . \quad . \quad . \qquad (16)$$

provided the flow (relative) is still from 1 to n.

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* Or, Fig. 24 may be conceived to be changed in this respect: that B is still the receiving-tank, and A the source of supply, but that B is at a *higher elevation than A*.

CHAPTER IV.

IMPULSE WHEELS.

43. Definition of Impulse Wheels.—Water-wheels furnished around the rim with small buckets, or cups, or curved vanes closed in on the sides, and receiving the action of a "free jet" of water directed tangentially to the rim or nearly so, are called "Impulse Wheels"; sometimes "tangential wheels". By a "free jet" is meant one which is formed "free" in the atmosphere, flowing from the extremity of a nozzle, often of a converging conical shape, though sometimes of rectangular cross-section.

We shall first consider that the greatest radial width of each cup or bucket is small compared with the radius of the rim of the wheel, in which case the movement of a cup may be considered to be one of translation.

44. Pressure of Free Jet upon a Fixed Solid of Revolution, when the Axis of the Solid is Coincident with the Axis of the Jet.—See Fig. 26. Here the jet is deviated smoothly and sym-

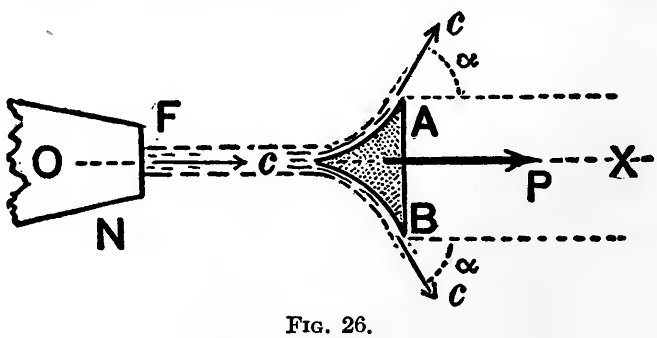


FIG. 26.

metrically on all sides of the axis, OX, of the jet; OX is also the axis of the *fixed* solid AB. The filaments of the jet meet

Used where supply is not very great and the head is large.

the surface of the solid tangentially, the center of the latter being pointed, as shown. The particles of the water, if we neglect friction, have the same velocity on leaving the outer edge of the solid at A, or B, as they had on leaving the tip of the nozzle at F, viz., c ft. per second; but the directions of their motion at A, being tangent to the solid, make an angle α with the original direction OX. This angle is evidently the same in value for all particles as they pass off the solid at A, the edge of the circle whose diameter is AB and whose plane is perpendicular to OX.

The resultant pressure, P lbs., of the jet against the solid during this steady flow, is found by considering that in a small time Δt seconds a small mass Δm of water has had its velocity in the direction of OX diminished from a value of c, to $c \cos \alpha$, ft. per sec. Hence a force equal and opposite to the force P has occasioned a negative acceleration $p = \dfrac{c - c \cos \alpha}{\Delta t}$ in the component of velocity parallel to OX, of the mass Δm.

$$\therefore \; P, = mass \times accel., = \Delta m \left[\frac{c - c \,.\, \cos \alpha}{\Delta t} \right]. \quad . \quad . \quad (1)$$

Now if Q is the volume, per second, of water issuing from the nozzle (being *also* the volume per second acting on the solid; since the latter is held at rest), we have for the mass passing over it in Δt seconds $\Delta M = \left(\dfrac{Q\gamma}{g} \right) \Delta t$, and therefore may write

$$P = \frac{Q\gamma c}{g}(1 - \cos \alpha). \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

1 in. in diam

For example, with a jet of 1 sq. inch sectional area having a velocity of 40 ft. per sec., the angle α being 45°, we have

$$Q = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 \times 40 = 0.218 \text{ cub. ft. per second};$$

$$\therefore \; P = \frac{0.218 \times 62.5 \times 40}{32.2}[1 - 0.707] = 4.96 \text{ lbs.}$$

45. Pressure of Free Jet on Solid of Revolution when the Latter is in Motion away from the Jet.—As before, the solid is one of revolution with its axis coinciding with that of the jet and its motion is assumed to have a uniform velocity v (less than that, c, of the water in the jet) and to be directed along the axis OX. (See Fig. 27.)

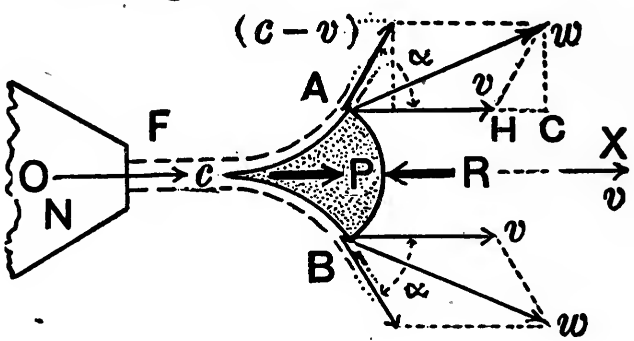


Fig. 27.

Here, for a given volume per second Q' passing over the solid the resultant pressure P of the water against the solid will be, of course, less than before, for the same angle α; but since the solid is now in motion P is a working force, for it; and to prevent acceleration of the motion of the solid a resistance R lbs. equal to P and in same line, but oppositely directed, is supposed to be furnished. It is required to find the value of P for a given v and c and sectional area, F sq. ft., of the jet.

The velocity of a particle of water is c just before encountering the point of the solid. On leaving the further edge of the solid, as at A, its velocity relatively to the solid (since friction is neglected and A is not appreciably higher or lower than the nozzle) is the same as before, viz., $c-v$; but the direction of this relative velocity is at an angle α with the former direction $O..X$, and the point A has itself a velocity v parallel to $O..X$. Hence the absolute velocity (i.e., relatively to the earth) is $w=(\overline{Aw}$ in figure), the diagonal of the parallelogram formed on the relative velocity, $c-v$, and $\overline{AH}, =v$, as sides. Hence the loss of velocity of the particle in the direction $O...X$ is equal to c diminished by $\overline{AC}$, the projection of w on OX. But evidently this projection, or velocity-component, $\overline{AC}$ is made up of v and $\overline{HC}$, $\overline{HC}$ being equal to $(c-v)\cos\alpha$.

Therefore the loss of velocity, in direction $O \ldots X$, of the small mass ΔM passing over the solid in the time Δt is

$$c-[v+(c-v)\cos\alpha], \quad \text{or} \quad (c-v)(1-\cos\alpha).$$

As before, $P = \text{mass} \times \text{accel.} = \Delta M \, . \, \dfrac{(c-v)(1-\cos\alpha)}{\Delta t}$.

But the value of ΔM is $\dfrac{Q'\gamma}{g}\Delta t$, where Q' is the volume of water passing per second *over the solid* (and not that, Q, issuing from the nozzle). Hence

$$P = \frac{Q'\gamma(c-v)(1-\cos\alpha)}{g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the work done by this working force on the solid every second is

$$Pv, \text{ or } L', = \frac{Q'\gamma(c-v)(1-\cos\alpha)v}{g} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

ft.-lbs. per sec., and is expended in overcoming the resistance R through v ft. each second; that is, $Pv = Rv$. Evidently if α is made greater than 90°, the solid of revolution becomes a cup, concave to the jet.

46. Impulse Wheels.—It is to be specially noted that in eq. (4) Q' denotes the volume (say cub. ft.) passing per second over the solid of revolution, so that $Q' = F(c-v)$; and not $Fc, = Q$, which is the volume per sec. issuing from the nozzle. But if a motor be constructed consisting of a series of such solids of revolution, or cups, or of equivalent curved vanes, coming into position successively and endlessly, which would be the case if they were placed on the rim of a wheel of large radius, more work per second could be done and in proportion to the water used; since more than one solid or cup would be in action at certain times, the portion of jet intercepted between two consecutive cups being able to finish its action on the cup in front, while new work is being done on the adjacent hinder cup. With this arrangement, therefore, all the water issuing from the nozzle would be used, and for Q' we may substitute Q and thus obtain for the power of a motor provided with such a *series* of cups the expression

$$L = \frac{Q\gamma(c-v)[1-\cos\alpha]v}{g} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

ft.-lbs. per sec. The corresponding average working force is

$$P = \frac{Q\gamma(c-v)(1-\cos\alpha)}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

lbs. It is evident from (5) that for a given water-supply, Q cub. ft. per second, the value of the power L depends on both v and the angle α, becoming zero both for v = zero (stationary cup) and for $v=c$ (in which case the jet does not overtake the cup). It is also zero for $\alpha=0°$.

For any constant v, L is evidently a maximum for $\alpha=180°$, i.e., for $\cos\alpha=-1$; and then takes the form

$$L = \frac{2Q\gamma(c-v)v}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

This value of the angle α may be attained by giving to the solid of revolution the form of a ring-shaped cavity the tangents to whose outer rims are parallel to the jet so that both the relative veolcity $c-v$, and absolute velocity w, of the water leaving the solid (or cup, as it may now be called) are parallel to the original jet. The pointed center of the cup provides for a gradual deviation of the water from its original path and prevents eddying and consequent internal fluid friction. (See Fig. 28.) When such a series of cups is fastened on the edge of a large wheel, however, the center of each cup does not remain accurately in the axis of the jet when under its action, since this center is moving in the arc of a circle. For the point, therefore, a sharp ridge is substituted whose edge lies in the plane of rotation, thus providing for a gradual deviation of the water at all times during the action of the jet on any given cup. This dividing ridge separates the cup or bucket into two lobes, thus giving rise to the general form adopted

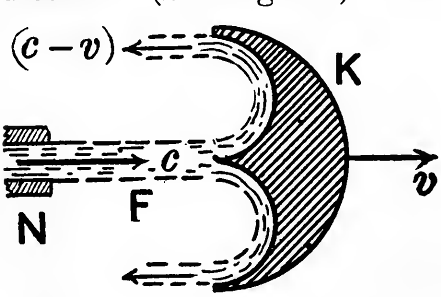


Fig. 28.

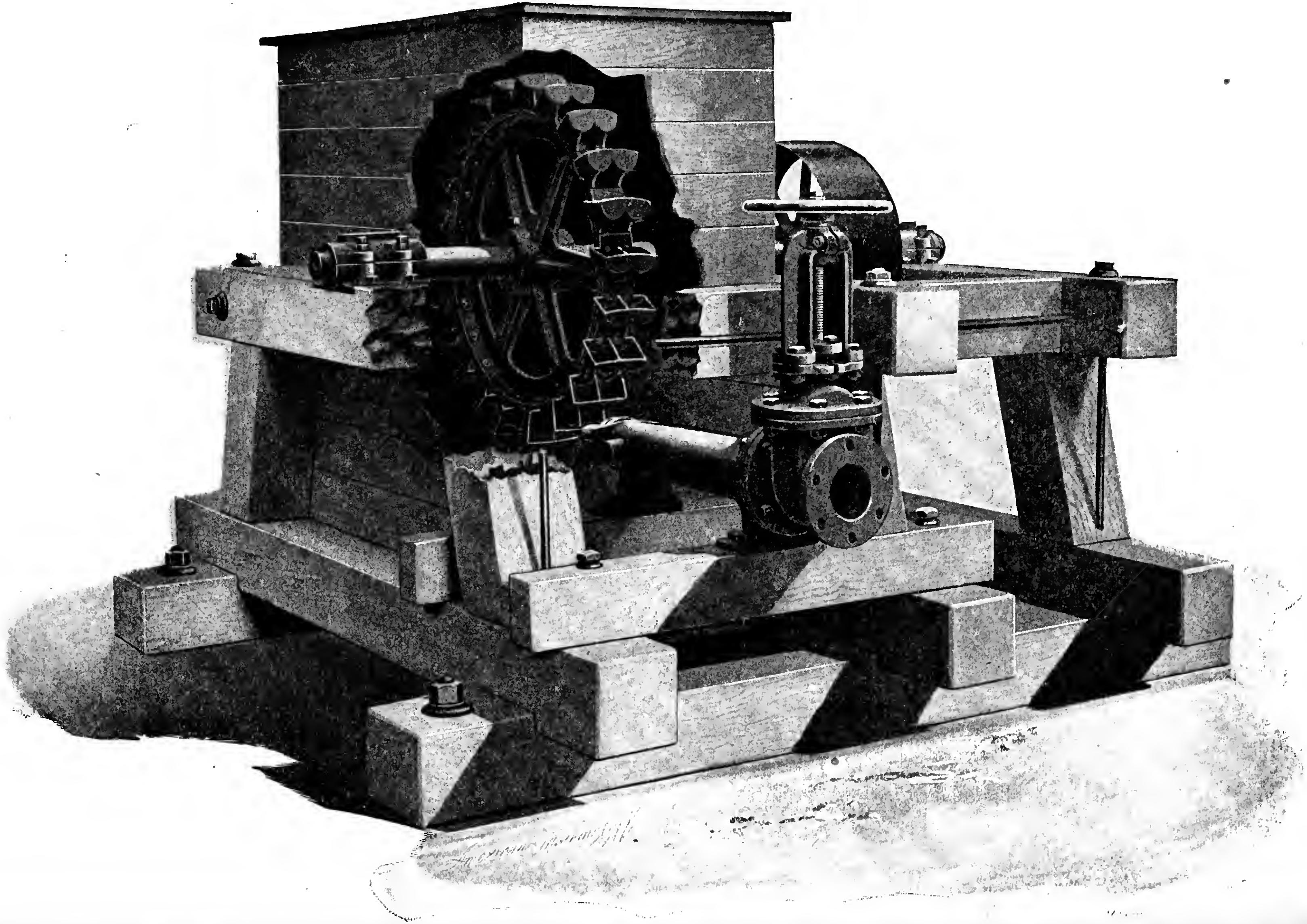


FIG. 30. Pelton Impulse Wheel.

in the Pelton and Doble impulse wheels, as shown in Figs. 30 and 31 (opposite pp. 67 and 69).

Fig. 29 represents a simple impulse wheel of this kind. A resistance, R' lbs., is shown, acting tangent to the edge of a pulley of radius r' keyed upon the shaft of the water-wheel. Without such resistance, of course, the wheel would "speed up" until the velocity of the cups reached a value equal to that, c, of the water in the jet. The working force would then disappear, and while a display of high speed might be made, no power would be obtained, the jet passing on as if the wheel were not present.

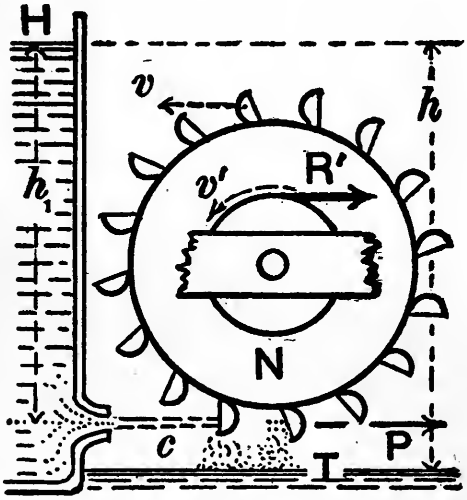


FIG. 29.

It still remains to determine the special value of v, the cup velocity, for which the power, L, is a maximum. Since from eq. (5a)

$$L = (\text{a constant}) \times (c-v)v, \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

we find that by putting $\frac{dL}{dv}$ equal to zero, i.e., $c-2v=0$, a value of $v=\frac{c}{2}$ gives a maximum L. The substitution of this special value of v for the v of eq. (6) results in the following expression for the maximum power, viz.,

$$L_{\text{max.}} = \frac{2Q\gamma}{g}\left(c-\frac{c}{2}\right)\frac{c}{2} = \frac{Q\gamma}{g}\cdot\frac{c^2}{2}. \quad . \quad . \quad . \quad . \quad (7)$$

47. Efficiency of the Impulse Wheel.—It is to be noted that in eq. (7) $Q\gamma \div g$ is the *mass* of water flowing per second from the nozzle and $c=\sqrt{2gh}$ if there is no friction at edges of the nozzle (and h_1 be considered equal to h, Fig. 29), so that $L=Q\gamma h$, theoretically; from which it is seen that the efficiency of this wheel, run at proper speed, should be 100 per cent. if it were of perfect

construction and if all friction could be avoided. This is as it should be, since the absolute velocity of the water as it leaves the outer rim of a bucket [this velocity having in general a value = Velocity of cup − relative velocity of water at rim, i.e., $= v-(c-v)$, or $2v-c$] would in this case be $2\times\frac{c}{2}-c;\ =$ *zero*. In other words, the water possesses no kinetic energy on leaving the cup. At the beginning of its path on the cup it has kinetic energy, but no potential nor pressure energy (i.e., none above that due to atmospheric pressure), and at exit none of any kind. It has given up its whole stock of energy.

On account of imperfect guidance of the water by the walls of the bucket and the friction of the water on itself and on the surfaces of the bucket, (aside from the fact that the value of the angle α cannot be made exactly 180°,) the efficiency of these wheels is brought down in practice to values ranging from 70 to 90 per cent., according to circumstances. (See test quoted on pp. 809 and 810, M. of E.)

In Fig. 29 the head h_1 is the depth of still water just behind the center of the nozzle; but in practice a conical nozzle is generally employed, attached to a pipe or other source of steady supply, and the efficiency of the wheel is generally referred to the total head (above atmosphere) at the base of the nozzle where the pressure is (say) p_1 lbs. per square inch (above the atmosphere) and the velocity (much less than that of the jet, since the sectional area F_1 is much larger than that of the jet) is c_1.

c_1

In such a case we may write, therefore, in place of h_1

$$\frac{p_1}{\gamma}+\frac{v_1^2}{2g};\ \text{and hence } c=.95\sqrt{2g\left[\frac{p_1}{\gamma}+\frac{c_1^2}{2g}\right]}$$

if $\phi,\ =0.95$, is the coefficient of the nozzle. If the efficiency is 80 per cent. (for instance), we have for the useful power (neglecting axle friction)

$$R'v'=\frac{(0.80)Q\gamma}{g}\cdot\frac{c^2}{2(0.95)^2}=(0.80)Q\gamma\left[\frac{p_1}{\gamma}+\frac{c_1^2}{2g}\right]. \quad . \quad (8)$$

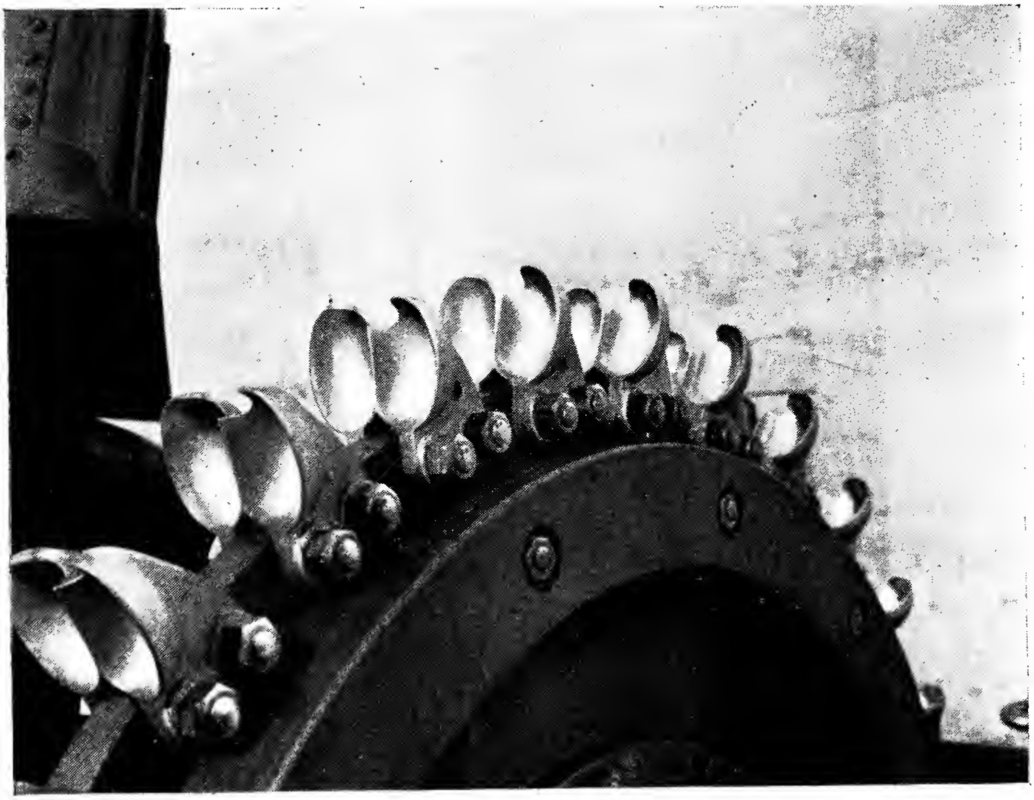


FIG. 31. Doble Impulse Wheel.

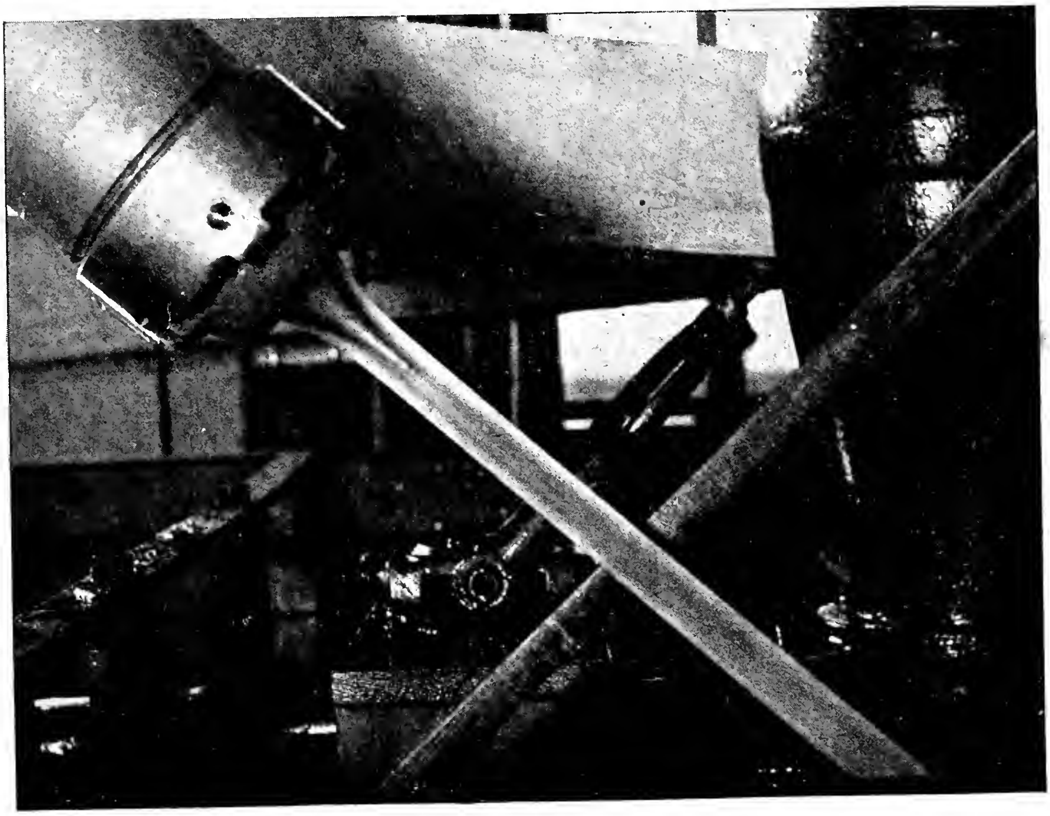


FIG. 35. Jet from Doble Nozzle.

1 ft. head corresponds to 0.434 lbs per sq in.
1 lb per sq in " " 2.304 feet head

48. Numerical Example, Impulse Wheel.—Compute what resistance, R', can be continuously overcome, tangent to a pulley of a radius $r'=2$ ft. keyed upon the shaft of a Pelton wheel, whose cup centers form a circle of $r=3$ ft. radius; if the available water-supply is $Q=1.50$ cub. ft. per sec., issuing in a "free jet" from a conical nozzle at whose base the fluid pressure is measured and found to be $p_1=130.2$ lbs. per sq. in. (above the atmosphere). The diameter of the base of the nozzle is $d_1=4$ inches, and that of the part of jet where *its filaments have become parallel* is $d=1.44$ in. The wheel is to be run at best speed and an efficiency of 80 per cent. is counted on.

Solution.—$c_1=Q\div\dfrac{\pi d_1^2}{4}=17.2$ ft. per sec., and hence $\dfrac{c_1^2}{2g}=4.6$ ft. Also $c=Q\div\dfrac{\pi d^2}{4}=133$ ft. per second; while $\dfrac{p_1}{\gamma},=\dfrac{130.2\times 144}{62.5}$, $=300$ ft. We may therefore compute the coeff. ϕ from $c=\phi\sqrt{2g\left[\dfrac{p_1}{\gamma}+\dfrac{c_1^2}{2g}\right]}$, obtaining $\phi=0.95$, the "coefficient of the nozzle." Substitution in eq. (8) now results as follows:

$$R'v'=0.80\times 1.5\times 62.5[300+4.6]=22{,}845 \text{ ft.-lbs. per sec.};$$

or 41.5 horse-power.

The proper speed of the cups or buckets should be $v=c\div 2$, $=66.5$ ft. per sec. The value of v' is $\frac{2}{3}$ of v and $=44.3$ ft. per sec. Finally, therefore, for R' we have

$$R'=L\div v'=22845\div 44.3=516 \text{ lbs.}$$

This force R' may be the tension in a cable winding upon a drum or the tangential component of the pressure between the teeth of a pinion on another shaft and those of a gear-wheel on the shaft of the Pelton wheel. Of course, if the radius r' of the drum or gear-wheel is changed, the value of R' will be altered in inverse ratio.

49. Flat Plates instead of Cups.—If flat plates were substituted for the cups of the impulse wheel, the highest theoretical power would be, as with the ordinary undershot, only

$$L=\frac{Q\gamma}{g}(c-v)v; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which, with $v = c \div 2$ for best effect, would give for the power

$$L = (0.50)\frac{Q\gamma}{g} \cdot \frac{c^2}{2g} = 0.50 Q\gamma h, \quad . \quad . \quad . \quad . \quad (10)$$

as the highest theoretical performance; reduced in practice to 25 to 35 per cent. efficiency, in place of the 0.50 of eq. (10).

50. American Impulse Wheels.—Early in the second half of the nineteenth century simple impulse wheels were constructed in California provided with flat plates as buckets. These so-called "Hurdy-gurdies," though of low efficiency, were easily and cheaply made, and the speed of rotation could be easily varied by a change of radius. The substitution of approximately hemispherical cups for the flat plates brought about a great improvement in performance, and later the invention of the dividing ridge, the main feature of the Pelton bucket, raised the efficiency to a high figure; and this improved type of impulse wheel is now widely used throughout America and Europe.

The three principal forms of impulse wheel with buckets characterized by the dividing ridge, or its equivalent, as made in the United States, are those manufactured by the Pelton Water-wheel Co. of San Francisco and New York, the Abner Doble Co. of San Francisco, and the James Leffel Co. of Springfield, Ohio. Perspective views of the three wheels made by these companies are shown in Figs. 30, 31, and 32, opposite pp. 67, 69, and 70, respectively, of this book. As will be seen from these representations, the two lobes of the Pelton bucket are rectangular in form, while those of the Doble wheel, called "ellipsoidal" by the makers, are oval, with notches cut out at the point of first impingement of the jet. In the "Cascade" wheel made by the James Leffel Co., the "lobes," or half-buckets, are set "staggering," or "breaking joint," on the two sides, and near the rim, of a thin circular disc, whose sharp edge serves the same purpose as a dividing ridge to split the jet. Fig. 33 (opposite p. 72) shows the Escher-Wyss type of impulse wheel, made in America by the Allis-Chalmers Co. of Milwaukee.

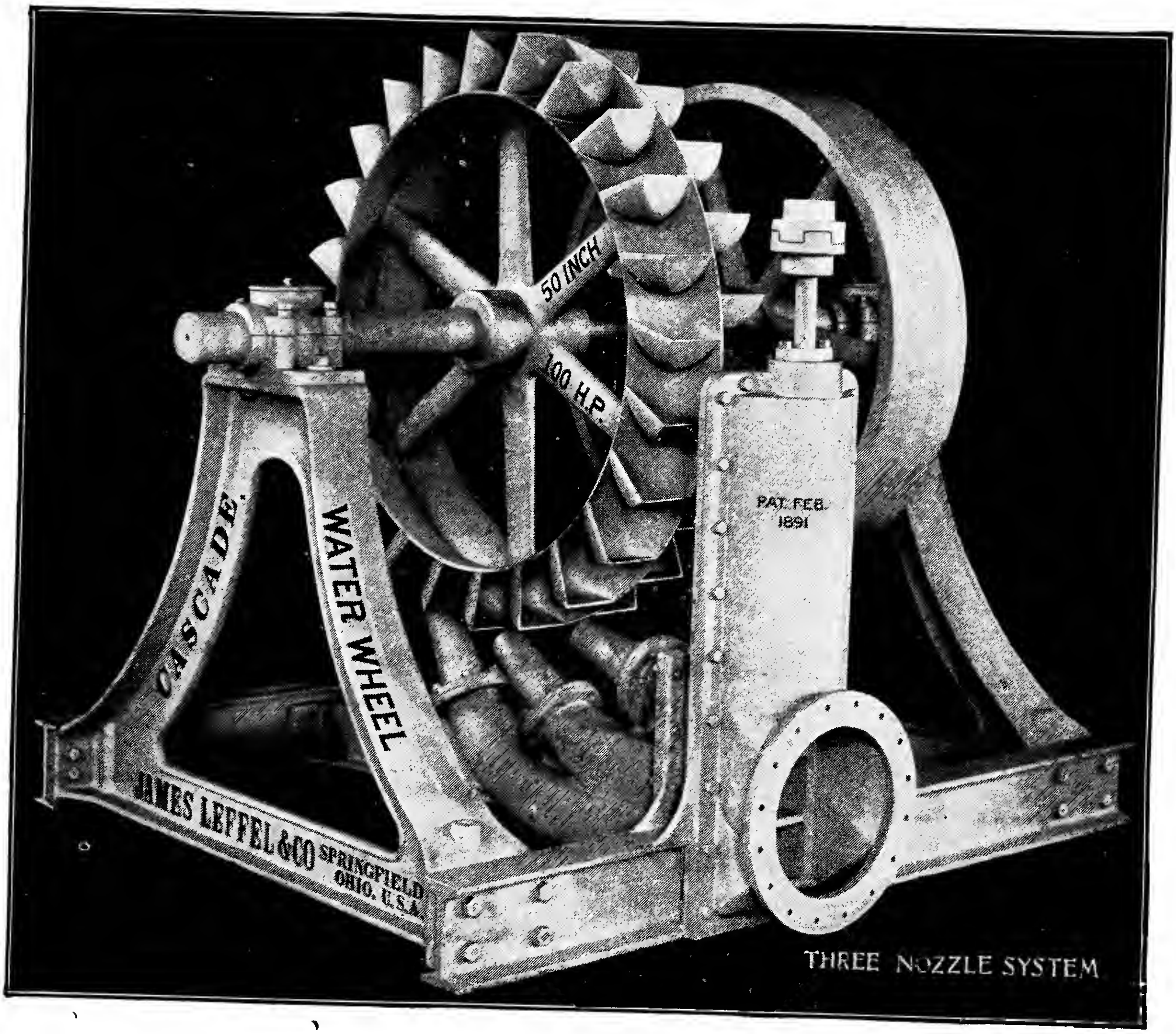


FIG. 32. The "Cascade" (Leffel) Impulse Wheel.

51. Regulation of Pelton Impulse Wheels.—A conical nozzle (one or more) furnishing a cylindrical jet of circular cross-section is generally employed with impulse wheels of the Pelton type. At the base of the cone the pressure, p_1, is high and the velocity, c_1, is small, in regular running; the energy being chiefly in the pressure form at that point of the flow. When the resistance R' is reduced below its usual value, unless the working force P on the buckets is reduced in the same proportion, the velocity of the wheel will be accelerated; and this is usually undesirable, especially in the running of electric generators. A reduction of P can be effected by reducing the size of the jet, or by reducing its velocity, or by diverting the direction of jet sufficiently so that only a portion acts on the buckets.

To reduce the velocity requires a reduction in the value of p_1 at the base of the nozzle; and this is frequently effected by the partial closing of a valve-gate in the pipe just up-stream from the nozzle. But this necessitates a loss of head in the supply-pipe due to the sudden enlargement of section experienced by the water in passing from the narrow section under the valve-gate to the full section of the pipe, and the efficiency of the wheel is much reduced. This loss of efficiency is due to two causes: *First*, the jet velocity and that of the bucket no longer have the proper relation for best effect. *Secondly*, the effective head, $\frac{p_1}{\gamma}+\frac{c_1^2}{2g}$, ($p_1$ is here the pressure in excess of the atmosphere,) at the base of the nozzle, has less than its usual value. This so-called "throttling" of the flow to produce the diminution of jet-velocity is therefore a very wasteful expedient in cases where economy in the use of water is of importance.

Diversion of the jet (so that only a portion acts on the buckets) without throttling is also wasteful of water, though often resorted to in situations where, on account of the extreme length of the supply-pipe, a checking of the flow would produce dangerous "water-hammer" (see later, in § 125).

To diminish the value of the working force P without materially altering the jet-velocity, and thus retain the proper

relation between the latter and that of the buckets, requires a reduction in the sectional area of the jet. A common way of doing this at the present day, with impulse wheels of the Pelton type, is by the use of an internal conical "stopper," or "spear-head," of brass, frequently called a "needle," partially closing the base of the conical nozzle and capable of longitudinal movement. Fig. 34 shows a longitudinal section

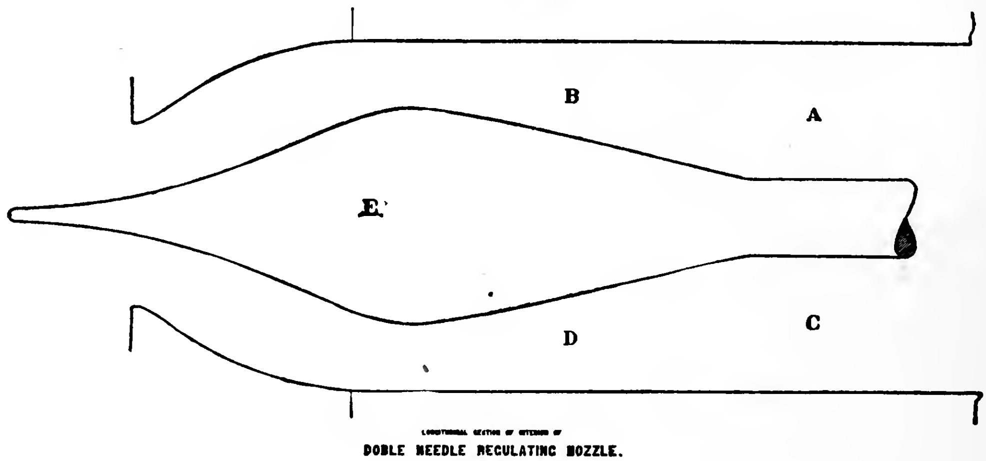


FIG. 34.

of the Doble Needle Regulating-nozzle as used with the Doble impulse wheel. The water passes through the space *AB* (and *CD*) toward the left. By the advance of the "needle" *E* toward the left the ring-shaped space between it and the edge of the nozzle opening is progressively diminished in sectional area.

The filaments of water converge toward and beyond the point of the "needle" and finally form a solid cylindrical jet of circular section, "a clear, transparent, polished stream," in the words of a thesis by Messrs. H. C. Crowell and G. C. D. Lenth of the Mass. Inst. of Technology, published in June 1903. Fig. 35 (opposite p. 69) is from a photograph of a jet issuing from one of these Doble nozzles, and is taken from the thesis mentioned. The size of the jet depends on the position of the "needle," and not on the head of water

52. Girard Impulse Wheels, or "Impulse Turbines."—Another form of impulse wheel may be formed by two flat, parallel,

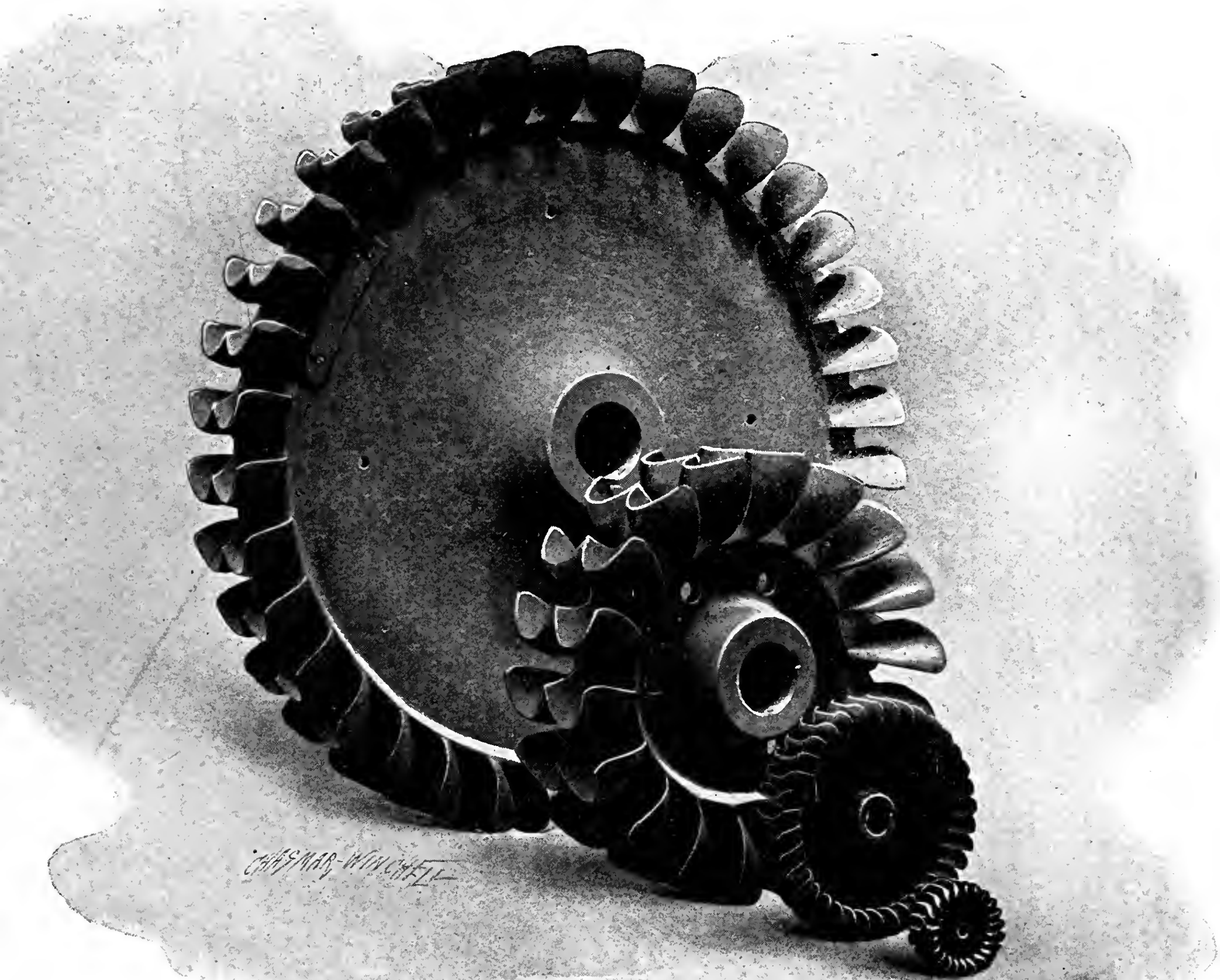


Fig. 33. Escher Wyss & Co. Impulse Wheels.

and concentric rings, or "crowns," between which are inserted numerous curved blades, or "vanes" (sometimes bent from flat plates and therefore cylindrical); and is shown in vertical and horizontal sections in Fig. 36. The wheel receives water in a "free jet" from a nozzle fitted to a pipe P, placed either on the inside of the wheel, as in this figure (and then the wheel is called an "outward-flow impulse wheel), or on the outside (an "inward-flow" wheel).

This form of wheel, now called in Europe a "Girard Impulse Wheel," was invented by Poncelet in 1826 and first applied practically by Zupinger. The wheel revolves in the direction shown in the figure, and the water in passing through it along a vane does not fill the entire space between the two consecutive vanes and is therefore exposed to atmospheric pressure on one side throughout its whole course. The shapes and positions of vanes are to be so designed, and a proper speed of wheel so determined, as to give a power* ($R'v'$) to be expended in overcoming some constant resistance, R', tangent to some pulley or gear-wheel (keyed upon the same shaft as the water-wheel), the velocity of a point in whose outer edge is v' ft. per sec.

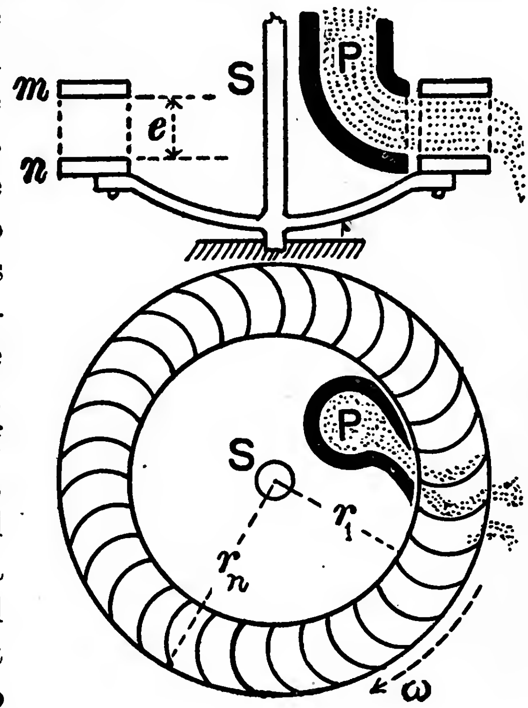


FIG. 36.

The deviation of the water of the jet from its original direction, and the progressive reduction of its absolute velocity, are accomplished gradually after entrance upon the vane; and each particle is considered to move in a plane parallel to the crown plates, the vertical thickness of the jet being equal to the distance apart of those plates (m and n in Fig 36) $=e$.

* Maximum power.

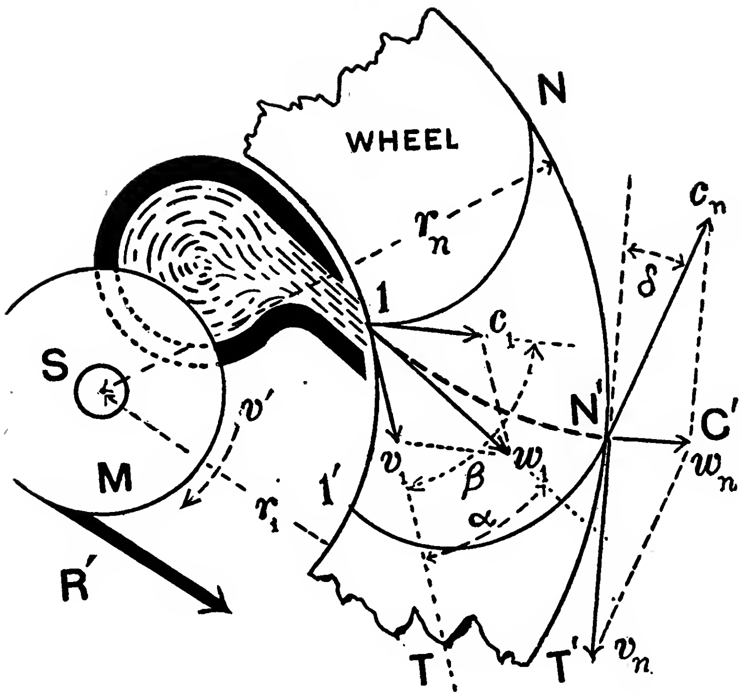


FIG. 37.

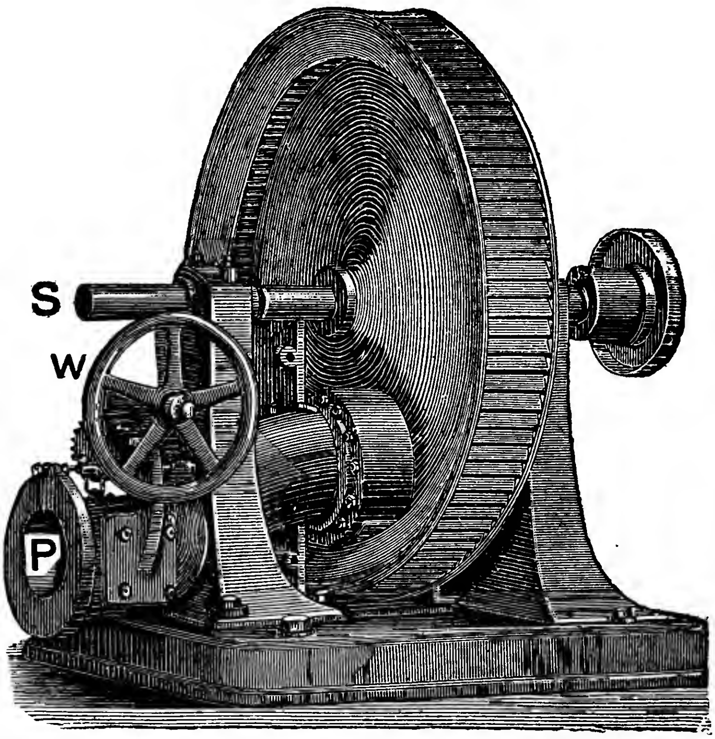


FIG. 38.

53. Best Speed of the Girard Wheel.—In the horizontal section of a portion of an inward-flow Girard wheel shown in Fig. 37, the water of the jet entering at point 1 has an absolute velocity of w_1 ft. per sec., at an angle of α with a tangent, $1..T$, to the inner wheel-rim, the velocity of this inner rim itself being constant at some value v_1. Since the jet is a free jet, the value of w_1 is not dependent (beyond a slight extent) on the presence, or velocity, of the wheel.

With w_1 as the diagonal of a parallelogram of velocities (p. 87, M. of E.), and v_1 as one side (both springing from the point 1), a parallelogram is constructed whose other side, c_1, will be the *relative* velocity of the water at 1, i.e., relatively to that point of the inner rim of the wheel). We assume that whatever the "best speed" of the wheel proves to be, the tangent at 1 to the wheel-vane $1..N$ is made to coincide with the direction of c_1, the relative velocity, making some angle β with v_1, i.e., with the rim-tangent $1..T$.

In this way the water will glide smoothly upon the vane $1..N$ without "shock" or eddying (which is always to be avoided, since it causes waste of energy). The vane is curved backward from 1 to N so as to produce (if its motion is not too rapid) a deviation of the motion of the water particles from the rectilinear path they would otherwise pursue into a curved path, $1..N'$ (*absolute path*). This deviation is accompanied by a gradual diminution of the absolute velocity of the water. (If the vane were stationary, there would be practically no change in the absolute velocity.) On the arrival of the water particles at the outer rim of the wheel the outer extremity N of the vane has come to the position N'. The relative velocity at N' (i.e., relatively to the outer end of vane) has now a different value, c_n, from that at the point of entrance and is, of course, tangent at N' to the vane curve. The point N' of the outer rim of wheel has a velocity v_n equal to $\frac{r_n}{r_1}v_1$ (from the proportion $v_1 : v_n :: r_1 : r_n$); and a parallelogram formed on v_n and c_n as two sides, will determine the value of w_n, the abso-

lute velocity of the water at exit, as being the diagonal $N'C'$ of this parallelogram.

It is seen that w_n is much smaller than the corresponding value w_1 at entrance. Denote by δ the angle between c_n and a line, $N'T'$, drawn tangent, at N', to the outer rim of wheel.

To determine the best value (i.e., conducive to greatest efficiency) for the velocity v_1 (or v_n) we must note that the kinetic energy carried away per second by the water at exit, viz., $\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}$, should be as small as possible; and this means that w_n should be as small as possible. Inspection of the parallelogram of velocities at N' shows that a small value for the angle δ, and equality between v_n and c_n, conduce to a small w_n. Now the angle δ cannot be made equal to zero, but may usually be made as small as 15°; it is quite feasible, however, to have

$$c_n = v_n; \quad \ldots \ldots \ldots \quad (1)$$

which assumption will therefore now be made and the result noted.

Bernoulli's Theorem without friction (eq. (13), § 42) may now be applied to the steady flow between 1 and N in the *rotating channel* here presented (it being noted that the water is under atmospheric pressure both at 1 and N so that the pressure-heads cancel out), leaving

$$c_n^2 - c_1^2 = v_n^2 - v_1^2; \quad \ldots \ldots \ldots \quad (2)$$

in which if we put $c_n = v_n$ from eq. (1) there results $c_1 = v_1$, which shows that the parallelogram at point 1 must be made a *rhombus*. Hence, from trigonometry,

$$v_1 = \frac{w_1}{2 \cos \alpha}, \quad \ldots \ldots \ldots \quad (3)$$

as the best value for the inner wheel-rim velocity. The resistance R' should therefore have a corresponding value (in connection with v') to prevent acceleration of the velocity beyond this speed. Evidently β must be made equal to 2α.

54. Power of the Girard Impulse Wheel.—The power of the wheel due to the action of the water, at this special speed, is

now obtained by using the formula in eq. (10) of § 34, viz.:

$$\therefore \ L = \frac{Q\gamma}{g}[u_1 v_1 - u_n v_n]; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which u_1 and u_n are the "velocities of whirl" of the water at entrance and exit respectively; i.e., the *projections* of the absolute velocities w_1 and w_n upon the tangents to the two wheel-rims.

Evidently $u_1 = w_1 \cos\alpha$. As to u_n, note that since c_n is to be equal to v_n the triangle $N'C'T'$ is isosceles and that hence the angle $C'N'T'$ between w_n and v_n is $90° - \frac{\delta}{2}$; hence $u_n = w_n \cos\left[90° - \frac{\delta}{2}\right]$, or $u_n = w_n \sin\frac{\delta}{2}$. We have, also, from the same triangle, $w_n = 2v_n \sin\frac{\delta}{2}$; substitution of all of which values in eq. (4), with $v_1 = \frac{w_1}{2\cos\alpha}$, and $v_n = \frac{r_n}{r_1}v_1$, gives rise to the relation

$$L, \text{ or } R'v', = \frac{Q\gamma}{g}\cdot\frac{w_1^2}{2}\left[1 - \frac{r_n^2}{r_1^2}\cdot\frac{\sin^2\delta/2}{\cos^2\alpha}\right]. \quad . \quad . \quad . \quad (5)$$

(N.B.—This is identical with what would be obtained in another way, viz., by deducting the kinetic energy $\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}$ carried away each second by the water at exit, from the kinetic energy $\frac{Q\gamma}{g}\cdot\frac{w_1^2}{2}$ arriving each second at the entrance 1. There is no change in pressure energy, nor in potential, between entrance and exit.)

If the whole head, h, of the mill-site be considered as producing the entrance absolute velocity w_1 (or $w_1 = \sqrt{2gh}$), friction being thus entirely ignored in the supply-pipe and nozzle, just as it has been, so far, in the wheel itself, eq. (5) may be written

$$R'v' = Q\gamma h\left[1 - \left(\frac{r_n}{r_1}\right)^2\left(\frac{\sin\delta/2}{\cos\alpha}\right)^2\right]. \quad . \quad . \quad . \quad (6)$$

(5) $L = \frac{Q\gamma}{g}\left[\frac{w_1\cos\alpha\cdot w_1}{2\cos\alpha} - 2\frac{r_n}{r_1}\frac{w_1}{2\cos\alpha}\sin\frac{\delta}{2}\sin\frac{\delta}{2}\frac{r_n}{r_1}\frac{w_1}{2\cos\alpha}\right] =$

It is seen from eq. (6) that the theoretical efficiency

$$\eta=\frac{R'v'}{Q\gamma h}=1-\left(\frac{r_n}{r_1}\right)^2\left(\frac{\sin\delta/2}{\cos\alpha}\right)^2, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which it is evident that not only does a small value for δ, but for α as well, conduce to an increase of efficiency; though a value less than 20° for α is rarely used.

On substitution of the values $\alpha=20°$, $\delta=15°$, and $r_n \div r_1 =1.25$ we obtain $\eta=97$ per cent.; but in actual practice it rarely rises over 80 per cent., on account of friction and imperfect guidance of the water.

For inward-flow Girard wheels the theory does not differ from the foregoing, but the angle δ at the exit-point must be taken a little larger.

55. Numerical Example. Girard Impulse Wheel.—With a head of $h=144$ ft. and a water-supply of $Q=2$ cub. ft. per sec., it is required to design an outward-flow Girard wheel with parallel crown-plates, taking $\alpha=25°$, $\delta=20°$, and the ratio $r_n \div r_1=4\div 3$. The foregoing theory will be applied, with no account of friction, at first, except in the nozzle. There being supposed to be no loss of head between the surface of head-water and the jet, except in the nozzle itself, we have

$$w_1=0.95\sqrt{2gh}=0.95\sqrt{64.4\times 144}=91.4 \text{ ft. per second.}$$

The best velocity for the inner rim will then be, from eq. (3),

$$v_1=\frac{w_1}{2\cos\alpha}=\frac{91.4}{2\times 0.906}=50.5 \text{ ft. per sec.}$$

(With friction considered, this might be reduced to 47 or 48 ft. per sec.)

If it be desired that the wheel make 240 revs. per minute, or 4 per sec., we obtain a value for r_1, the inner radius, by writing $4\times 2\pi r_1=50.5$; obtaining $r_1=2.01$ ft.; and hence $r_n=(4:3)r_1$, $=2.68$ ft.

As to e, the proper distance apart of the two flat crown-plates, or rings, if the "free jet" at point 1 (Fig. 37) is given a horizontal thickness of $t_0=\frac{3}{4}$ inch, the vertical dimension of its rectangular cross-section will be e, and we may write $Q=et_0w_1$,

whence

$$e=\frac{2}{(\frac{3}{4}\cdot\frac{1}{12})50.5}=0.634 \text{ ft.}, =7.6 \text{ inches.}$$

While the theoretical efficiency would be

$$\eta=\left[1-\left(\frac{r_n}{r_1}\cdot\frac{\sin\delta/2}{\cos\alpha}\right)^2\right], =1-\left(\frac{4}{3}\cdot\frac{0.174}{0.906}\right)^2=0.93,$$

the actual performance would probably be in the neighborhood of from 75 to 80 per cent. On the basis of 75 per cent. the useful power would be

$$L, =R'v', =0.75Q\gamma h=0.75\times2\times62.5\times144,$$
$$=13500 \text{ ft.-lbs. per sec.}; =24.5 \text{ H.P.}$$

If the radius r' of the pulley (on same shaft as water-wheel), to whose circumference the resistance R' is to be applied, is $r'=1$ ft., the velocity of a point in that circumference would be $v', =\frac{r'}{r_1}v_1, =\frac{1}{2.01}\times50.5, =25.2$ ft. per sec.; and therefore the necessary value of R' would be

$$R'=\frac{L}{v'}=\frac{13500}{25.2}=536 \text{ lbs.}$$

56. Bell-mouthed Profiles.—When the distance between the two crown-plates of an outward-flow Girard wheel is the same at outlet as at entrance of the space between two adjacent vanes a small value of the angle δ may occasion too narrow a passageway between the vanes at exit. If, however, the crowns diverge toward exit, making what is called a "*bell-mouthed*" profile, the stream of water becomes thinner perpendicularly to the vane, on account of lateral spreading along the surface, and choking of the passageway is prevented.

Openings are frequently made in the crowns to facilitate the escape of air, with the same object in view.

57. Practical Construction of Girard Wheels.—The Girard wheel is a favorite type in Europe, some motors of this kind developing as much as 1000 horse-power.

Several are working at the Terni Steel Works, in Italy,

from 50 to 1000 H.P., under a head of nearly 600 ft. One of the smaller of these is shown in Fig. 38 (on p. 74) in which W is a hand-wheel for opening the gate in the main supply-pipe, P. The wheel revolves on a horizontal shaft S and is seen to be of outward-flow and "bell-mouthed" design.

The larger wheels at Terni are practically of the same general design. In the case of the 800-H.P. wheel which drives the rolling-mill machinery, frequent stopping and starting being necessary, a lateral pipe 8 ins. in diameter is provided, opening out of the main supply-pipe, whose diameter is 24 ins., the gate of the smaller pipe being so connected with the gates admitting water to the wheel that when the latter are closed the former is opened and *vice versa*. In this way the motion of the water in the main supply-pipe, which is very long, is not entirely checked when the water is shut off from the wheel, but finds a vent through the smaller pipe; and thus "water-hammer" (i.e., excessive rise of pressure) in the main pipe is prevented. (See § 125.) The outer diameter of this wheel is 9 ft. 5 ins.; the inner, 8 ft. 2.4 ins. The distance between crowns at entrance is 4.91 ins.; that at exit, 16.14 ins.; and the quantity of water used is $Q = 16$ cub. ft. per second, while the normal speed is 200 revs. per min.

In Fig. 40 is shown a vertical section, through the axis of shaft and also of supply-pipe, of a 1000-H.P. Girard wheel at Vernayaz, Switzerland; one of six in an electric power-station, each of 1000 H.P. and working under a head of 1640 ft. The velocity, v_n, of outer rim is normally 184 ft. per second. The outer diameter is of the wheel 2.150 meters, or about 6.5 ft.; and that of the supply-pipe, 0.30 meters. To prevent too rapid "speeding up" of the wheel when the resistance, R', or "load," is diminished, two heavy steel rings are shrunk on the wheel on the outside (these are seen in section in Fig. 40), and thus form a fly-wheel. As is evident from the figure, the profile between crowns is "bell-mouthed."

Fig. 39 gives a cross-section at right angles to the shaft and midway between crowns, and shows the nature of the nozzle and of the regulating apparatus. Through action of

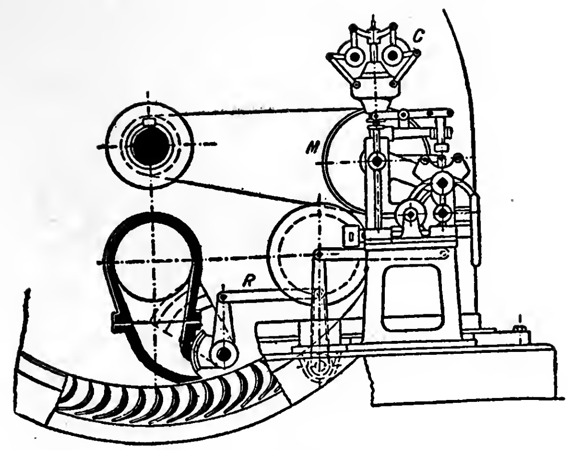

C
M
R
L

FIG. 39.

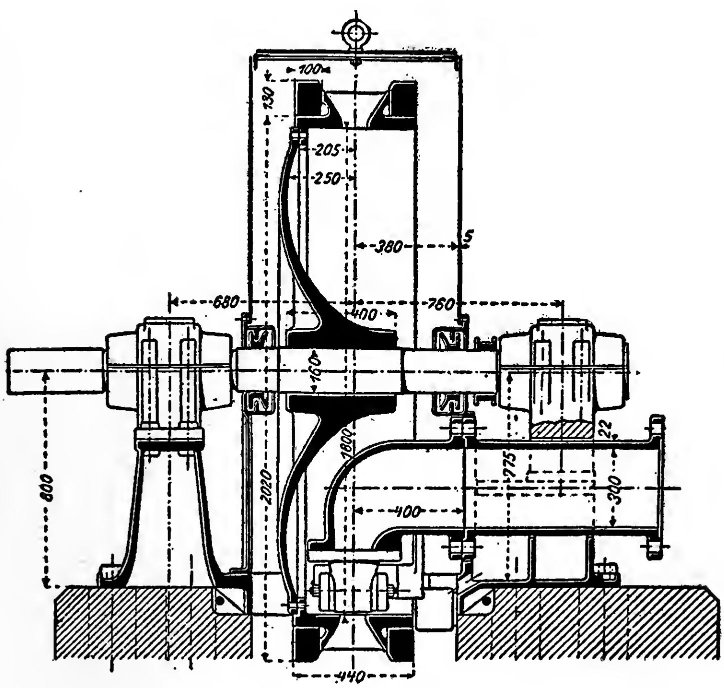

100
130
205
250
380
5
680
400
760
160
1800
2020
800
775
22
300
400
440

FIG. 40.

the centrifugal governor-balls at *C*, and the intervening mechanism, when the "load" on the wheel changes from the normal, and a slight change of speed is thus brought about, one edge of the rectangular opening which forms the jet is caused to move, and thus to diminish or increase the thickness of the jet, and thus vary the amount of the working force acting on the wheel.

CHAPTER V.

Turbines and Reaction Wheels.

58. "Reaction Turbines."—A turbine proper, or "reaction turbine," is a hydraulic motor consisting generally of two crown-plates or shells (surfaces of revolution) mounted on an axle, the space between the shells being divided by rigid curved blades or vanes ("buckets") into numerous curved passageways, or channels, distributed regularly around a circumference. The mouths of these channels receive water simultaneously, and all around the periphery, from the extremities of certain *fixed* guide-channels and discharge it at the turbine-channel exits either into the atmosphere or into a space filled with water (whose internal pressure is frequently less than that of the atmosphere).

The special feature of the turbine as distinguished from Girard wheels is that *all* of its channels or passageways are simultaneously in action and are completely filled with water, flowing under pressure. By the proper design of the wheel or turbine, and restriction of its velocity of rotation (as accomplished by the imposition of a certain resistance), the course of the water is so deviated from the path it would take if the wheel were not present that its absolute velocity is gradually reduced, and its internal pressure brought to an equality with that of the space into which it is discharged; so that the water exerts pressure or working forces against the vanes, thus enabling the turbine to maintain its uniform motion notwithstanding the resistance. In steady operation the flow of the water is "steady," or permanent, as already defined.

59. The Reaction Wheel, or Barker's Mill. Theory. (This

theory will now be given, as preliminary to that of the modern turbine.)—A simple form of reaction wheel consists of a single rigid casing secured transversely to a vertical axle, and is provided with two small orifices in the vertical sides of the casing, each facing backwards as regards direction of motion, and equidistant from the axle. (See Fig. 41.) A water-tight joint at n prevents leakage when water is passing from reservoir W, by the fixed pipe $m..n$, to, and through, the moving pipe and casing $n..AB$. The area, F, of each orifice is supposedly small

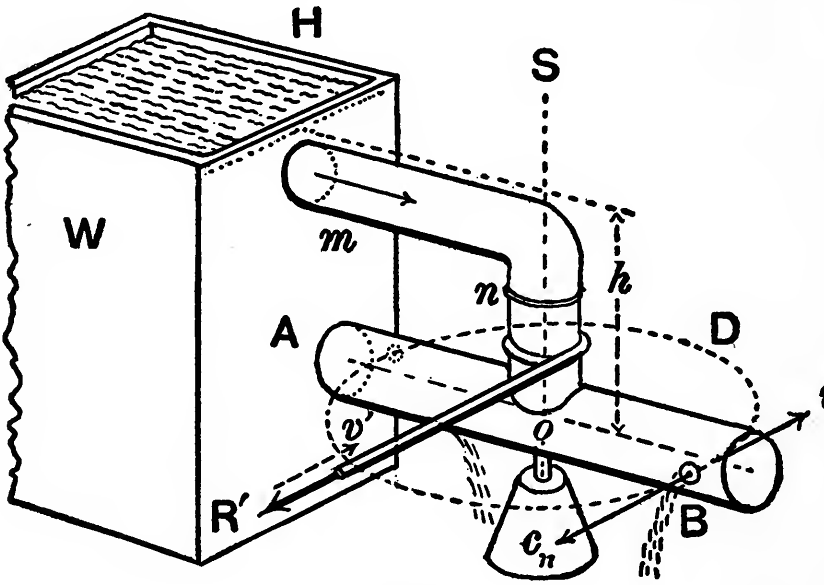


Fig. 41.

compared with the cross-section of the casing, AB, so that the relative velocity of the water in the main body of the latter may be considered to be zero. The horizontal plane of rotation of the centers of the orifices is h feet below the reservoir surface, and the absolute velocity of the water at the point o in the casing (in the axis of motion of the latter) is so slight that the internal fluid pressure there is practically $p_a+h\gamma$ (where p_a denotes atmospheric pressure). As above indicated, the relative velocity c_1 of the water at o is to be taken as zero. A moderate resistance, R' lbs., being provided (tension in a rope winding on drum, say, as shown), and the two orifices being opened (whole apparatus originally full of water), a flow

begins and the motion of water and motor soon adjusts itself to some constant speed of rotation, the linear velocity of the center of each orifice, at distance r_n from the axis, assuming some value v_n, corresponding to which a point in the rope, or periphery of the drum, has a velocity v', $=\frac{r'}{r_n}v_n$, where r' is the radius of the drum. In other words, a steady flow for the water, and a uniform rotary speed for the motor, have set in. It is now required to find the proper value of v_n that the useful power, $R'v'$, may be a maximum, considering friction at the orifice (only).

The absolute velocity of the jet of water at exit B (in the contracted vein, where the filaments have become parallel and are therefore under atmospheric pressure) is evidently

$$w_n = c_n - v_n, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c_n is its velocity relatively to the orifice.

Noting that AB is a uniformly rotating pipe, and taking o as an up-stream point where the relative velocity is zero and the pressure is $p_a + h\gamma$, and B (jet in air) as a down-stream point where the relative velocity is c_n and the pressure $= p_a$, these two points being at the same level, and considering the one loss of head h'', $=\zeta\frac{c_n^2}{2g}$ at the orifice, we may apply Bernoulli's Theorem for such a case (rotating casing; see eq. (13a) in § 42) and obtain

(13a), pg 60

$$\frac{c_n^2}{2g} + \frac{p_a}{\gamma} = 0 + \frac{p_a + h\gamma}{\gamma} + \frac{v_n^2 - 0}{2g} - \zeta\frac{c_n^2}{2g}. \quad . \quad . \quad . \quad (2)$$

Here ζ is a "coefficient of resistance" for the orifice and is found by experiment to have a value of about 0.125, or $\frac{1}{8}$, for the present case; the orifice being in thin plate, or rounded.

This reduces to

$$h = (1 + \zeta)\frac{c_n^2}{2g} - \frac{v_n^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The weight of water passing per second in steady flow being $Q\gamma$, let us apply the equation of "angular momentum," eq. (10) of § 34, to this case, viz.:

$\frac{c_n^2}{2g} + \cancel{\frac{p_a}{\gamma}} = \cancel{\frac{p_a}{\gamma}} + h + \frac{v_n^2}{2g} - \zeta\frac{c_n^2}{2g}$

$h = \frac{c_n^2}{2g} + \zeta\frac{c_n^2}{2g} - \frac{v_n^2}{2g} = (1+\zeta)\frac{c_n^2}{2g} - \frac{v_n^2}{2g}$

$$R'v' = \frac{Q\gamma}{g}(u_1 v_1 - u_n v_n);\ \text{(ft.-lbs. per sec.)}, \quad . \quad . \quad (4)$$

u_1 and u_n being the projections of the two absolute velocities w_1 and w_n (at entrance and exit) upon the "wheel-rim" velocities v_1 and v_n. Now, in the present case, at the entrance-point o (see Fig. 41) the absolute velocity of the water is practically zero (large passageway), and the velocity of that point of the motor is $v_1 =$ zero. At the point of exit (jet in the air) the velocity of the mid-point of the orifice is v_n and the projection of the absolute velocity upon the line of v_n is w_n itself, which is numerically equal to $c_n - v_n$; but since this projection points backward with respect to the motion of the wheel we write it negative in the substitution; and hence eq. (4) reduces to

$$R'v' = \frac{Q\gamma}{g}(0 - [-(c_n - v_n)]v_n); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or, $$R'v' = \frac{Q\gamma}{g}(c_n - v_n)v_n, \ = \frac{Q\gamma}{g}(c_n v_n - v_n^2). \quad . \quad . \quad . \quad (6)$$

Now the efficiency of the motor is $\eta = R'v' \div Q\gamma h$; hence, substituting from (6), and the value of h from (3), we have

$$\eta = \frac{\frac{Q\gamma}{g}(c_n v_n - v_n^2)}{Q\gamma\left((1+\zeta)\frac{c_n^2}{2g} - \frac{v_n^2}{2g}\right)} = \quad \eta = \frac{2(c_n v_n - v_n^2)}{(1+\zeta)c_n^2 - v_n^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In (7) we have η as a function of *two* variables, c_n and v_n, but it is of such a character that it can be reduced to a function of the *one* variable x, if x denote the ratio $c_n : v_n$; that is, if for c_n we write xv_n, (7) becomes

$$\eta = \frac{2(x-1)}{(1+\zeta)x^2 - 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

By obtaining $d\eta/dx$ and placing it equal to zero, we derive $(x^2 - 2x)(1+\zeta) = -1$; and, finally, taking plus sign of radical,

$$x = 1 + \sqrt{\frac{\zeta}{1+\zeta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

as the special value of x that makes the efficiency a maximum.

With $\zeta = 0.125$, or $\frac{1}{8}$, we find, from (9), $x = \frac{4}{3}$, whence $c_n = \frac{4}{3}v_n$;

and also, from (8), a value of $\eta=\frac{2}{3}$, or $66\frac{2}{3}$ per cent., as the maximum efficiency.

For this maximum efficiency to be obtained it is necessary that v_n be regulated to a value of $v_n=\sqrt{2gh}$, as obtained from eq. (3) when for c_n we write $\frac{4}{3}v_n$. At this special speed we find also that the maximum power for a given Q is $R'v'=\frac{2}{3}Q\gamma h$; and that the absolute velocity at exit, $=w_n$, $=c_n-v_n$, $=\frac{1}{3}(\sqrt{2gh})$; so that $\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}=\frac{1}{9}Q\gamma h$.

That is to say, of the whole power of the mill-site (viz., $Q\gamma h$ ft.-lbs. per sec.) two thirds is usefully employed in overcoming the resistance R', one ninth is carried away in the effluent jet in the kinetic form, while the remaining two ninths is lost in friction at the edges of the orifice (when the speed is regulated as above stated for maximum effect). In order that the whole available flow, Q cub. ft. per sec., may be utilized at this special speed, the aggregate sectional area of the two jets, *in the contracted vein where the filaments are parallel and relative velocity is* c_n, must have a value of $2F=Q\div c_n$.

If *no friction whatever* were considered, ζ would be zero in eq. (9), giving $x=1$, or $c_n=v_n$, and $\eta=$unity from (8). But this is impossible since from eq. (3), which gives $c_n=\sqrt{2gh+v_n^2}$ when ζ is zero, c_n is always greater than v_n. It is evident, however, that as greater and greater speed is permitted, the ratio $c_n\div v_n$ decreases towards a value of unity, and since h is constant, may be made to differ as little as we please from unity by a proper increase of v_n. While, mathematically, the efficiency would not become unity except for $v_n=$infinity, it would be high for values of v_n which are not excessive; e.g., for $v_n=\sqrt{2gh}$, $\sqrt{4gh}$, and $\sqrt{8gh}$, we should find η to be 0.83, 0.90, and 0.94, respectively. This is, of course, for the ideal case of no friction. With great speeds of rotation fluid friction increases fast, as also the resistance of the air to the motion of the motor.

Weisbach's experiments with a small reaction wheel some 3 ft. between the two orifices under a head of $h=1.3$ ft. confirmed the above theory where friction at the orifice has been considered, with $\zeta=0.125$.

In the foregoing theory it has been virtually supposed that Q was constant at all speeds of rotation; which implies a varying size of orifice, since $Q=mFc_n$, where m is the number of orifices and F the sectional area of the contracted vein of jet; that is, a different F would apply to each different speed. If the value of F were fixed, Q would be variable, depending on the speed; and the outcome of the theory would be different. However, if a special value of Q is desired to be used at any particular speed, a proper size of orifice is easily computed to secure this result, since eq. (6) is independent of the size of orifice so long as the latter is small compared with the sectional area of the casing.

60. Reaction Wheel. Theoretical Points.—The reaction wheel, though now obsolete, presents some interesting theoretical features. The expression for the useful power, $R'v'$, as already derived, and stated in eq. (6), may be transformed as follows.

It may be written thus:

$$R'v' = \frac{Q\gamma}{2g}[2c_nv_n - v_n^2 - v_n^2]. \quad . \quad . \quad . \quad . \quad (10)$$

We may then, in the bracket, add the quantity $2gh+v_n^2-\zeta c_n^2$, and subtract its equal, c_n^2 (see eq. (3)); whence

$$R'v' = \frac{Q\gamma}{2g}[2gh+v_n^2-\zeta c_n^2-(c_n^2-2c_nv_n+v_n^2)-v_n^2]; \quad (11)$$

which, since $c_n-v_n=w_n$, reduces to

$$R'v' = Q\gamma h - \frac{Q\gamma}{g}\cdot\frac{w_n^2}{2} - Q\gamma\left(\frac{\zeta c_n^2}{2g}\right), \quad . \quad . \quad . \quad . \quad (12)$$

which is the same expression for the power as might have been derived by deducting from the whole theoretical power, $Q\gamma h$, of the mill-site, the kinetic energy carried away each second by the water in the effluent jets by virtue of its absolute velocity w_n and the power lost in friction at the orifice (i.e., the product of the lbs. of water flowing per second by the "friction-head," or "loss of head," due to the passage through the orifice.

61. Working Forces in Barker's Mill.—Another interesting matter is the nature and position of the actual working forces

or pressures which are exerted on the inside wall of the casing during steady operation, enabling the motor to keep up the motion uniformly notwithstanding the resistance R'.

Fig. 42 shows a horizontal section of the casing of Fig. 41, but it is now supposed to have vertical side walls; the two orifices indicated being under like conditions. The rotation is counter-clockwise, with a constant angular velocity ω, so that $v_n = \omega r$ = linear velocity of orifice. B is the jet, in the atmosphere, having, at the contracted section where the sectional area is F, a velocity c_n relatively to the orifice. The casing is so wide that at B', in the interior, just inside from the orifice, the relative velocity of the water is practically zero, while its absolute velocity at B' is v_n, =that of the orifice itself. At C the *excess of pressure* of the water against the vertical wall of casing over that on the corresponding portion of wall from which the orifice is cut out, or "*reaction*" of the jet on the casing, is a force P whose value, according to p. 800 of M. of E., is $P = 2\phi^2 Fh\gamma$, friction at the orifice being considered. But the h of this expression was equal to the $(v^2 \div 2g)$ of p. 800, v being there the velocity of the jet relatively to the vessel $(= c_n$ in our present notation), and ϕ the "coefficient of velocity," which is the same as $1 \div \sqrt{1+\zeta}$ (see p. 706, M. of E., and eq. (2) of the foregoing). That is, at C we have a working force of

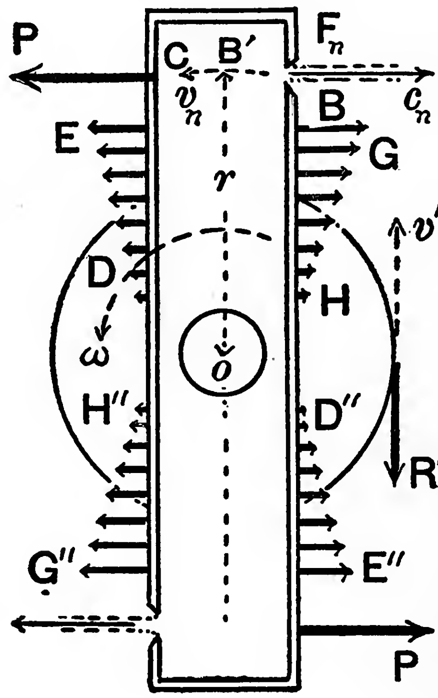


FIG. 42.

$$P = 2F\gamma\frac{c_n^2}{2g}, \quad = \frac{Q\gamma c_n}{g}; \ldots \text{(lbs.)} \quad . \quad . \quad . \quad (13)$$

and a similar, equal, force in connection with the other orifice.

At first sight it might seem that all other horizontal pres-

sures on the inside walls of the casing were balanced; but since the water is being caused to travel out from the center o toward the position B', acquiring an increasing absolute velocity as it proceeds, the casing has to act as a centrifugal pump to that extent and consequently must encounter resisting forces due to this cause. This resistance consists in the fact that the water pressures along GH (and $G''H''$), the rear vertical walls of the casing, are greater than those along DE (and $D''E''$), the front walls. These pressures constitute a couple in a horizontal plane whose moment, M', may be found from eq. (9a) of § 34, i.e.,

$$M' = \frac{Q\gamma}{g}[u_1 r_1 - u_n r_n] \ldots \text{(ft.-lbs.)}. \quad . \quad . \quad . \quad (13a)$$

Here u_1 and u_n are the tangential components of the absolute velocities of the water at the two points in the rotating casing between which the forces in question act, viz., o and B' in Fig. 42, and r_1 and r_n the two corresponding radii. Evidently u_1 is zero and $u_n = v_n$; therefore

$$M' = -\frac{Q\gamma}{g} v_n r_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The negative sign shows that this couple tends to retard the motion of the casing instead of furnishing working forces.

We are now able to formulate the net power (ft.-lbs. per sec.) due to the two working forces P and P and the resisting forces constituting the couple whose moment is M'; remembering that the work done per second by the couple is the product of its moment by the angular velocity ω of the casing, i.e.,

$$R'v' = 2Pv_n - \omega\left[\frac{Q\gamma}{g} v_n r_n\right]. \quad . \quad . \quad . \quad . \quad (15)$$

Substituting v_n for ωr_n and, for P, its value as found in eq. (13), we have

$$R'v' = \frac{Q\gamma}{g}(c_n v_n - v_n^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

ft.-lbs. per second, as before obtained; see eq. (6).

The foregoing applies equally well to a casing of any form (orifice small, however) when in place of the pressures on the

vertical sides of the present form we substitute the components, in plane of rotation and tangent to motion) of the actual pressures on interior walls.

62. Development of the Turbine.—Barker's Mill was improved by Whitelaw and given a form resembling that shown in Fig. 43, called the Scotch turbine; furnished with three orifices, which were made adjustable in size by movable flaps, to provide regulation of the quantity of water used and power developed.

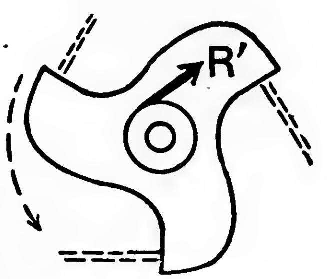


Fig. 43.

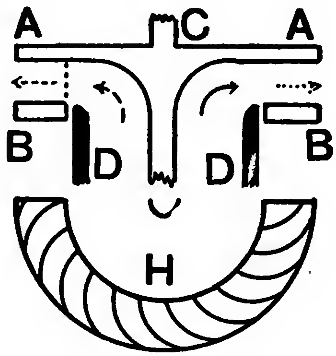
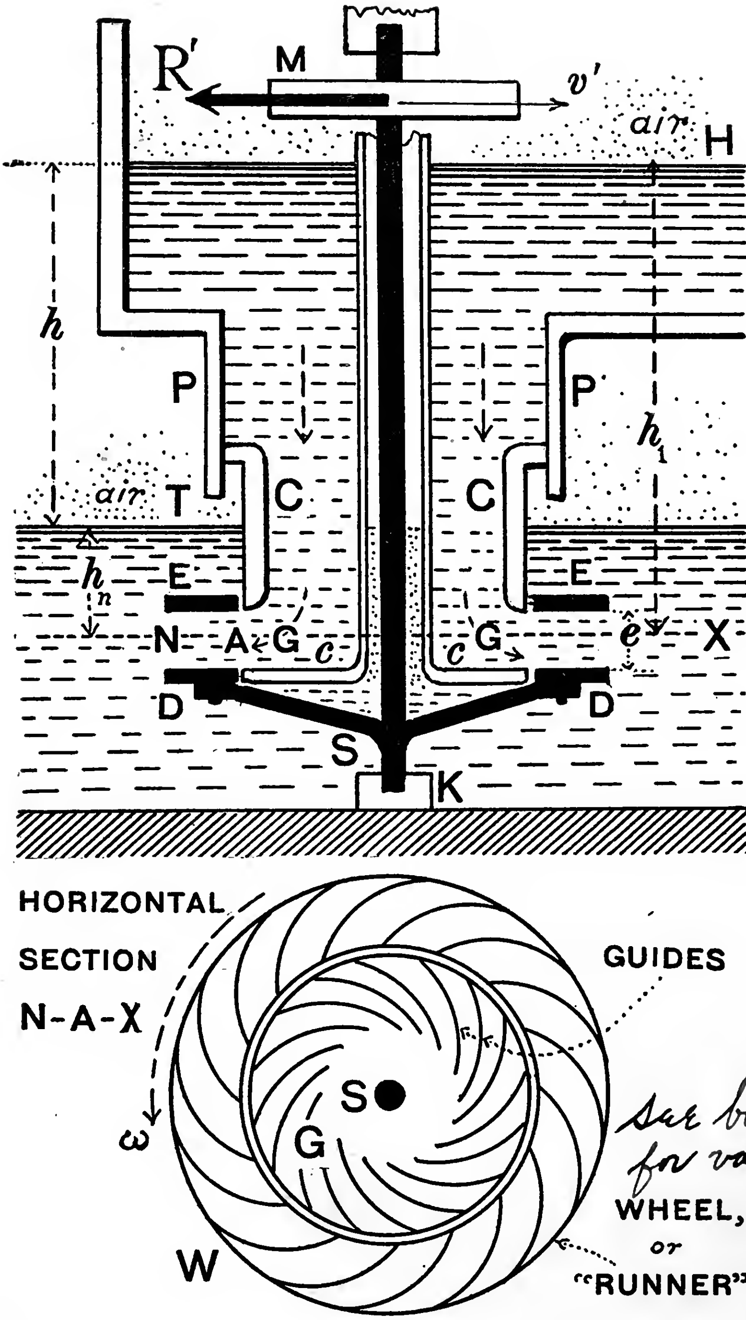


Fig. 44.

We next find in Combe's turbine (Fig. 44) many jets, occupying the entire circumference, guided between vanes, or blades, fixed in a ring attached to a shaft, the water being supplied from underneath through a fixed pipe or tube. No interior fixed guides were provided to direct the water at any special angle upon the moving vanes. Fig. 44 shows a vertical section of the wheel and vertical shaft, viz., *BACAB*; and supply-pipe *DD*; also a horizontal section, *H*, of one half of the wheel. Passing from the fixed pipe *DD* outwardly through the wheel, the water completely fills the passages of the latter and is discharged at the outer rim, around the entire circumference, with a relatively small absolute velocity into the atmosphere. The Cadiat turbine was practically the same as Combe's, but the supply-pipe was placed above.

In 1826 the French engineer Fourneyron improved the Cadiat turbine by placing fixed guide-blades just inside the wheel-ring around the entire circumference, by means of which the water received a forward direction of motion before enter-



See bottom of p201 for value of h_1.

FIG. 45.

ing the channels of the moving turbine. This rendered attainable a very low value of the absolute velocity of the water at exit from the outer rim of the wheel-ring. Also, the wheel being operated under water, the complete filling of the wheel-channels was insured when properly designed. This was the first modern turbine: a motor which, as varied and improved by Fontaine, Henschel, Jonval, and others in Europe, and by Boyden and Francis and their successors in America, has grown in popular favor and, together with the impulse wheels already described, has almost entirely supplanted the old forms of vertical water-wheels so long considered as giving the highest efficiency.

It is the peculiarity of the turbine proper (or "reaction turbine," as distinguished from a Girard impulse wheel or "Girard turbine") that the power to be transmitted to the wheel by the water is present at entrance partly in the form of pressure energy and partly in that of kinetic; since the pressure of the water at entrance is usually above that of the atmosphere.

63. Description of a Simple Fourneyron Turbine.—Fig. 45 shows in the upper part a vertical, and in the lower part a horizontal, section of a simple design of a turbine of the Fourneyron type (or "outward-flow, radial turbine"). A case has been chosen of a "low-pressure" turbine, or one for which no long supply-pipe or penstock is necessary, the turbine being placed at the bottom of an open wheel-pit.

The water from the head-bay or head-water, H, descends slowly through the tube, or short penstock, PP, which is firmly supported and is provided with a prolongation, CC, or cylindrical gate, movable vertically and having rounded edges on its lower periphery. This lower edge is also slotted to receive the curved stationary guides which are shown (in the horizontal section) at G and which are rigidly attached to the fixed plate $c..c$. This plate is supported from above by means of a pipe enclosing the shaft of the turbine and serves also to protect the lower shell DSD of the turbine from the pressure of the water in space CG.

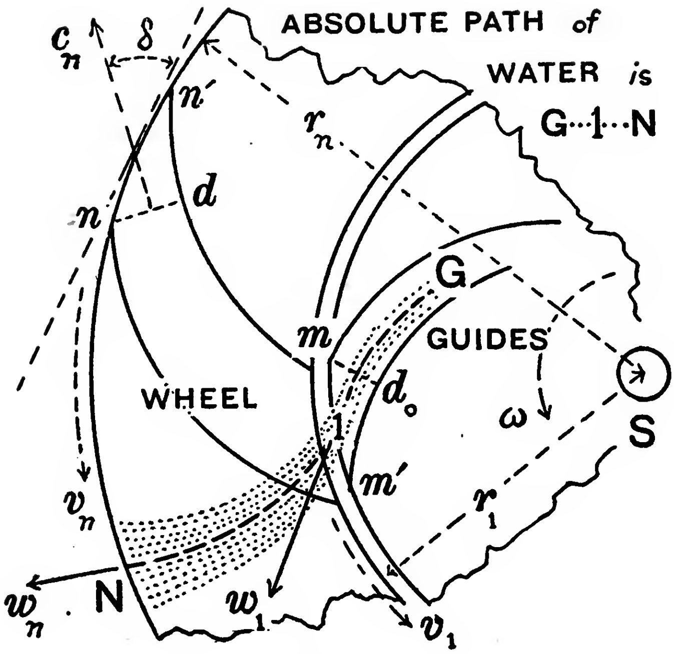

ABSOLUTE PATH of
WATER is
G···1···N
WHEEL
GUIDES
G
N
S

FIG. 47.

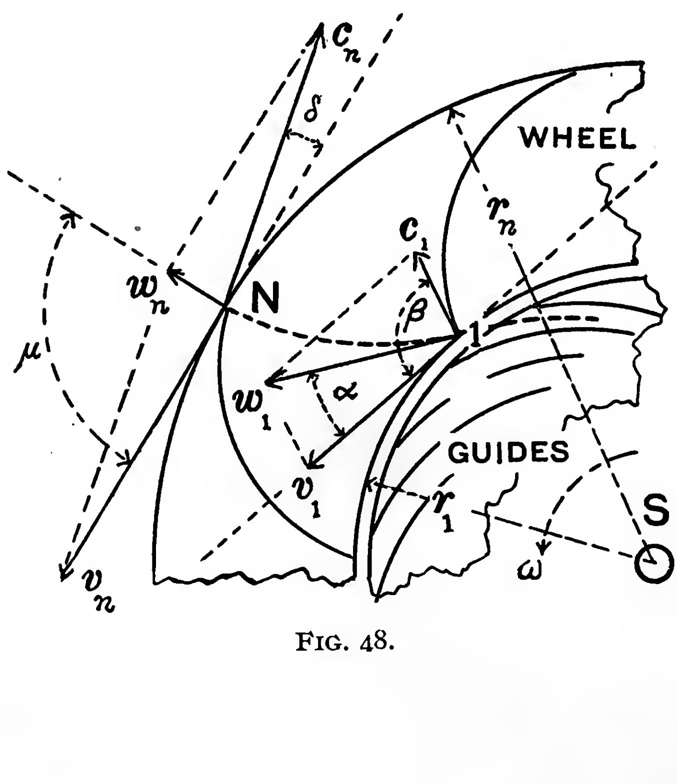

WHEEL
GUIDES
N
S

FIG. 48.

The turbine itself and its shaft are shown in vertical section by solid black shading; viz., *EDKDE*. *EE* and *DD* are the two crowns, or horizontal rings, between which are inserted the curved vertical vanes shown in outline in the horizontal section at *W*. The lower shell of the turbine provides for the rigid connection of the turbine proper (or crowns and vanes) with the shaft, and may be lightened by perforations. The turbine illustrated in Figs. 17, 18, and 19 (opp. p. 42) is practically of this design. The resistance R', which the wheel is overcoming, is shown in Fig. 45, as acting at edge of pulley *M*, keyed on shaft of wheel. The velocity of the edge of the pulley is v' ft. per sec.

Fourneyron placed a number of horizontal partitions between the crowns, thus dividing the turbine into several stories, for the purpose of preventing in some degree the loss of head, and consequent loss of power, resulting from the sudden enlargement of passageway which would occur when the turbine is operating at "part gate," if this device were not adopted. In turbine parlance, "full gate," or "whole gate," refers to the fact that the spaces between the fixed guides, *G*, are fully open, the gate being then fully drawn up, as in Fig. 45; its lower edge being even with the upper crown. In Fig. 46 is shown

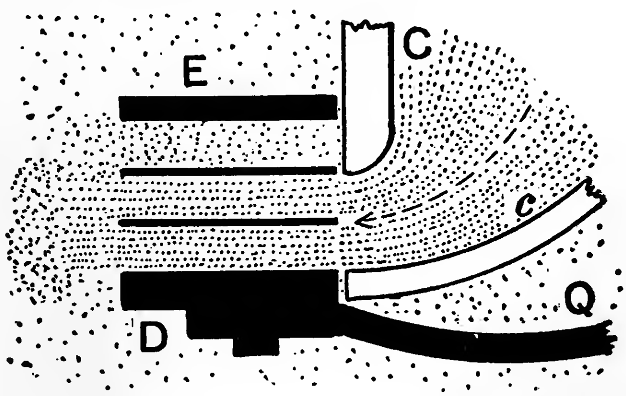


Fig. 46.

a section of a Fourneyron turbine furnished with horizontal partitions of the kind mentioned. When the lower edge of the gate is even with one of these partitions only those portions of the channels which are below this partition are in action,

and the efficiency of the turbine, when thus working at "part gate" and using less than the usual quantity of water, is not materially changed.

64. Notation for Theory of Fourneyron Turbine.—In Fig. 47 is represented a portion of the turbine, and corresponding guides and guide-channels, in horizontal section. The turbine channels mn, etc., are so many closed pipes, supposed completely filled by the water when the turbine is in operation. Let F_n = the sum of all the sectional areas like nd of the turbine channels at the outer circumference; and F_o the sum of all those like md_o between the stationary guides, where the water is just leaving them to enter the turbine or wheel.

Let w_1 denote the absolute velocity of the water leaving the guides at point 1, Fig. 47 (and at A in Fig. 45). Also, in Figs. 47 and 48, let w_n denote the absolute velocity of the water leaving the wheel at the exit-rim, N, being represented in amount and position by the diagonal of the parallelogram formed on c_n, the relative velocity at N, and v_n, the velocity of the outer rim of the wheel itself.

Similarly, at point 1, the absolute velocity, w_1, of the water entering a wheel-channel is the diagonal of a parallelogram formed on its relative velocity at that point and the velocity, v_1, of this inner rim of the wheel. Note that in each case the diagonal meant is the one which springs from the *same corner* as the c and the v. (For relative and absolute velocity, see p. 89, M. of E.)

If the wheel is run at the proper speed and the angle β has been given a corresponding suitable value, such that the tangent to the vane curve at 1 coincides in position with the relative velocity c_1 (velocity of the water leaving the guide extremities relatively to the point 1 of the inner wheel-rim), there will be no "elbow" or sharp turn in the absolute path of the water as it enters the wheel, but that path will be a smooth curve throughout its whole extent. See curve $G..1..N$ in Fig. 47. In this way, impact or "shock" at entrance is avoided and the corresponding loss of energy due to the internal friction of the water.

This relation being stipulated, it follows that the absolute velocity of the water just entering a wheel-channel at 1 is practically the same as the absolute velocity that it has on leaving the guides; both being therefore designated by w_1.

Fig. 49 shows by a vertical section the notation used for vertical heights. That from the surface of head-water to that of tail-water, $=h$; while the heights of these surfaces above the horizontal plane passed through a point of turbine half-way between the two crowns are h_1 and h_n respectively. The radii of the inner and outer edges of the wheel are r_1 and r_n respectively; see Fig. 48. The height of wheel, or vertical distance between crowns, is e; the same in this case both at entrance and exit of a wheel-channel. The meaning of the angles α, β, μ, and δ is evident in Fig. 48. Q denotes the number of cub. ft. of water used per sec., in steady flow. Let p_1 be the internal pressure of the water at entrance of the wheel; and p_n that at exit from the wheel, i.e., at N.

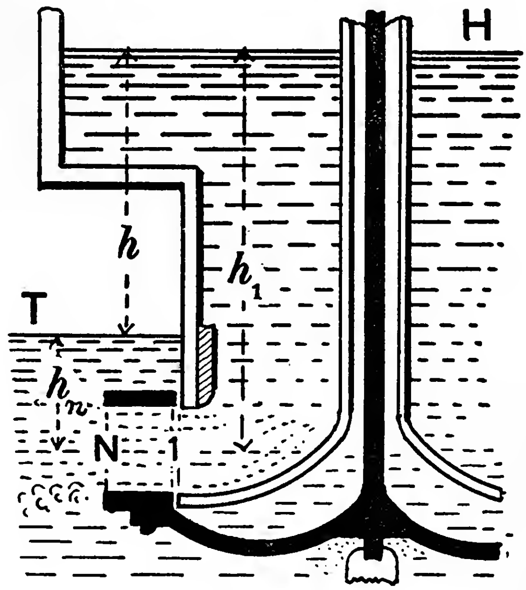


FIG. 49.

65. Theory of the Fourneyron Turbine. Friction Disregarded.—The quantities Q, h_1, h_n, γ, r_1, r_n, α, and δ, being given; it is required to determine the "best" value for the velocity v_n of outer wheel-rim (i.e., inducing the highest efficiency); and the proper height, e, between crowns that the whole available rate of flow, Q, may be used. We shall find that in the relations to be written out nine unknown quantities are involved, viz., v_1, v_n, w_1, w_n, e, c_1, c_n, p_1, and p_n, and it is evident that for a complete solution nine independent and simultaneous equations will be needed. For the present all friction will be disregarded and the simple design already

shown in Figs. 45 to 49 inclusive will be the one treated. We suppose the cylindrical gate raised to its full height ("full gate").

The necessary equations are the following:

From the parallelogram of velocities at entrance or point 1:

$$c_1^2 = w_1^2 + v_1^2 - 2w_1v_1 \cos \alpha. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Similarly, from the parallelogram of velocities at exit, or N,

$$w_n^2 = c_n^2 + v_n^2 - 2c_nv_n \cos \delta. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thirdly, applying Bernoulli's Theorem for a *stationary* rigid pipe and steady flow of water (see p. 654, M. of E.) to the surface of the head-water, as up-stream position, and the point 1, where the water leaves the guides, as down-stream position, we have

$$\frac{p_1}{\gamma} + \frac{w_1^2}{2g} = b + h \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(where b is the height of the water barometer).

In its progress through a wheel-channel from 1 to N the water is flowing with steady flow through a closed pipe rotating uniformly in a horizontal plane and we may therefore apply *Bernoulli's Theorem for Steady Flow in a (uniformly) Rotating Casing* to this part of the path of the water; hence (see eq. (13), § 41)

$$\frac{c_n^2}{2g} + \frac{p_n}{\gamma} = \frac{c_1^2}{2g} + \frac{p_1}{\gamma} + \frac{(v_n^2 - v_1^2)}{2g}. \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Since the kinetic energy carried away per second by the water at exit is $\frac{Q\gamma}{g} \cdot \frac{w_n^2}{2}$, and this may be made small by making $v_n = c_n$ in the parallelogram of velocities at N (in connection with a small value for the angle δ) (see also § 53), we shall write

$$v_n = c_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The aggregate sectional area, F_n, of the wheel-passages at exit may be expressed thus: The area of cross-section of any one channel, taken at right angles to the vane, at exit, is (see Fig. 47) $F' = e \times \overline{nd}$; but $\overline{nd}$ is $= \overline{nn'} \cdot \sin \delta$, and hence $F' = \overline{nn'} \cdot e \cdot \sin \delta$; but the sum of all the short linear arcs like $\overline{nn'}$

making up the entire outer periphery of the turbine (if we neglect the thickness of the vanes) is $2\pi r_n$, whence it readily follows that $F_n = 2\pi r_n \sin\delta \times e$. Similarly, we have $F_0 = 2\pi r_1 \sin\alpha . \times e$ But $Q = F_n c_n$, and also $= F_0 w_1$; therefore we have

$$[2\pi r_1 e \cdot \sin\alpha] w_1 = [2\pi r_n e \cdot \sin\delta] c_n, \quad . \quad . \quad . \quad . \quad (6)$$

as also

$$Q = [2\pi r_n e \cdot \sin\delta] c_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Since $v_1 = \omega r_1$ and $v_n = \omega r_n$ (ω being the angular velocity of wheel), it follows that

$$v_1 \div v_n = r_1 \div r_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Also (see below) $$p_n = \gamma h_n + p_a. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

66. Combination of Foregoing Equations.—Since the water is supposed to leave the wheel at N in parallel filaments, the outer of these filaments being subjected to the hydrostatic pressure $\gamma h_n + p_a$ (where p_a is the pressure of the atmosphere) from the surrounding still water in the receiving pool or tail-water, the internal pressure of the water at this place may be taken as $p_n = \gamma h_n + p_a$; i.e., $\frac{p_n}{\gamma} = h_n + b$ (where $b = \frac{p_a}{\gamma}$ is the height of the water barometer). This value being substituted in eq. (4), as also the value of $\frac{p_1}{\gamma}$ obtained from (3), eq. (4) becomes

$$\frac{c_n^2}{2g} - \frac{c_1^2}{2g} = \frac{v_n^2 - v_1^2}{2g} + h_1 - h_n + b - b - \frac{w_1^2}{2g}. \quad . \quad . \quad (10)$$

This last equation, on substitution of the value of c_1^2 from (1), reduces to

$$2w_1 v_1 \cos\alpha + c_n^2 - v_n^2 = 2g(h_1 - h_n). \quad . \quad . \quad . \quad (11)$$

But $h_1 - h_n = h$; and, from (5), $c_n = v_n$, so that (11) becomes

$$w_1 v_1 \cos\alpha = gh. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Now, from eq. (6), $w_1 = (c_n r_n \sin\delta) \div (r_1 \sin\alpha)$; and, from ~~(7)~~ 8, $v_1 = r_1 v_n \div r_n$; also $c_n = v_n$ from (5); therefore (12) becomes

$$\textit{Velocity of outer rim for a max. efficiency} = v_n = \sqrt{\frac{gh\tan\alpha}{\sin\delta}}. \quad (13)$$

As will be seen later, this value may be reduced by 8 per cent. of itself to allow for friction, and the resulting reduced value used in eq. (6) for the determination of w_1, the relation $c_n = v_n$ being practically independent of the consideration of friction. We are then in a position to find the angle β at entrance, the angle α being given and the values of w_1 and v_1, $=(r_1 \div r_n)v_n$, being now available.

This angle β determines the position which the vane-tangent at entrance should have to avoid impact or "shock" at that point; i.e., the vane-tangent at 1 should follow the direction of the relative velocity c_n. The vane-tangent at exit, N, must make the given angle δ with a tangent to the outer wheel-circumference at that point. The form of curve to be given to the vane between points 1 and N is theoretically immaterial, so long as the curvature is smooth. Two circular arcs may be used, the radius of the part near 1 being about one half of that of the other part. To a guide-blade is generally given the form of a single circular arc.

67. Shorter Proof of Foregoing Eq. (12). (See Figs. 46–49 inclusive.)—There being no loss of energy considered to take place between the surface, H, of head-water and the entrance, 1, of the wheel-channels, and also no loss due to friction in those passages themselves, the difference between the aggregate energy (of the weight $Q\gamma$ flowing per second) of the three kinds (see § 9) at that upper surface and that at the point, N, of exit from the wheel-channels, should represent the power, L, (ft.-lbs. per sec.,) exerted by the water on the turbine.

The horizontal plane through N will be taken at datum-plane for the potential energy. At H the weight $Q\gamma$ has $Q\gamma h_1$ ft.-lbs. of potential energy, zero of kinetic energy, and $Q\gamma b$ of pressure energy; while at N its potential energy is zero, kinetic energy $\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}$, pressure energy $=Q\gamma\cdot\frac{p_n}{\gamma}$, $=Q\gamma(b+h_n)$, $=Q\gamma(b+h_1-h)$. Subtracting the sum of the latter three items from that of the former three, we have

$$L = Q\gamma h - \frac{Q\gamma}{g}\cdot\frac{w_n^2}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

ft.-lbs. per second; and this should be equal to the work done per second by the "equivalent couple" (see eq. (7), § 34) on the turbine, an expression for which work per second we have in the "angular momentum" equation, eq. (10) of § 34, viz.,

$$L=\frac{Qr}{g}(u_1v_1-u_nv_n) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

(and this relation holds true, also, when friction is considered).

But u_1, the projection of w_1 on the inner wheel-tangent, is $w_1 \cos\alpha$; and similarly, at the outer rim, $u_n=w_n \cos\mu$. Hence (15) becomes

$$L=\frac{Qr}{g}(w_1v_1 \cos\alpha-[w_n \cos\mu]v_n). \quad . \quad . \quad . \qquad (16)$$

The right-hand members of eqs. (14) and (16) being equated, there results

$$w_1v_1 \cos\alpha-(w_n \cos\mu)v_n=gh-\frac{w_n^2}{2}. \quad . \quad . \quad . \quad . \qquad (17)$$

Let now the parallelogram of velocities at the exit-point N be reproduced in Fig. 50 (the direction of rotation (clockwise) of the turbine is contrary to that of previous figures). The condition that $c_n=v_n$ for best effect has been introduced into this figure by making it a rhombus, with side DN equal to side NE. The diagonals bisect each other *at right angles*, and BN represents $\frac{1}{2}w_n$. Hence the intersection, B, of the diagonals, lies in the circumference of the semicircle described on NE (or v_n) as a diameter. Hence, if BO be drawn perpendicular to NE, BN is a mean proportional between NO and NE; or $\overline{BN}^2=\overline{NO}\cdot\overline{NE}$; i.e.,

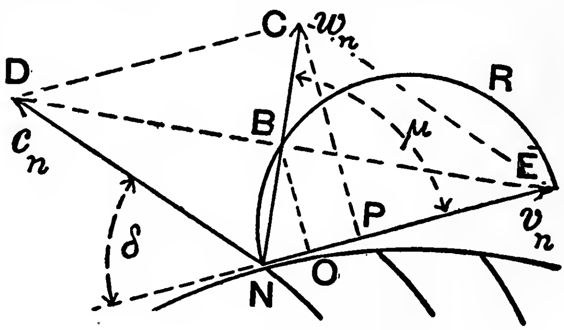


FIG. 50.

$$\frac{w_n^2}{4}=\left(\frac{w_n \cos\mu}{2}\right)v_n; \quad \text{or,} \quad (w_n \cos\mu)v_n=\frac{w_n^2}{2}. \quad . \quad . \qquad (18)$$

Substituting from (18) in (17) we obtain

$$w_1v_1 \cos\alpha=gh, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

as holding good when the turbine (frictionless) is running with speed of maximum efficiency, the same as eq. (12).

68. Note.—It is to be noted that this same demonstration for eq. (19), with same result, will hold good whatever the positions of the planes of the parallelograms of velocities at points 1 and N, entrance and exit, of the turbine; since the projections u_1 and u_n would always be in the same lines as the wheel-rim velocities, v_1 and v_n, respectively. Eq. (19) holds, therefore, for *all kinds of turbines*.

69. Theoretical Efficiency of the Fourneyron Turbine.—It is evident that the value of h_n or depth of the wheel below the surface of the tail-water is immaterial, since h_n is offset by an equal portion of the height h_1; hence we may formulate the power transferred to the wheel by the water (on the present basis; friction disregarded; i.e., no loss of head either in the penstock between surface of head-water and entrance of turbine, nor in the turbine itself) by supposing h_n to be zero. That is, this power, L, (ft.-lbs. per second,) equals the whole theoretic power of the mill-site $Q\gamma h$ less the kinetic energy carried away per second by the water leaving the wheel at N, or

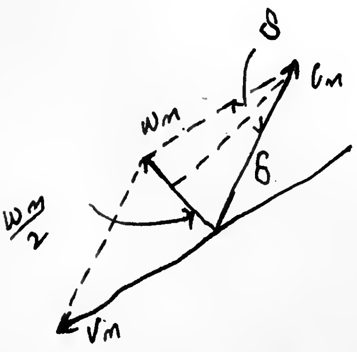
$$L, =R'v', =Q\gamma h-\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}. \quad . \quad . \quad . \quad . \quad (18a)$$

Since the condition that $c_n=v_n$ makes the parallelogram of velocities at N consist of two isosceles triangles (see also Fig. 50) we have $w_n=2v_n \sin\frac{\delta}{2}$, in which if the value of v_n for best effect as derived in eq. (13) be substituted, and the result so obtained for w_n placed in (18a), we have

$$L, =R'v', =Q\gamma h\left[1-\frac{2\tan\alpha\sin^2\frac{\delta}{2}}{\sin\delta}\right]. \quad . \quad . \quad (19a)$$

In this case the efficiency, η, $=R'v'\div Q\gamma h$, whence

$$\eta=1-\frac{2\tan\alpha\sin^2\frac{\delta}{2}}{\sin\delta}. \quad . \quad . \quad . \quad . \quad . \quad (20)$$



$\frac{1}{2} w_n \div c_n = \sin\frac{\delta}{2}$

$\frac{w_n}{2} = c_n \sin\frac{\delta}{2}$ $\qquad c_n = v_n$

$w_n = 2 v_n \sin\frac{\delta}{2}$

From the details of this expression we gather that the smaller the angles α and δ can be made, the greater the efficiency. In practice α is taken from 20° to 30°; and δ from 15° to 20°.

With the values of $\alpha=25°$ and $\delta=15°$ we obtain $\eta=0.92$ from eq. (20); but in actual practice this figure is reduced to 80 per cent. or less (unless in exceptional cases) on account of fluid friction, axle friction, and imperfect guidance of the water. 75 per cent. is a fairly good performance.

70. Numerical Example. Fourneyron Turbine.—Given $h=$ 60 ft. and the available water-supply $Q=150$ cub. ft. per sec., and assuming radii of $r_1=2$ and $r_n=2.5$ ft.; with angles α and δ, 20° and 15°, respectively; it is required to design a Fourneyron turbine having parallel crowns, etc., as in Fig. 46; i.e., to find the proper value of the outer-rim velocity, v_n, for best effect, that of the angle β for the vane tangent at entrance and the proper distance, e, between crowns, that all the water available may be used (at full gate).

Up to this point the effect of fluid friction has not been represented in any of the formulæ, but a fair allowance for it may be made (see § 71) by deducting 8 per cent. of itself from the value of v_n for best effect, as given by eq. (13) in § 66; i.e., with $\tan 20°=0.364$ and $\sin 15°=0.259$, we have

$$v_n=0.92\sqrt{\frac{32.2\times 60\times 0.364}{0.259}}=48 \text{ ft. p. sec.}$$

With $r_n=2.5$ ft., this means that the wheel should be run at an angular velocity of $\omega=\frac{48}{2.5}=19.2$ radians per sec., or at

$$(19.2\div 2\pi)\times 60=183 \text{ revs. per minute.}$$

(Should it be wished to run the wheel at a different *angular* velocity, a different value of the radius r_n could be selected, so long as the value of the *linear* velocity v_n of the outer rim is kept unchanged.)

Since $c_n=v_n$ we have, from eq. (6),

$$w_1(2\pi r_1 e \sin\alpha)=v_n(2\pi r_n e \sin\delta)$$

(which holds good whether friction be considered or not); and

hence for the absolute velocity of the water leaving the guides

$$w_1=\frac{v_n r_n}{r_1}\cdot\frac{\sin\delta}{\sin\alpha}=\frac{48\times2.5}{2.0}\cdot\frac{0.259}{0.342}=45.4 \text{ ft. p. sec.};$$

also,

$$v_1=(r_1 v_n)\div r_n=(2\div2.5)48=38.4 \text{ ft. p. sec.}$$

To determine the vane-tangent angle, β, at point 1, i.e., the position of the relative velocity c_1, v_1 and w_1 being now known and angle α being given, we have only to solve the triangle ABC in the parallelogram of velocities concerned; see Fig. 51. Here we have two sides (w_1, v_1) and the included angle (α); the other two angles being ζ and θ, ($\theta=180°-\beta$.) Hence

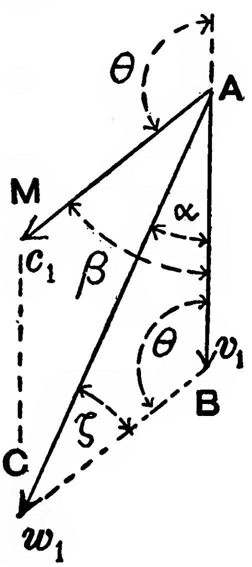


FIG. 51.

$$\tan\tfrac{1}{2}(\theta-\zeta)=\frac{(w_1-v_1)\tan\frac{1}{2}(\theta+\zeta)}{w_1+v_1}$$

$$=\frac{7\times\tan 80°}{83.8}=0.473;$$

$$\therefore\ \tfrac{1}{2}(\theta-\zeta)=25°\ 19'.$$

Hence

$$\theta,=\cancel{\tfrac{1}{2}}(80°+[25°\ 19'])=105°\ 19';$$

and $\beta,=180°-\theta,=74°\ 41'$.

(N.B. Another method of solving the triangle and finding β is illustrated in § 94 and Fig. 77.)

To find e, the distance between crowns, i.e., the common height of all parts of all wheel-passages (at full gate), we have from eq. (7) $Q=2\pi r_n e(\sin\delta)c_n$, and again write v_n for c_n and obtain $e=150\div(2\pi\times2.5\times0.259\times48)=0.768$ feet.

As no account has been taken of the thickness of the guides or vanes, this value for e would need to be increased somewhat, perhaps 10 per cent. in some cases (see § 91 for further details on this point).

As to the horse-power to be expected from the turbine when run at the proper speed deduced above (183 revs. per minute), assuming an efficiency of 75 per cent. (not an extravagant figure), we have for the useful power

$$R'v' = 0.75Q\gamma h = 0.75 \times 150 \times 62.5 \times 60,$$
$$= 421{,}500 \text{ ft.-lbs. per sec.}; = 766 \text{ H.P.};$$

equivalent to the continuous raising (vertically) of a weight of R', $=42{,}150$ lbs., at a uniform speed of $v'=10$ ft. per sec.

71. Theory of the Fourneyron Turbine, when Friction is Considered.—Let us now consider that in the steady flow between the head-water surface H and the outlet, A, of the guides (Fig. 45) a loss of head occurs of the form $\zeta_0 \frac{w_1^2}{2g}$, and introduce it into eq. (3) of § 65; and furthermore that another loss of head occurs in the wheel-channels, between entrance A and exit N, of an amount $\zeta_n \frac{c_n^2}{2g}$ (i.e., proportional to the square of the relative velocity at exit) to be placed in eq. (4) of § 65. These two losses of head are the h' and h'', respectively, of §§ 40, 41, and 42; ζ_0 and ζ_n are abstract numbers (coefficients of resistance; see p. 704, M. of E.). Adopting, as before, the relation that for best effect v_n should be placed equal to c_n, and combining the forms now assumed by eqs. (3) and (4) with the other equations of § 65 (which remain unchanged in form), we finally obtain

$$v_n = (\sqrt{2gh}) \div \left(\sqrt{\frac{2 \sin \delta}{\tan \alpha} + \zeta_0 \left(\frac{r_n}{r_1} \cdot \frac{\sin \delta}{\sin \alpha}\right)^2 + \zeta_n}\right) \quad . \quad (21)$$

as the value of v_n for best effect; that is,

$$v_n = \left(\sqrt{\frac{gh \tan \alpha}{\sin \delta}}\right) \div \left(\sqrt{1 + \frac{\zeta_0}{2} \cdot \frac{r_n^2}{r_1^2} \frac{\sin \delta}{\sin \alpha \cos \alpha} + \frac{\zeta_n \tan \alpha}{2 \sin \delta}}\right). \quad (22)$$

According to Weisbach, a value of 0.05 to 0.10 may be taken for each of the coefficients ζ_0 and ζ_n. If the larger value, 0.10, be taken and substituted in eq. (22), with ordinary values of the ratio $r_n : r_1$, and the angles α and δ, there results

$$v_n = 0.92 \left(\sqrt{\frac{gh \tan \alpha}{\sin \delta}}\right); \quad . \quad . \quad . \quad . \quad . \quad (23)$$

which explains the 8 per cent. reduction, as an allowance for friction, mentioned in the foregoing paragraphs.

The revolving wheel encounters friction from the adjoining tail-water and also at its own axle. These various frictions and the fact of leakage of water through the space between the edges of the wheel-crowns and fixed guides render any refined analysis out of the question. Only approximate results can be reached, short of actual test.

72. Efficiency of the Fourneyron Turbine. Friction Considered.—If we deduct the losses of head just mentioned from the whole head h (Fig. 45), and also the velocity head due to the absolute velocity w_n of the water at exit, we have, for the net power (ft.-lbs. per sec.),

$$R'v'=Q\gamma\left[h-\frac{w_n^2}{2g}-\zeta_0\frac{w_1^2}{2g}-\zeta_n\frac{c_n^2}{2g}\right]; \quad . \quad . \quad . \quad (24)$$

and therefore, for the efficiency,

$$\eta=\frac{R'v'}{Q\gamma h}=\left[h-\frac{w_n^2}{2g}-\zeta_0\frac{w_1^2}{2g}-\zeta_n\frac{c_n^2}{2g}\right]\div h. \quad . \quad . \quad (25)$$

For example, if we substitute in this equation the values occurring in the last numerical problem (§ 70), viz., $h=60$ ft.; $w_1=45.4$, $w_n=2v_n \sin\frac{\delta}{2}=12.5$, and $c_n=v_n=48$, ft. per sec.; with 0.10 for both ζ_0 and ζ_n; we obtain

$$\eta=\frac{60-2.5-3.2-3.6}{60}=\frac{50.7}{60}=0.84,$$

or 84 per cent. But the power lost in axle friction ($R''v''$) and that spent on the resistance of tail-water on the outside surfaces of the crowns, etc., would probably reduce this to some 78 or even 75 per cent. (See §§ 99, etc., as to actual tests of turbines.)

73. Note.—Evidently the bracket in eq. (24) represents the work done per sec. for each unit of weight of water used; thus, in the numerical instance above, from each pound of water are derived 50.7 ft.-lbs. of work per sec., out of the total theoretical 60 ft.-lbs. for each pound of water. Of the total loss (9.3), 2.5 ft.-lbs. per sec. is due to residual kinetic energy

at exit, 3.2 to fluid friction in the penstock or wheel-pit, and 3.6 to fluid friction in the wheel-channels.

74. Fourneyron Turbines at Niagara Falls, N. Y.—During the years 1894 to 1903 the Niagara Falls Power Co. constructed a water-power "installation" about a mile above the falls at Niagara Falls, N. Y., involving two power-houses containing twenty-one turbines, and a tunnel (as a tail-race) some 6700 feet in length and 490 sq. ft. in sectional area, on a grade of 7 ft. per thousand, and at a depth at its upper end of some 146 ft. below the level of the upper river. The tunnel is of a horseshoe form in section, is lined with hard brick, and empties at the base of the cliff a short distance below the American Fall. The velocity of the water in it is sometimes as great as 25 ft. per second.

In "Power House No. 1" each of ten vertical shafts carries two Fourneyron turbines, each such (double) wheel or "unit" furnishing 5000 H.P. and working under a mean head of 136 ft., at 250 revolutions per minute, and using about 440 cub. ft. of water per second. The wheel-pit under the power house is an immense slot excavated in the rock, the lower part discharging the water after its passage through the turbines into the upper end of the tunnel. Each of these double wheels has a separate penstock into which water is admitted from a wide canal, leading out of the upper river. These turbines were built and installed by the I. P. Morris Co., of Philadelphia; from designs by Faesch and Piccard of Geneva, Switzerland.

Fig. 52 gives a side view, or elevation, of one of these penstocks with its corresponding wheel-casing, shaft, etc. The steel penstock, *P*, is 7.5 ft. in diameter, conducting water under pressure to the wheel-casing, *e*. At the upper and lower extremities of this casing revolve the two wheels, the discharge from which issues at *a* from the upper, and at *T* from the lower, turbine. *T* shows also the level of the tail-water at the bottom of the wheel-pit. The height of its surface is variable, depending on the number of turbines in action at any time. Although each turbine works in a position above the tail-water, discharging into the atmosphere, its design is

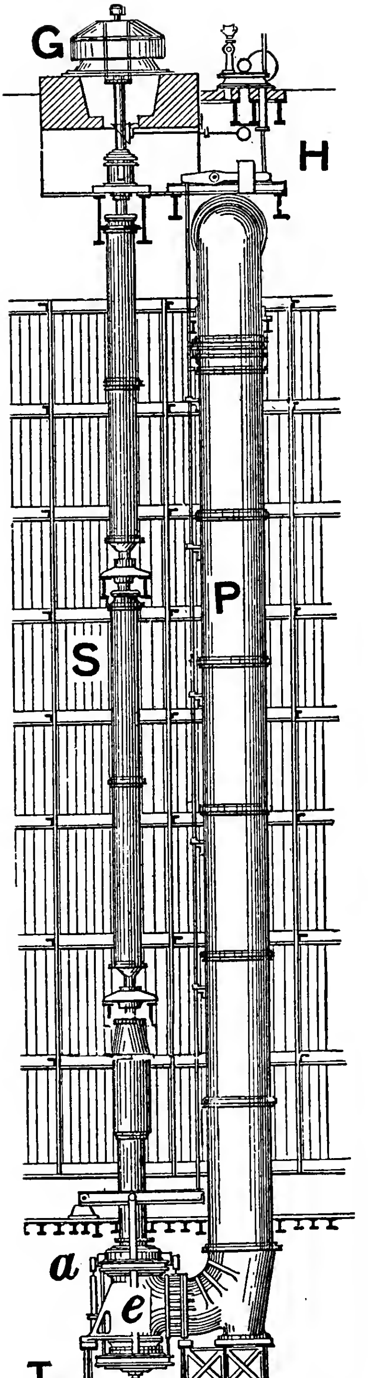


FIG 52.

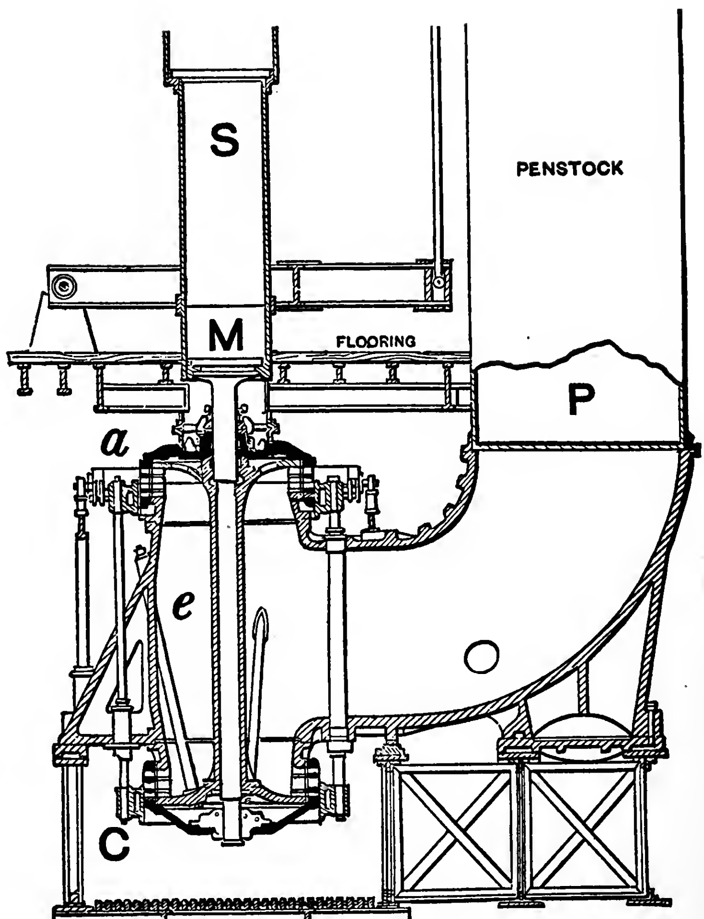


FIG 53

such that all the channelways are completely filled with water, so that it operates as a "reaction turbine" and not as an impulse, or "tangential," wheel.

S is the shaft, which for strength and lightness combined is mainly hollow, consisting of segments of steel tubing 38 inches in diameter, connected to each other at intervals by short solid portions, 11 in. in diameter, running in bearings. These bearings, however, provide only lateral support. On the upper end of the shaft is fixed the re olving part, G, of an electric generator, which has sufficient mass, with that of the two turbines themselves, to serve as a fly-wheel.

In Fig. 53 is given a section, on a larger scale, of the lower end of the penstock and of the wheel-casing and turbines. Although the velocity of the water in the penstock is about 10 ft. per second, the fluid pressure in the casing e differs but slightly from the hydrostatic pressure due to the whole head of 136 ft. The two turbines, and their supporting shells extending out from the shaft, are indicated by solid black shading (better shown in a subsequent figure). Rigidly attached to the shaft S is a disc M, the space underneath which is in communication with the water of the penstock, while the upper face is open to the atmosphere. The lifting effort thus exerted on the shaft serves to sustain the greater part of the weight of the shaft, turbines, and generator. In other words, the friction of a solid disc on a *liquid* is substituted for that of a journal, or pivot, in a solid bearing; a gain both in convenience and power. The excess (or deficiency) of this hydrostatic pressure is taken up by a special thrust-bearing at the upper end of the shaft.

The lower turbine of one of these double wheels (or "units") is shown in vertical section in Fig. 54, where the solid black shading indicates the revolving part, or turbine ("runner") itself. Between the crown-plates E and D are placed two horizontal partitions, thus practically dividing the turbine into three separate turbines (see Fig. 46 in this connection). Corresponding partitions are also placed at G between the guides. The extreme outside diameter of the turbine is 6 ft.

2 in., and the inner diameter of the crowns is 5 ft. 3 in. The vertical distance, e, between crowns is about 12 inches. A portion of the turbine and guides is shown in horizontal section in Fig. 55, where it is seen that the middle portions of the wheel-vanes are thickened, and in such a way as to secure more gradual changes of cross-section in the wheel-passages than would otherwise be the case.

The regulating-gate is a vertical cylinder placed outside of the turbine. It is shown in horizontal section in Fig. 55; and in vertical section, at C, in Fig. 54, in which latter figure the gate is entirely closed. A downward motion of the rods R, R, is required to open it. The corresponding gate of the companion turbine at the upper part of the casing (at a in Fig. 53) is moved simultaneously by the same rods. In this way one or more of the spaces between the horizontal partitions of each turbine is opened for the action of the water. Though this method of regulation is usually accompanied by a low efficiency at "part gate," the effect is here much improved by the presence of the horizontal partitions. The great hydrostatic pressure on the stationary disc m (Fig. 54) forming the floor of the wheel-casing is sustained by the rods K, K, (see also Fig. 55,) whose upper ends are fastened to the sides of the casing. Each turbine contains 32 vanes (or "buckets"), while the number of guides is 36. The angles α, β, and δ in these wheels have values of about 20°, 110°, and 13°, respectively.

Tests of one of these double turbines have shown an efficiency as high as 82 per cent., the useful power being measured electrically; and the consumption of water determined by current-meters held at the entrance of the penstock.

All of the ten (double) turbines in Power House No. 1, each of 5000 H.P., are situated in a common wheel-pit and deliver their water into a common tail-race which empties into the upper end of the great tunnel. Each is regulated to a fairly constant speed by a governor of special design, any slight change of speed affecting the angular position of the centrifugal "fly-balls." With any increase of speed from

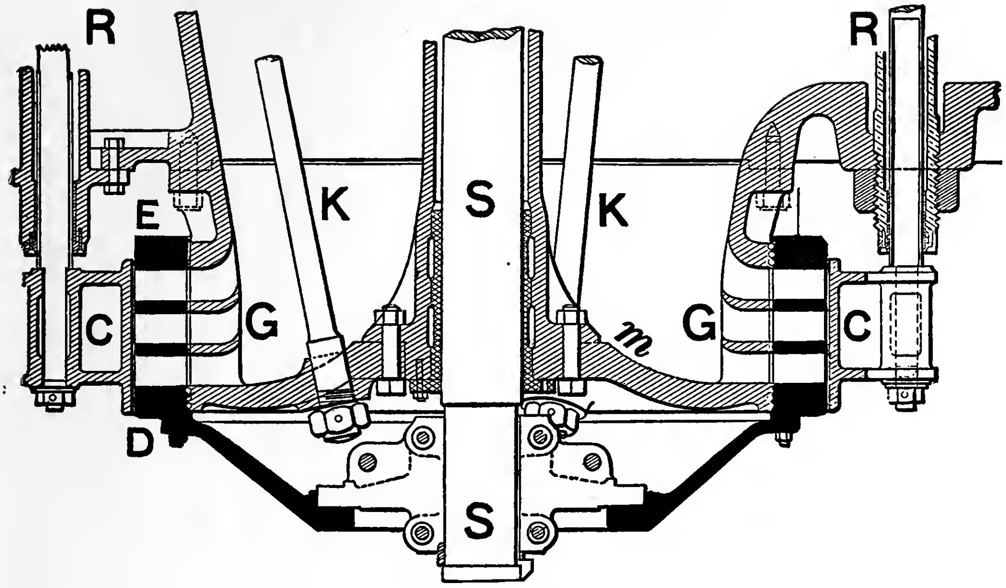


FIG. 54.

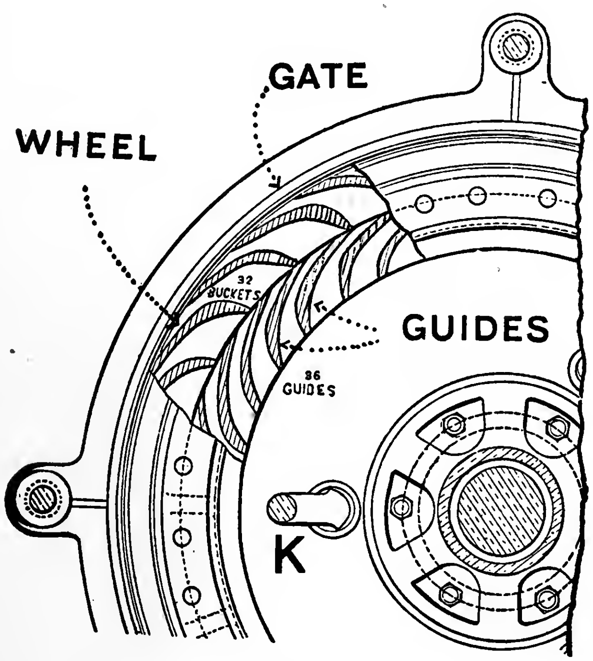


FIG. 55.

the normal the gate mechanism is thrown into gear with the turbine itself and the gate is partially closed until the speed returns to its normal value; and *vice versa*. In this way the speed does not vary more than 3 or 4 per cent. from the normal, even when as much as 25 per cent. of the "load" (R'), or resistance, is suddenly removed. (In Power House No. 2, of more recent construction, the turbines are of another type; see § 78.)

The turbines just described are made chiefly of cast iron, with some smaller parts of steel.

75. The Fall River Turbine.—The Fourneyron turbine, made at Fall River, Mass., by Kilburn, Lincoln, and Co., is shown in Figs. 17, 18, and 19 (opp. p. 42). The nest of guides fits within the inner hollow of the wheel, or "runner," while the cylindrical gate is movable vertically between. Fig. 56 gives a view of the exterior of wheel-case, etc. The penstock is attached at P. The turning of the small shaft H, by means of intervening screw-gearing, causes motion of the four vertical rods to which the gate G is attached. At T is seen the turbine itself. By bevel-gearing the turbine shaft communicates motion, at E, to the horizontal shaft S, for the driving of machinery, etc. One of these wheels was tested in 1870 by Mr. Clemens Herschel,

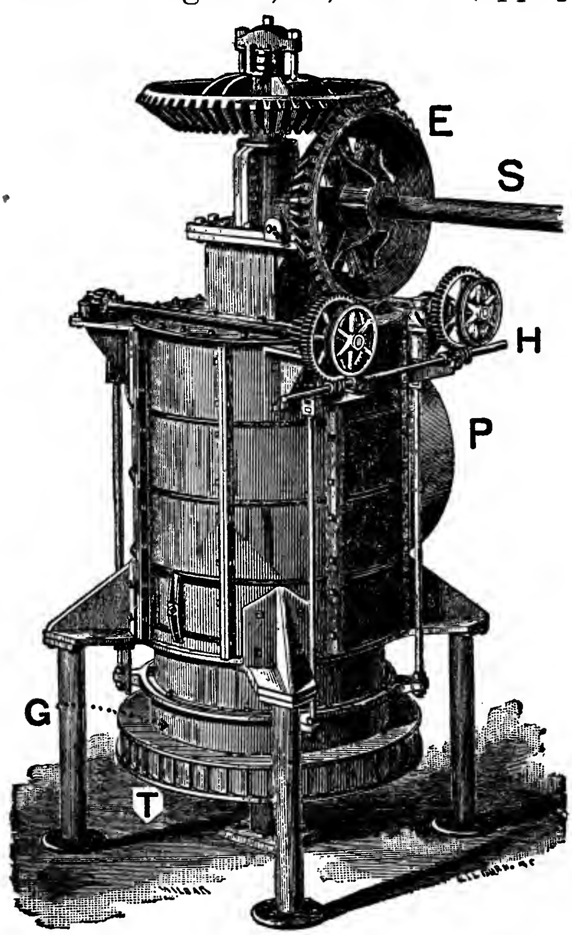


FIG. 56.

and gave an efficiency of practically 80 per cent. under a head of $h=19.6$ ft.; developing 130.3 H.P. at its "best speed" of 92.5 revs. per min., and using $Q=58.6$ cub. ft. of water per sec. The diameter of the turbine was 5 ft. 8 in., and height of wheel-passages $e=6.4$ in. These wheels are made with either iron or bronze buckets.

76. Classification of Turbines.—A general definition of a turbine may be thus stated, viz.: A water motor consisting of a number of short curved pipes set in a ring attached rigidly to a shaft upon which it revolves, and receiving water at all parts of its circumference from the mouths of other and fixed pipes or passageways; the cross-section of all of these curved pipes being completely filled with water during steady operation. The principal types of turbines are as follows:

I. Radial, Outward-flow, Turbines; in the working of which the general course of the water lies in a plane at right angles to the axis or shaft and is directed outward, away from the axis of rotation. (The Fourneyron turbine just treated is of this type). In this case the guide blades serving to form the fixed passageways are placed *within* the turbine and deliver water to the turbine channels at the *inner* edge of the turbine-ring.

II. Radial, Inward-flow, Turbines; in which the fixed guide-passages are situated on the *outside* of the turbine-ring, the general course (absolute path) of the water in the turbine channels lying in a plane perpendicular to the axis but directed radially *inward.* These are called Francis, or "center-vent," wheels.

III. Axial Flow, or Parallel Flow, Turbines; in which the absolute path of a particle of water lies substantially in the surface of a cylinder whose axis coincides with that of the turbine; that is, all points of this path are practically equidistant from the axis of rotation. (The "*Jonval*" Turbine.)

IV. Mixed Type, or Mixed Flow.—In case the water enters the turbine channels from the outside, having at first a radial and inward direction of motion, and is later so diverted as to leave those channels in a direction parallel to the axis, the

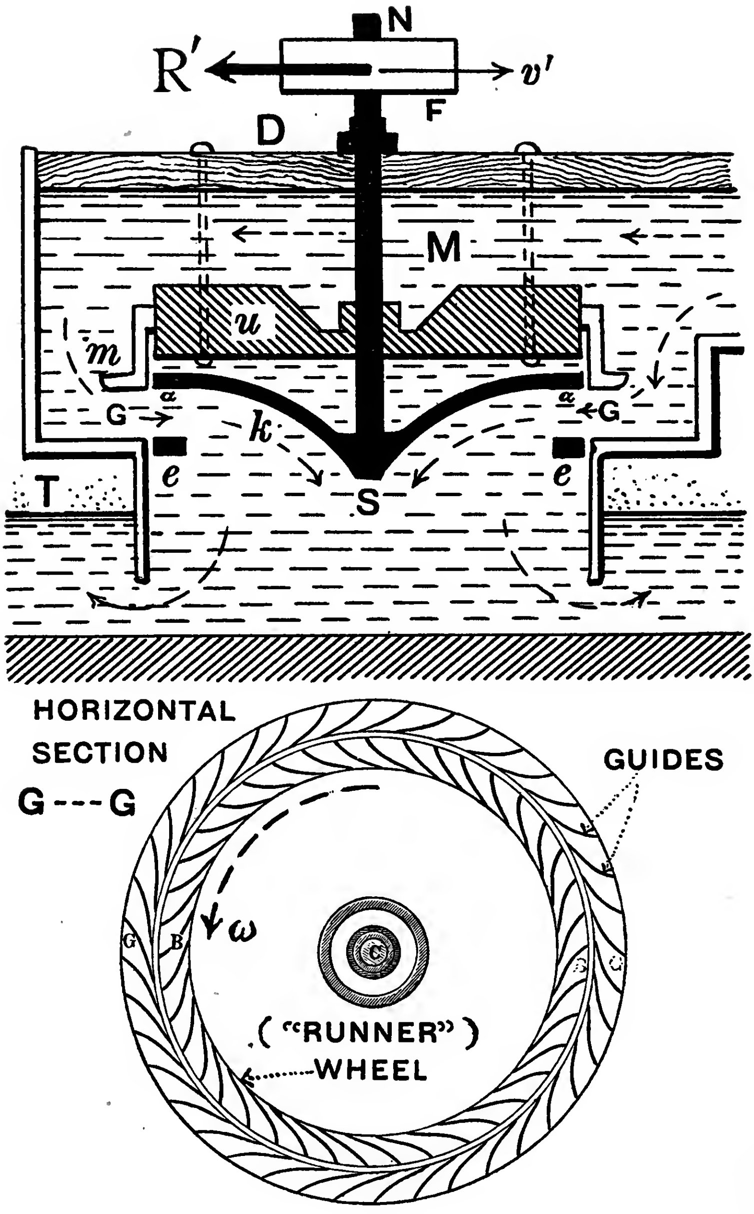

N
R′
v′
F
D
M
u
m
G
a
k
e
S
T
HORIZONTAL
SECTION
G---G
GUIDES
ω
G
B
C
("RUNNER")
WHEEL

FIG. 57.

turbine is said to be one of Mixed Type. Most American turbines belong to this type of wheel. (*"Inward and downward discharge."*)

77. Radial, Inward-flow, Turbine. (The *Francis* Wheel.)—A simple arrangement of this type of turbine is shown in Fig. 57. in the upper part of which is a vertical section of the wheel, shaft, casing, etc.; while below is a horizontal section taken through a point half-way between the crowns of turbine. The section of the turbine crowns, shaft, and supporting shell, k, are shaded in solid black. The fixed guides are placed in the space G, on the outside of the turbine, while the curved vanes of the turbine are situated between, and unite, the two crowns a and e. After passing through turbine-ring the water finds its way through the vertical tube eSe, and finally joins the tail-water at T. At the upper end of the shaft, which protrudes through the upper floor, D, of the water-tight wheel-casing M, is keyed a pulley, F, at whose circumference a resistance, R' lbs., is overcome at a velocity v' ft. per sec. By a downward movement of the ring m the sectional areas between the guides may be reduced; when less water is to be used. The horizontal plate u, supported by rods from above, serves to protect the revolving plate k of the wheel from the high pressure in the space M, where the water is slowly travelling toward the guide-openings at G. The surface of the head-water is not shown, being at an elevation above the upper floor D (of the casing), which is therefore subjected to considerable hydrostatic pressure from the water in space M underneath.

The theory of the inward-flow turbine does not differ essentially from that of the outward-flow type already given (see § 89, etc., where a general theory for all turbines will be given). It will be sufficient for the moment (see Fig. 58) to note the parallelograms of velocities at entrance (point 1), and at exit (point N) in the inner circumference of wheel. The same notation is used as in the case of the outward-flow turbine; that is, the subscript 1 refers to the point of entrance and N to that of exit. 1 . . . N is the absolute path of the

water in passing through the wheel-ring, and for best effect, after the "best" value of the exit wheel-rim velocity v_n has been determined, and the corresponding value of the absolute velocity w_1 at point 1 of leaving the guides, the tangent to wheel-vane at 1 must be so placed as to coincide with the position of the relative velocity c_1 as determined by the values of v_1 and w_1 already found. There will then be no sudden change of direction in the absolute path of the water at the point 1, at the entrance of the wheel-channels, and hence no "shock" and accompanying loss of energy.

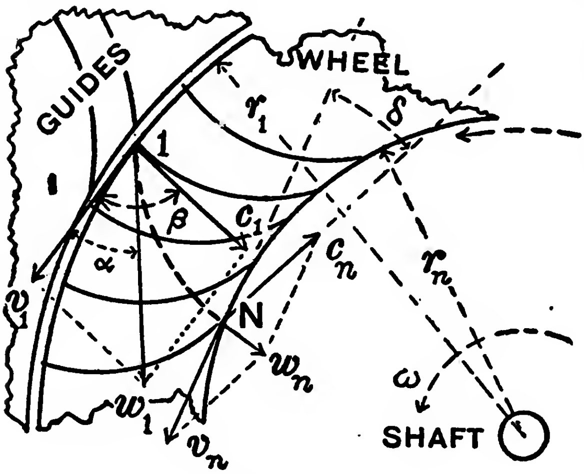


FIG. 58.

78. Francis Turbines at Niagara Falls.*—In their "Power House No. 2" the Niagara Falls Power Co. has recently installed eleven turbines of about 5500 H.P. each, substantially of the Francis type. Fig. 59 gives an end view of the wheel-pit showing the penstock, shaft, etc., of one of the turbines. S is the turbine shaft, chiefly tubular (3.28 in. in diameter; of metal $\frac{3}{8}$ in. thick), with occasional solid portions for lateral support, in bearings. Behind the shaft is seen the penstock, P, P, into which the water enters at H from the upper river. The penstock is made of steel plates $\frac{1}{2}$ in. in thickness and is 7 ft. 6 in. in diameter; and conducts the water to the turbine in the wheel-casing, E. After leaving the wheel, the water enters the upper ends of two "draft-tubes" (or "suction-tubes," as they are often called), from which it is finally discharged into the tail-water at T. These "draft-tubes" discharge *under water* that the air may not enter and thus prevent their flowing full. They act like water-barometers, except that the water

* See the Engineering Record, Nov. 1901, p. 500; also Nov. and Dec. 1903, pp. 616, 652, 691, and 763.

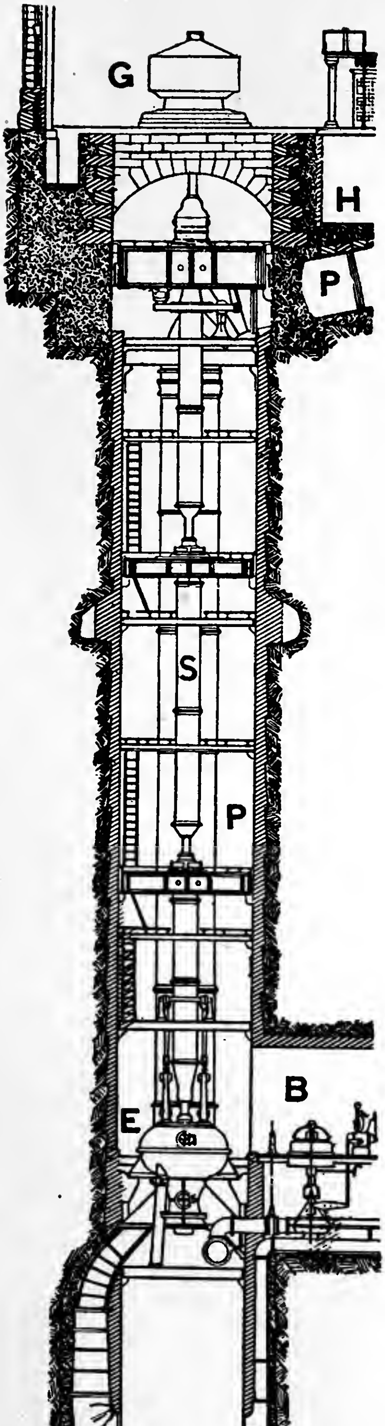


FIG. 59.

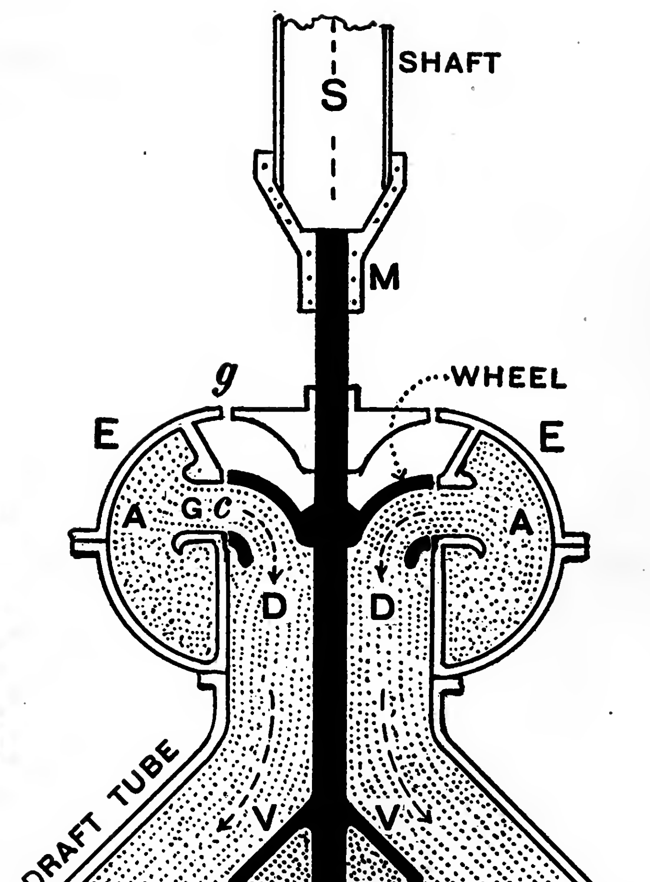


FIG. 60.

is in motion, the internal fluid pressure being less than one atmosphere at points of higher elevation than the surface of the tail-water. Their upper extremities are not more than about 20 ft. above the surface of the tail-water, so that the water continues to fill the tubes after the air has once been swept out of the tubes by the current. The draft-tubes are placed within the walls of the wheel-pit in order that they may not obstruct the flow of water from the other turbines on its way to the junction (at one end of the wheel-pit) with the great tunnel which serves as a tail-race for both power houses. By this arrangement, also, the whole head of some 146 ft. from the head-water to the surface of the tail-water is made effective.

The interior of the wheel-case and draft-tubes, etc., is shown by the vertical section of Fig. 60 (largely diagrammatic). The water from the penstock fills the annular chamber A under nearly hydrostatic pressure, passes through the guide-passages at G, and enters the wheel-channels at c under reduced pressure and with high velocity. The revolving turbine, shaft, and attachments are shown in solid black shading (except the portion, S, of the first tubular part of shaft). There are 25 guide-blades in the ring G surrounding the wheel; the blades and ring being of bronze, cast in one solid piece. The wheel itself, also of bronze and cast in one piece, contains 21 vanes or buckets in the space extending from c about half-way to D, is 5.25 ft. in diameter, and is operated at a speed of 250 revs. per min. The water leaving the turbine-channels enters the space D with both low (absolute) velocity and low pressure, the pressure being practically that corresponding* to the height of D above the tail-water surface (which is, however, variable in position). At the lower extremity of the shaft, while lateral support is provided by the bearing or step at R, a great lifting force is furnished by the admission of water under the full penstock pressure to the space U, U, on the under side of the conical shell, or piston V, V, or "balancing

* E.g., if that height were 22 ft., the pressure would be about 5 lbs. per sq. in., only.

disc" keyed upon the shaft and revolving with it. The pressure on the upper surface of this piston is small, of course, being that of the water in the upper end of the draft-tube. In this way the larger part of the weight of the wheel, shaft, and armature of the electric generator is supported by fluid pressure. The diameter of this piston or "balancing disc" is 4.9 ft. The turbine was cast, in one piece, of manganese bronze and weighs 4000 lbs. nearly. The weight of the whole revolving mass, including that of the armature of the generator, is 71 tons, to sustain which the pressure underneath the "balancing disc" provides an upward force of some 66 tons, leaving about 5 tons to be sustained by a "suspension bearing" at the upper end of the shaft.

The gate of the turbine is a cast-steel ring or cylinder moving vertically in the narrow space c (Fig. 60) between guides and wheel, and operated by rods through the space g. It is not shown in the figure. These eleven turbines were installed by the I. P. Morris Company of Philadelphia after designs of Escher, Wyss and Co. of Zurich, Switzerland. Other large Francis turbines (10,000 H.P. each) are in use by a branch company on the Canadian side at Niagara Falls.

79. Other Large Francis Turbines.—The Shawinigan 10,500-H.P. turbine was designed and constructed in 1904 by the I. P. Morris Company, and installed at Shawinigan Falls, in the Province of Quebec, Canada. It is also of the Francis inward-flow type. A view of the wheel-case and the upper segments of the draft-tubes is given in the frontispiece of this book. The penstock joins the wheel-case at the lower left-hand corner. The wheel-case is of spiral (or "volute") form, the space for the water being progressively narrowed in the circuit around the ring containing the guides. As evident from the figure the turbine revolves in a vertical plane, its shaft being horizontal. The water leaving the turbine toward the center passes into the two draft-tubes, the upper curved segment of one of which is seen in the figure. The hydraulic cylinders at the top furnish power for moving the regulating apparatus. In this design the guide-blades are

movable about their inner ends, as in the Thomson Vortex Wheel (see § 80), and by their change of position the area of cross-section of the guide-channels is varied and thereby the quantity of water per second controlled. The movable guide-vanes are operated by circular rings, and these rings by the pistons of the hydraulic cylinders. The turbine or "runner" is cast, in one piece, of an alloy of about 88 parts copper, 10 parts tin, and 2 parts zinc, and has an external diameter of 7 ft. It operates under a head of 135 ft. The I. P. Morris Co. is also building (June 1905) four turbines, each of 13,000 H.P., for a Canadian company at Niagara Falls. Each of these consists of two wheels of the Francis type mounted on one shaft and discharging into one central "draft-chest." Each "runner" is fitted with solid cast guides, with cast-steel cylinder-gates and bronze wheels, the inside diameter of the cylinder-gate being about 5 ft. 5 in. The diameter of the supply-pipe or penstock is 10 ft. 6 in., and that of the draft-tube 9 ft.

The two wheels above described are probably the largest turbine "units" that have been built, up to the present date (Sept. 1905).

In Fig. 60*a* is shown a 3000-H.P. turbine intended for a power station at Glommen, Norway; designed and constructed by Escher, Wyss and Co. of Zurich, Switzerland. The runner itself is on the right. This engraving is from a pamphlet published by the Allis-Chalmers Company, American agents for the above-mentioned Swiss firm and manufacturers of its designs. (See also Fig. 60*b*, opp. p. 124.)

80. The Thomson Vortex Wheel is also of the radial inward-flow type, and was invented by Prof. James Thomson. It is remarkable for its excellent device for regulating the flow of water and for the fact that the outer radius is made from two to four times as great as that of the inner, or discharge, circumference. Fig. 61 shows a view of one of these wheels, one-half of the upper plate of the wheel-case being removed. From the space within the outer casing the water finds its way into four gradually contracting passages, *A*, *A*, etc., leading

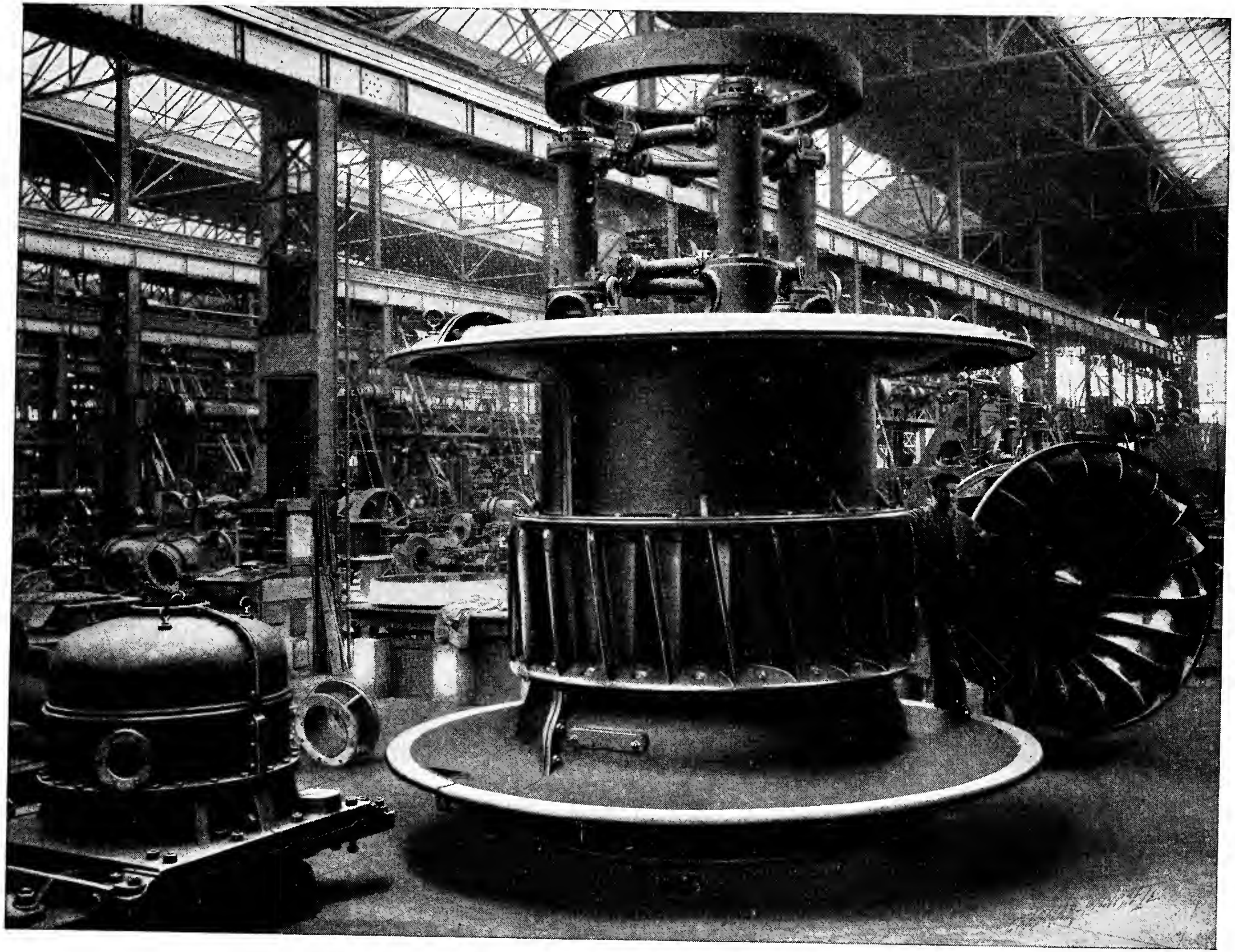


FIG. 60a. 3000 H.P. Francis Turbine for Glommen, Norway. (Escher Wyss & Co.)

to the wheel-entrances; i.e. there are only four guide-blades, like *RG*. Each of these guide-blades is pivoted at *G*, very near the extremity, so that when the blades are turned on these pivots, the water way may be diminished in sectional area; without, however, sensibly altering the general form of the stream, thus avoiding any sudden enlargement of its section at entrance of wheel with the consequent loss of energy. The water leaves the wheel at *E*.

This wheel is very efficient, and is to a certain extent self-regulating in the matter of speed; for if, through lightening of load, the speed becomes augmented, the "centrifugal action" of the water between the wheel-vanes tends to "oppose the

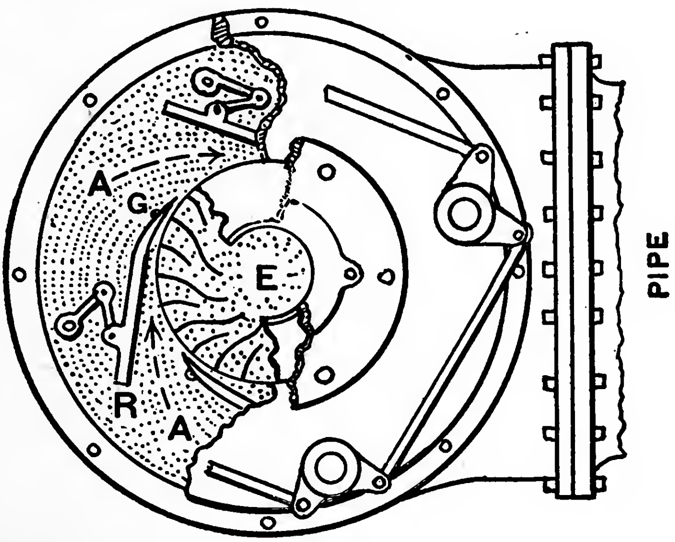


FIG. 61.

entrance of water from the supply-chamber"; and *vice versa* (from a report of Prof. Rankine on this wheel).

81. The Parallel-flow (or Axial) Turbine, usually called the *Jonval* wheel. (It is sometimes named, however, after Fontaine, Henschel, and Koecklin, according to slight differences in minor details.)—In this turbine, as in the two preceding types, fixed guides deliver the water without impact into the wheel-passages, whose vanes are curved in such a manner (in connection with a proper speed of wheel) that the final absolute velocity w_n is as small as possible; but the water passes through the wheel in cylindrical surfaces sub-

stantially parallel to the axle or shaft. Hence this type of wheel resembles somewhat a screw-propeller of numerous blades, bounded by two concentric cylindrical shells.

In Fig. 62 is shown a vertical section of the shaft, penstock, and discharge-tube (or "draft-tube," if turbine is above tail-water) of a parallel-flow turbine. The sides of the discharge-tube T, T, are in this case rigid prolongations of those of the (vertical) penstock or tube P, P. S, S, is the shaft, to which the turbine W is rigidly attached. G is the side view of the guide-box or fixed ring containing the stationary guide-channels formed by the guide-blades and two concentric cylindrical walls, BE and AC. $\overline{AB}$ $(=e_0)$ is the radial width of this ring. (In Fig. 64 the running part, wheel and shaft, is shown in wide black lines.) The mouths of the guide-channels are open all around the ring AB and deliver the water into the channels of the turbine, W, below. The turbine is itself a ring of channels receiving water above and discharging it below. In this figure the width, e_n, of the turbine ring at the point of exit is equal to that, e_0, at the point of entrance. But frequently e_n is made larger than e_0, thus producing a "bell-mouthed," or flaring, shape for the axial section of the wheel-passages.

Let now a cylindrical cutting surface, aa, having its axis in that of the shaft, be imagined to be passed through points half-way out, radially, between the vertical walls of the guide-ring; its radius is the "r" of Fig. 62.

The intersections made by this cutting surface with a few of the guide-blades and turbine-vanes (these sections being drawn in solid black lines) are shown, developed, in the middle of Fig. 62. The absolute path of a particle of water entering the guide-channel at H is $H \ldots I$, through a guide-channel; and $1 \ldots N$, through the moving wheel; whose velocity is supposed to be such, together with a proper value for the angle β, that there is no impact, or "shock," at 1, the entrance to a turbine channel; that is, that the whole absolute path $H \ldots 1 \ldots N$ (dotted) is a smooth curve, without sudden turn or elbow at point 1.

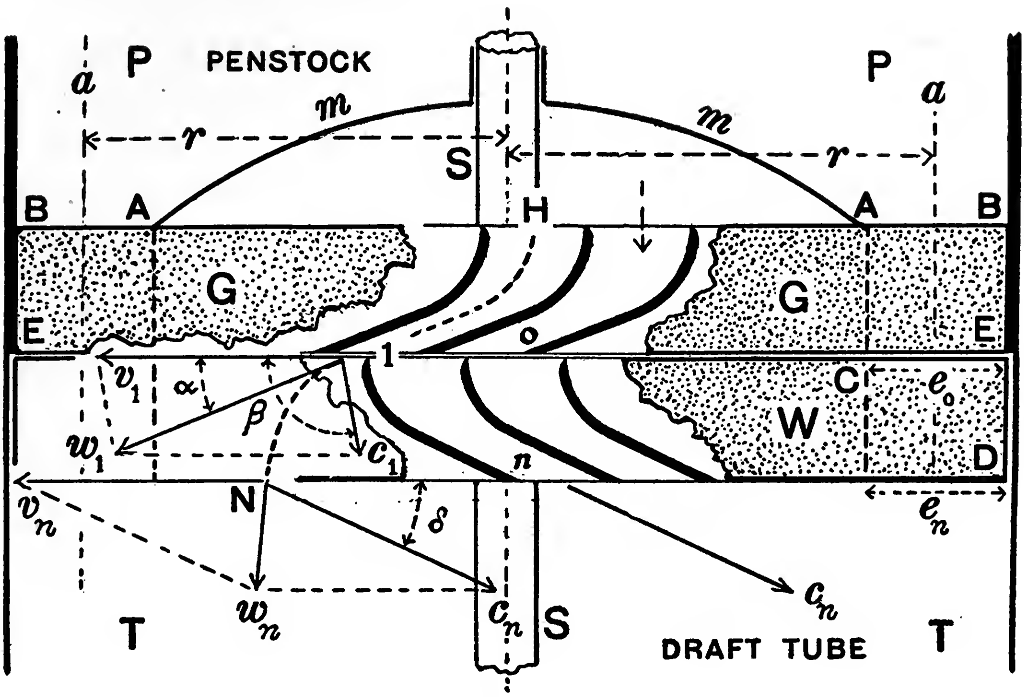


FIG. 62.

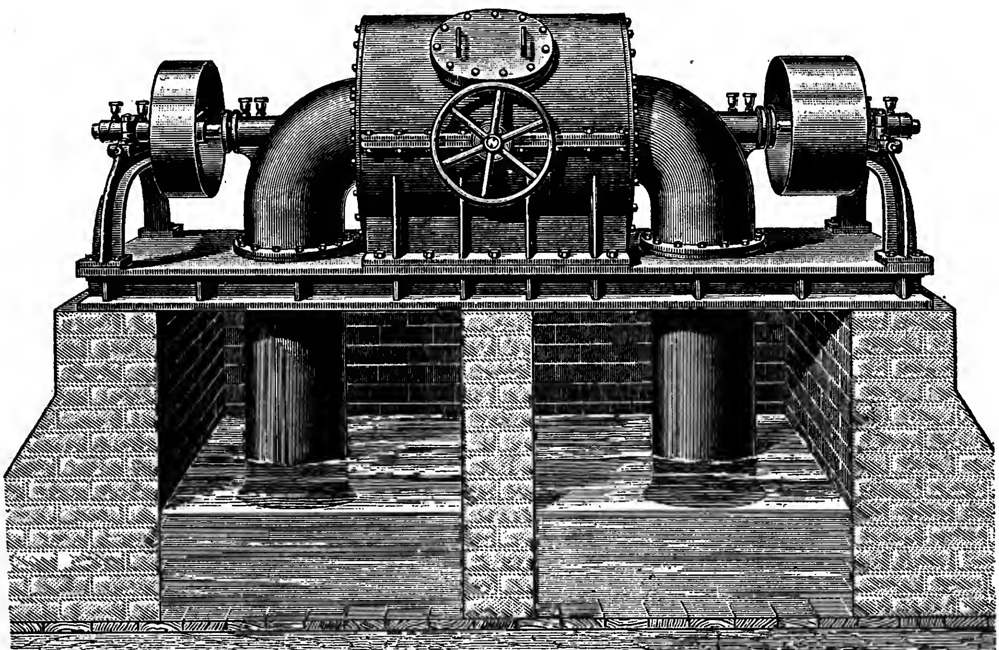


FIG. 63.

The points 1 and N are half-way out along the radial dimension of the turbine ring, being at a radial distance $=r$ from the axis. The linear velocities of these two points are of course equal; that is, $v_n = v_1$. The notation used in Fig. 62 for the parallelograms of velocities at entrance and exit is the same as in previous figures; w_1 and w_n being the absolute velocities; c_1 and c_n, the relative; while v_1 and v_n are the turbine (linear) velocities at the points in question. Each of these parallelograms lies in a plane (vertical, here) tangent to the cylindrical cutting surface above mentioned.

The curved plate or shell $m \ldots m$ prevents the passage of the water from the penstock to the turbine except through the guide-channels. See also Fig. 64, where a pulley, or gear-wheel, is shown keyed on the upper end of the shaft; the resistance R' acting at the circumference of this wheel is overcome through a distance v' each second by the action of the couple formed by the horizontal components of the water pressures on the turbine-vanes. The vertical components create a downward thrust on the turbine supports.

82. The Draft-tube.—Jonval was the first to discover that a turbine, especially his own, occupying so little space horizontally, would operate with practically the same efficiency when placed above the level of the tail-water and discharging its water into the upper end of a "*draft-tube,*" or air-tight tube opening below the water surface of tail-water. So long as the internal fluid pressure of the water can be kept greater than zero the tube will keep full, but for this result to be attained the turbine must not be placed more than about 25 ft. above the surface of the tail-water.

Draft-tubes are rarely made longer than 10 to 15 ft., their principal use being to render the turbine easily accessible for examination, repairs, etc. Fig. 63 (on p. 123) shows a wheel-casing receiving water from a penstock (not shown; entering the casing from behind) and containing two turbines fixed upon a common horizontal shaft. Each of these turbines discharges water into a separate draft-tube. On account of the symmetrical arrangement of the turbines the end thrusts

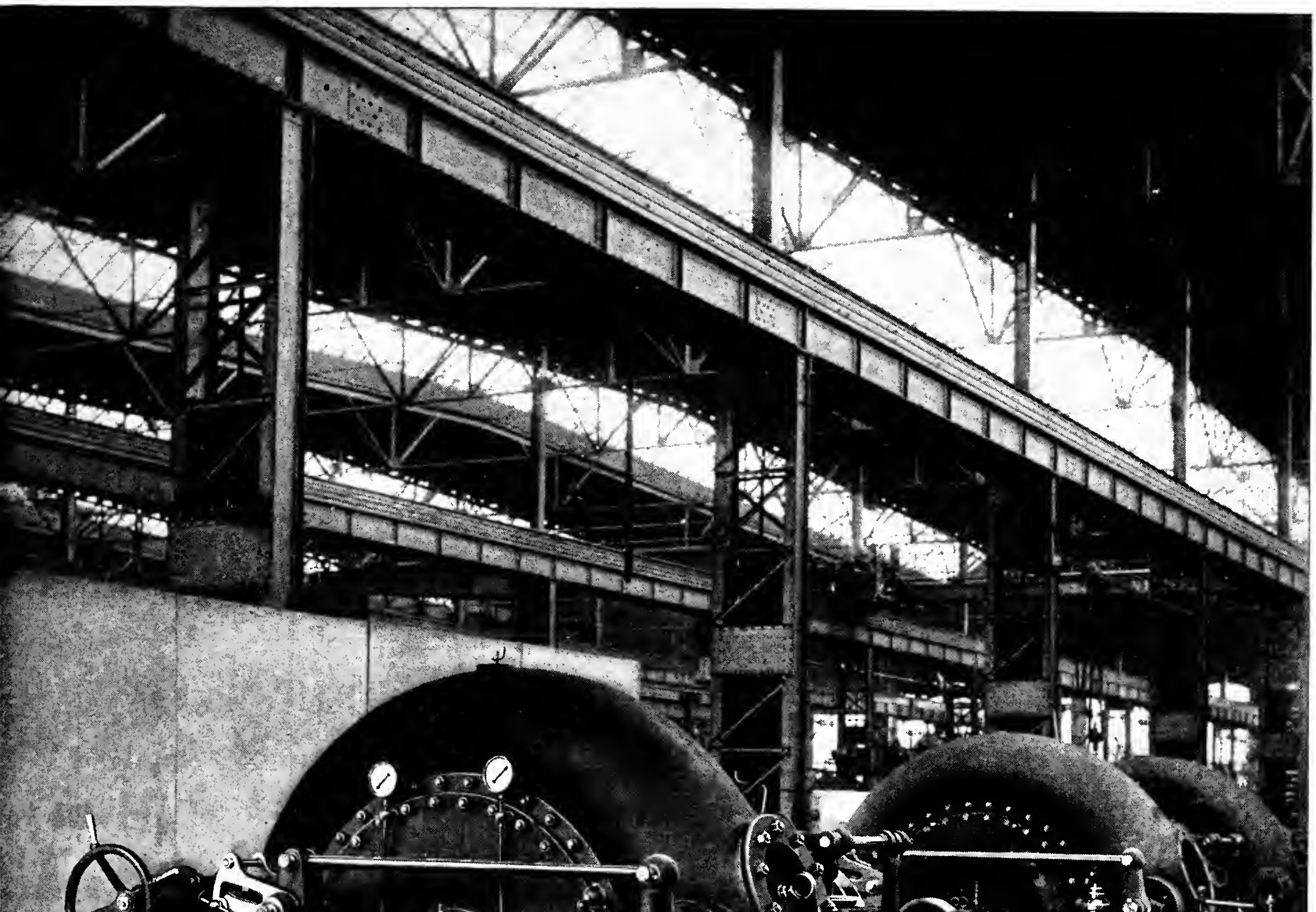


FIG. 60b. Escher Wyss & Co. Turbines at Spiez, Switzerland.

FIG. 70. Victor High-Pressure Runner.

along the shaft neutralize each other so that only lateral friction is occasioned in the shaft-bearings.

In the analysis of § 65 (Fourneyron turbine) it is noticeable that the results obtained are independent of the depth h_n of the wheel below the surface of the tail-water. A negative h_n would mean that the exit-point of the turbine is *above* the tail-water, and in order that the tube into which it discharges may flow full (after being once cleared of air) it is only necessary that its vertical length shall not exceed that of the water-barometer (or, rather, something less; since the water in the tube has a certain velocity and a loss of head due to skin friction occurs. (See § 532 on the siphon, p. 735, M. of E.)

When this air-tight tube is provided, the virtual surface of the tail-water is about 34 ft. (at sea level) higher than the actual, and a similar statement is true for the head-water. The "potential head" or "elevation head" apparently lost by the placing of the turbine above the actual surface level of the tail-water (within the limit indicated) is made good by the diminution of pressure (i.e., of "pressure-head") at the point where the water leaves the turbine channels.

The only additional source of loss of energy attending the use of the draft-tube, as compared with that occurring when the turbine discharges into a large water-filled space below the level of the tail-water, is the loss of head due to "skin friction" in the draft-tube itself, but this may be made quite small if the tube is sufficiently short and wide and the velocity of the water in it correspondingly slow. If the tube has the same width at the top (i.e., at the exit of the turbine channels) as elsewhere, there is thereby produced a "sudden enlargement" of section and a loss of head whose value is $\frac{w_n^2}{2g}$ (from Borda's Formula, p. 721, M. of E.); the same as if no draft-tube were used.

Draft-tubes may be employed with any class of turbine, though the Jonval and Francis types, with their modifications, are best adapted to its use, and have even been fitted to impulse-wheels of the Pelton and Girard designs. But in

this latter instance the wheel revolves in rarefied air within a strong casing forming the top of the draft-tube; the upper surface of the water in the draft-tube being maintained automatically just below the lowest sweep of the moving buckets. An advantage secured in such a case lies in the diminished resistance of the air.

83. The Diffuser.—In previous paragraphs, when the statement has been made that the water carries away with it at exit a kinetic energy of $\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}$ ft.-lbs. each second, it was with the understanding that the pressure at that point was that due to a head of 34 ft. (water-barometer height) in case the pressure around the jet was that of one atmosphere; or that due to a depth h_n of *still water* (with atmosphere on surface) between the point of exit and the surface of tail-water. But if the current leaving the turbine channels does not immediately enter a large body of comparatively still water, but is guided by the rigid walls of a *stationary* and *gradually enlarging* passageway, at the entrance of which the sectional area is equal to that of the current; then the internal fluid pressure at the point of exit from the turbine is *not* that corresponding to the position of this point (hydrostatically) with reference to the surface of the tail-water, but will be *less* (provided the tube conducting the water from the turbine-exit to the main body of the tail-water is of proper design). Such an apparatus to provide a gradual enlargement of section for the passage of the water after it leaves the turbine is called a *diffuser* and was first invented and used by Mr. Boyden of Boston, Mass., about 1845, in connection with a radial outward-flow turbine. Its use was found to increase the efficiency of the turbine some three per cent., by actual experiment. In the case of this type of wheel the diffuser consisted of two fixed conical zones flaring out opposite the outer edges of the turbine crowns, giving a "bell-mouthed" or divergent profile to the walls of the passageway at that point of the flow.*

* Somewhat as shown between n and m in Fig. 24, p. 50, but with a much more gradual increase of section.

(Kneass's book on the steam-injector gives an account of experiments on the flow of water in divergent tubes (i.e., in tubes of gradually enlarging longitudinal profile) which are interesting in this connection).

When a diffuser is provided, the "pressure energy" carried away by the water at exit from the turbine is smaller than otherwise; and the gain in that respect aids in offsetting the loss of energy due to the water leaving the wheel with an absolute velocity w_n. In brief, any prevention or lessening of loss of head, either in penstock, wheel-channel, or draft-tube, is a distinct gain to the efficiency of the turbine and its appurtenances.

84. Theory of the Draft-tube, with Diffuser. — Fig. 64 shows a vertical section of a Jonval turbine revolving on a vertical shaft and provided with a draft-tube, DM, and a diffuser (stationary), nKn. The revolving wheel and shaft are shown in solid black shading. It is revolving uniformly at best speed and the flow of the water is "steady"; the power developed being employed in overcoming the resistance R' lbs. through a distance v' each second at the periphery of the pulley, or gear-wheel, keyed upon the upper end of the shaft; v' being the linear velocity of the periphery of the pulley.

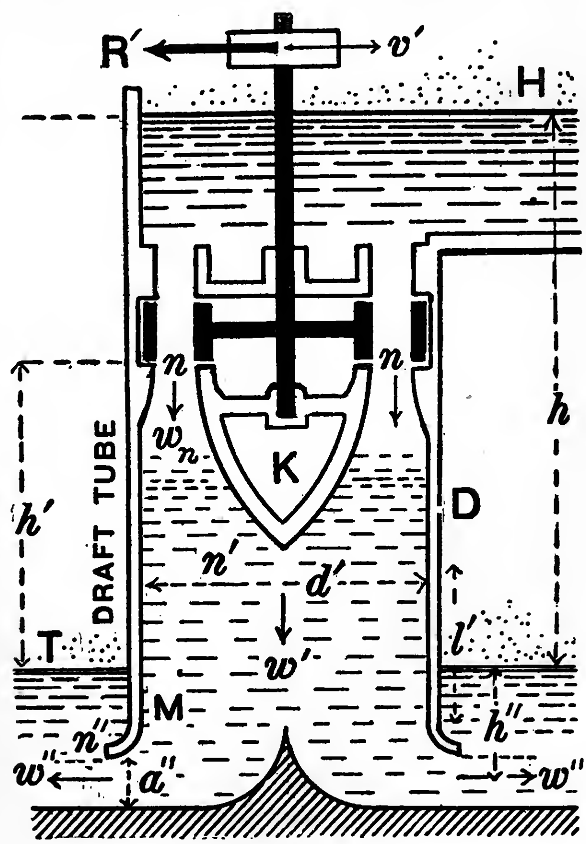


Fig. 64.

The absolute velocity of the water, which has a value w_n

at the point n where it leaves the turbine channels, is gradually reduced to a low value w' in the cylindrical part of the draft-tube; and the water finally leaves the tube at n'', beneath the surface T of the tail-water, through a vertical cylindrical opening with a velocity w'' (which should be small, the opening being large) and under a pressure $(p_a + h''\gamma)$ due to the depth h'' of still water below the surface T of tail-water. That is to say, at the point where the water leaves the whole apparatus, flowing into the full body of the tail-water, and where it is *under a pressure corresponding* (hydrostatically) *to the depth of this point below the* surface of the tail-water, its absolute velocity is (by proper design) smaller than that at the point n of exit from the turbine channels and the kinetic power thus carried away correspondingly small. The gain, however, in this respect would be more than offset by the loss of head between n and n'' if the change of absolute velocity from w_n to w' were not brought about gradually by the gradual change of sectional area of waterway between n and n'.

Let h and h'' denote the elevations so indicated in Fig. 64, d' and l' the diameter and length of the cylindrical portion of the draft-tube (the tube is not necessarily vertical), and f the coefficient of fluid friction in the same (p. 707, M. of E.); also let F''' $(=2\pi r e_n$ of Fig. 62) denote the sectional area of the horizontal ring at n (to which the direction of the velocity w_n is practically perpendicular in the regular running of the turbine at its best speed), and a'' the altitude of the cylindrical opening at n''. The small loss of head due to the gradual enlargement of waterway from n to n' may be represented by $\zeta\frac{w_n^2}{2g}$; while, as in a previous paragraph, $\zeta_0\frac{w_1^2}{2g}$ and $\zeta_n\frac{c_n^2}{2g}$ are those occurring in the guide-channels and wheel-channels respectively (see § 71 and Fig. 62). If we now deduct the total energy possessed by the flow per second at point n'' from that at point H, we obtain for the power spent in overcoming the resistance R' each second

$$R'v' = Q\gamma\left[h - \frac{\zeta_0 w_1^2}{2g} - \frac{\zeta_n c_n^2}{2g} - \frac{\zeta w_n^2}{2g} - \frac{4fl'}{d'}\cdot\frac{w'^2}{2g}\right]. \qquad (1)$$

In this connection we have, of course,

$$F'''w_n = \pi d''a''w'' = \frac{\pi d'^2 w'}{4}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(in which d'' is the diameter of the horizontal circle formed by the edge n'').

In order that the flow of water may fill all sections of the draft-tube, as supposed in the above analysis, it is necessary that during the steady flow the fluid pressure, p_n, at the point n of exit from the turbine channels be greater than zero; in other words, that the algebraic expression for this pressure must not lead to a negative result when applied in any numerical case. Since between points n and n'' the steady flow of the water takes place through rigid and stationary pipes, the application of Bernoulli's Theorem for such a case is warranted and leads readily to the following relation (with n'' as a datum plane; b denoting the height of the water-barometer or about 34 ft.):

$$\frac{p_n}{\gamma} + \frac{w_n^2}{2g} + h' + h'' - \frac{\zeta w_n^2}{2g} - \frac{4fl'}{d'} \cdot \frac{w'^2}{2g} = \frac{w''^2}{2g} + h'' + b, \quad . \quad (3)$$

or

$$\frac{p_n}{\gamma} = \left[b + \frac{4fl'}{d'} \cdot \frac{w'^2}{2g} - \frac{w_n^2}{2g}(1 - \zeta) - \frac{w''^2}{2g} \right] - h'. \quad . \quad . \quad . \quad (4)$$

The value of ζ may be taken as approximately 0.30 (see § 107). It is evident from eq. (4) that the value of the pressure p_n would be negative if the elevation h' were numerically greater than the quantity in the large bracket; that is, the greatest permissible value of h' would be, theoretically,

$$h' = b + \frac{4fl'}{d'} \cdot \frac{w'^2}{2g} - \frac{w''^2}{2g} - \frac{w_n^2}{2g}(1 - \zeta), \quad . \quad . \quad . \quad . \quad (5)$$

if flow with full sections is to be realized. But practically, since water-vapor might form in the upper part of the tube if the pressure were too low, especially in a warm climate, this value of h' should not be approached within (even) 5 or 6 feet.

If the diffuser were absent, that is, if no provision were made to secure a gradual enlargement of waterway between

n and n', the term $\frac{\zeta w_n^2}{2g}$, or $0.30\frac{w_n^2}{2g}$, would be replaced by $\frac{w_n^2}{2g}$.

85. Turbines of the Mixed-flow Type, or "Inward and Downward."—In turbines of this type the wheel-channels, while receiving the water from guides on the outside, so that the course of the water is at first radial and inward, toward the shaft, gradually curve in such a way that at exit the water has an absolute path nearly parallel to the axle. The axle being usually vertical, the course of the water may be rudely described as "inward and downward," and the turbine is said to be one of "mixed flow."

Most American turbines are of this class, a typical vertical section (or, rather, a section through the axis of the shaft) being shown diagrammatically in Fig. 65, where the solid black shading represents the revolving part, or "runner." The guides are placed in a ring surrounding the "runner" as in the Francis turbine; but the water leaves the turbine mainly in a direction parallel to the axle or shaft.

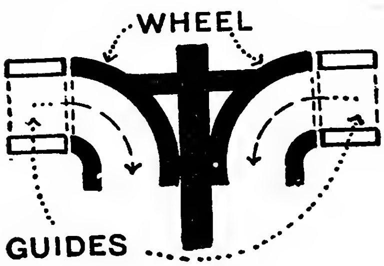


FIG. 65.

86. American Turbines. — A prominent and successful American turbine is made by the Risdon-Alcott Co., of Mount Holly, N. J. Fig. 66 shows the "runner," or turbine itself, which has a curved upper crown; the place of a lower crown being taken by a vertical cylindrical band (represented as transparent in the engraving). In Fig. 67 is seen an outside view of the wheel-case, etc. The guide-vanes, B, B, etc., are fixed upon the ring R. S is the short discharge-tube, intended to dip a few inches below the surface of the tail-water. The gate is a vertical cylinder, seen at C, and in this make of turbine is furnished with a number of horizontal extension pieces, such as D, D, etc., accompanying the gate in its vertical movement, and providing, therefore, a movable "roof" for each guide-channel. The turbine vanes or blades are warped sur-

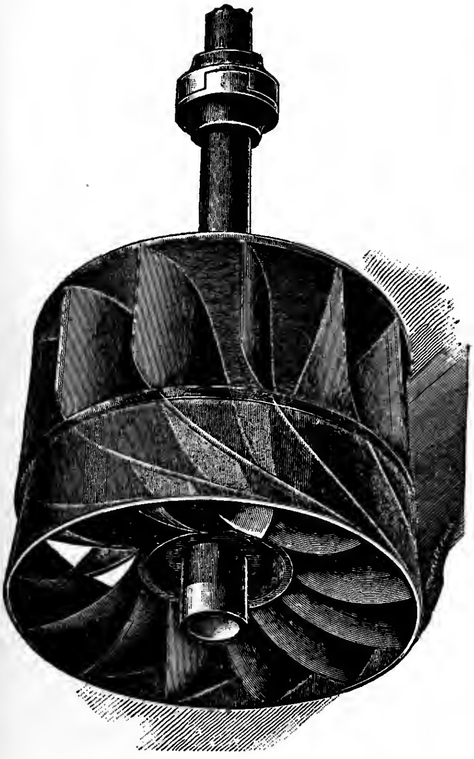


FIG. 66.

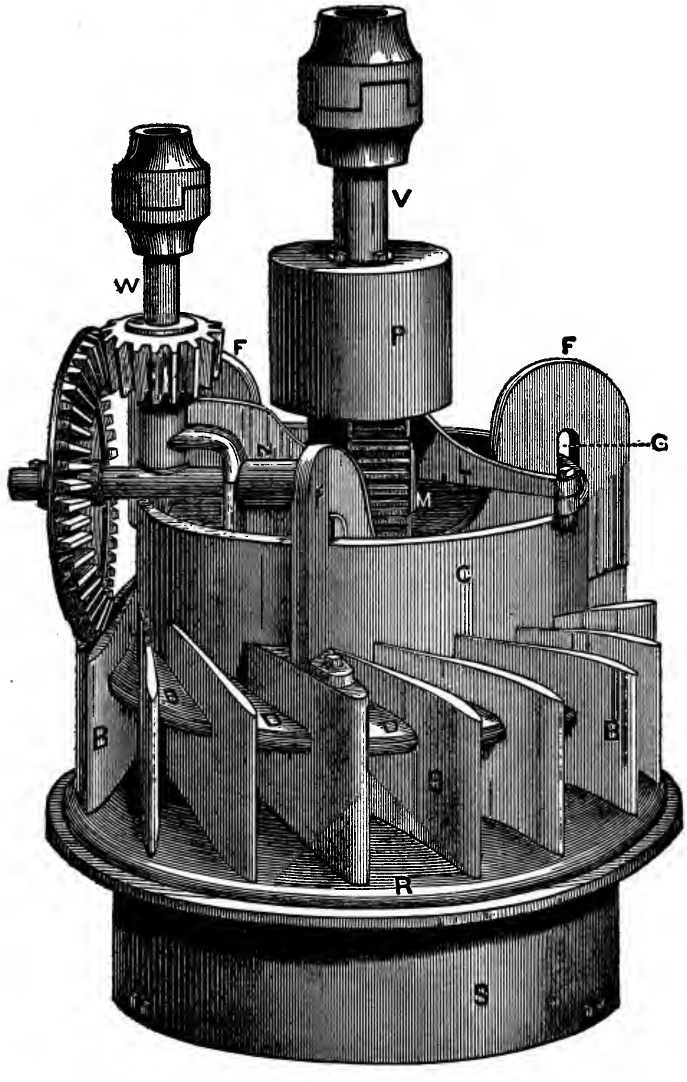


FIG. 67.

faces, their lower extremities suggesting the form of a spoon, or scoop. The turbine, generally of cast iron, though sometimes of bronze, is cast in a solid piece. In Fig. 67, V is the shaft of the turbine, while the smaller shaft W is intended to operate the gate; whose motion up or down, as may be needed, is brought about through the intervening gear-wheels and rack by the turning of shaft W. A wheel of this design made the highest record at the turbine tests conducted at the Centennial Exposition held at Philadelphia in 1876; its performance at part gate being remarkable for that date.

The Risdon-Alcott Co. also manufacture a turbine provided with a "*register*" gate; which consists of a cylindrical shell placed between the guides and outer edge of wheel and perforated with slots parallel to the shaft, somewhat like a gridiron. Fig. 67*a* shows such a gate. Its motion is circumferential instead of parallel to the shaft, the slots and intervening solid portions being of such dimensions, in connection with guide-vanes of considerable thickness, that while in one position the passage of the water is entirely obstructed, a comparatively small angular motion will leave the guide-passages fully open. The register-gate is used with several turbines of American make.

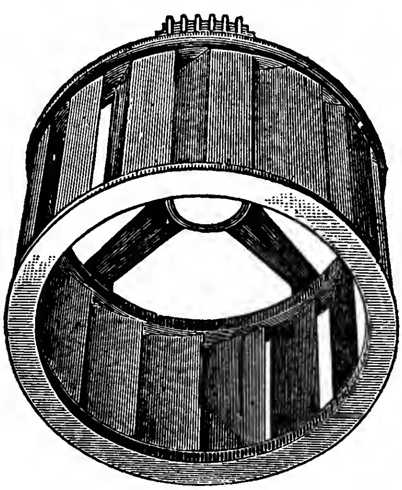


Fig. 67*a*.

Another prominent make of turbine in America is the "Victor Turbine," now (1905) manufactured by the Platt Iron-works Co., of Dayton, Ohio. Fig. 68 gives a view of the turbine itself, with its peculiar scooped-shaped vanes; while in Fig. 69 is shown the outer wheel-case, and guides, of one of these turbines, with its shaft directly connected to that of an electric generator vertically above. The engraving also shows the steel casing forming the lower terminus of the flume or penstock conducting water to the turbine, which is a "33-inch

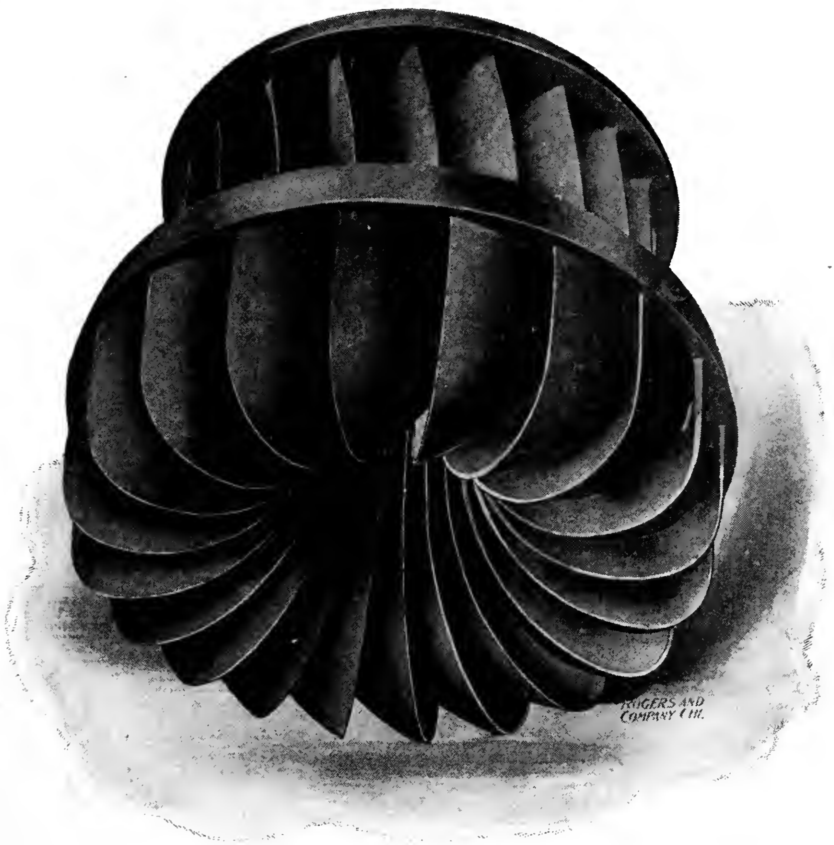


FIG. 68. Victor Cylinder Gate Runner.

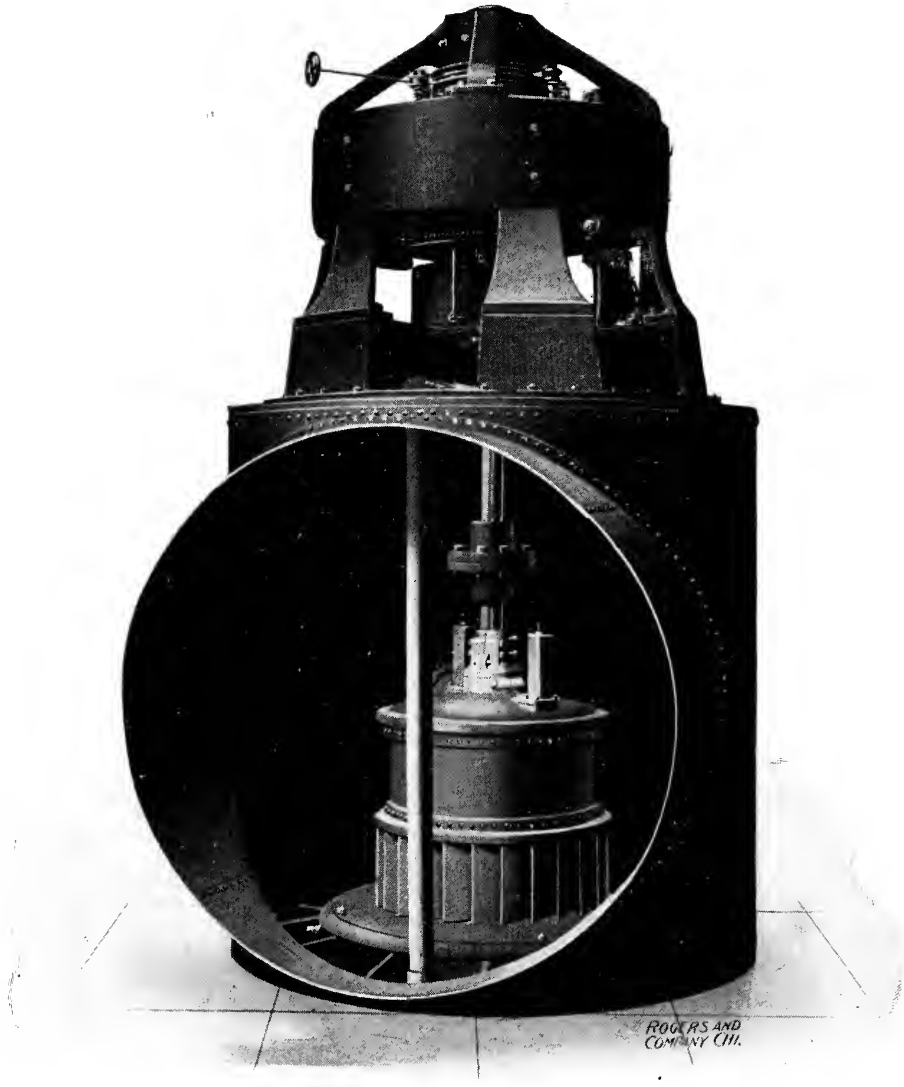


FIG. 69. Victor Turbine in Flume, Direct-connected to Generator.

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Cylinder-gate Victor Turbine." In Fig. 70 is shown the "Victor High-pressure Runner" intended for heads of from 100 to 2000 ft. All of these wheels are cast in one piece, of cast iron or bronze.

The "New American Turbine" manufactured (in 1905) by the Dayton Globe Iron-works, of Dayton, Ohio, is a prominent American motor of the "inward and downward" type. The "runner," or turbine itself, is shown in Fig. 71. Two kinds of gate are used with this turbine: the cylinder-gate, as already described in connection with other turbines, moving parallel to the shaft; and the "*wicket-gate,*" which consists in having a movable leaf on one side of each guide-channel, this leaf being pivoted at the extremity nearest the turbine and provided with a rounded shoulder at the other. By the swinging of this leaf the waterway of each guide-channel is varied in amount, and may be closed entirely.

Fig. 72, which gives a horizontal section made through the upper part of a New American Turbine (as made in 1890) and its guides, shows these movable blades or leaves, this arrangement of regulation being somewhat similar to that adopted in the Thomson Vortex Wheel (see § 80). In Fig. 73 is given a view of the wheel-case and shaft of a "wicket-gate" New American Turbine, set in the floor of a wooden flume. The small shaft on the right is for moving the guide-leaves, each of which is connected by an outside link (visible in the figure) with a horizontal disc. When the small shaft turns, the disc also turns and moves all the guide-leaves simultaneously and through the same extent of movement. As seen in the figure the discharge-tube, or short "draft-tube," as it may be called, has its lower edge somewhat below the surface of the tail-water.

Other prominent American turbines of the "mixed-flow" type, like those just described, are the "Hercules," made by the Holyoke Machine Co., of Holyoke, Mass.; the McCormick, made by the S. Morgan Smith Co., at York, Pa.; and the "Samson," by the James Leffel Co., of Springfield, Ohio. This last-named wheel is shown in Fig. 74, opp. p. 134, and is in

reality a double wheel, the upper portions of the wheel-passages being partitioned off as shown. These three wheels use the cylinder-gate, moving axially, i.e., parallel to the shaft.

The trade circulars of some of the makers of the foregoing turbines refer to tests of their wheels made at Holyoke, Mass., at the testing-flume of the Holyoke Water-power Co. (see § 97), where the highest head available is 18 ft. Some of the values of efficiency obtained in these tests not only greatly exceed 80 per cent., but in some cases approach 90 or over. Polishing and smoothing of the surface of the turbine-vanes has been found to increase the efficiency in several instances. In the case of several American turbines taken to Europe and there re-tested, European methods being followed, somewhat lower values of efficiency have resulted. It is thought that the discrepancy is due to the differing modes of measuring the water used.

The Jonval type of turbine, or "parallel-flow" variety, is manufactured by R. D. Wood and Co. of Philadelphia, Pa., and is sometimes made "*duplex*"; that is, the runner is provided with two concentric rings, each containing a set of vanes, the guide-ring being double also. When the supply of water is reduced, one ring alone is brought into action without sacrifice of efficiency.

As already mentioned, the firm of Kilburn, Lincoln and Co., at Fall River, Mass., manufacture an outward-flow turbine of the Fourneyron type. See § 75 and Figs. 17, 18, 19.

87. American Turbines. Historical. (See paper by Mr. Samuel Webber in Transac. Am. Soc. M. E. for 1905, abstracted in the Engineering News of Dec. 5, 1895; and also one by Mr. A. C. Rice, published in the Engineering News of Sept. 18, 1902, p. 208.)—During most of the first half of the nineteenth century the large mills of New England made use of the overshot and breast wheel for water-power; but in 1844 Mr. Uriah A. Boyden built and installed a Fourneyron turbine of 75 H.P. at Lowell, Mass., which on test yielded an efficiency of 78 per cent., a figure considerably greater than that furnished by the old-fashioned wheels in the neighboring factories.

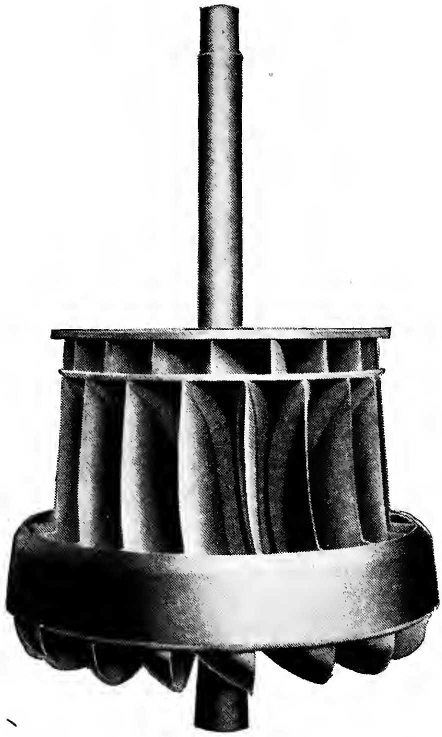

FIG. 74. The Samson Runner.

FIG. 72. A type of Wicket Gate. (1890).

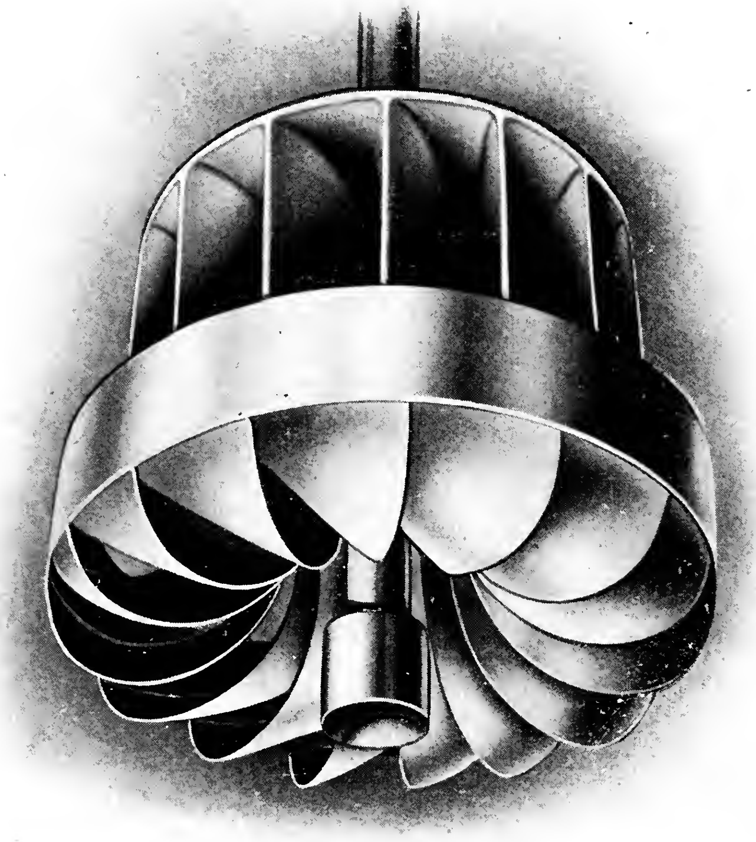

FIG. 71. New American Runner.

Several other turbines were then built, some of which were tested by Mr. J. B. Francis, hydraulic engineer, and the success of these motors stimulated imitation and invention in the United States; and turbines of the inward-flow (*Francis*) and parallel-flow (*Jonval*) types were constructed and put into service in many mills. About 1860 and later, the Swain and Leffel turbines were invented, combining the features of the Francis and Jonval types by securing an "inward and downward" discharge of the water (mixed type). The dimensions of the wheel-passages parallel to the shaft being made relatively great, a given quantity of water could be used with a less diameter of wheel; while the angular velocity (or revolutions per minute) was greater for a given linear velocity of the outer edge of wheel; that is, for a given head. These features conduced to cheapness of construction and speed in operation. High efficiencies were also obtained with these turbines; those at "part gate" being a notable improvement on previous results. The Leffel wheel was provided with the "wicket-gate" device (as at present in the "Samson" and "New American"). The Swain wheel had a form of gate which maintained a rounded aperture at all stages.

To quote from Mr. Webber's paper: "The Swain wheel had, however, given an excellent result as far back as 1862, and from that date down to about 1878 the number of turbines was legion, in all sorts of variations of curve of bucket and form of gate, but all containing the same general features of inward and downward discharge." "The general result of this change from the Fourneyron type, as first introduced, has been to furnish the public with turbines of equal power, in one-half the space and at one-fifth the cost, being single castings of iron or bronze instead of being built up of many parts."

In 1876 began the "new departure" in the design of American turbines, inaugurated by the high efficiency at part gate, and large capacity for its diameter, of a 24-in. "Hercules" wheel invented by Mr. John B. McCormick. The axial dimensions of this wheel were, relatively, greater than ever

before, and each bucket was provided with three sharp projecting ridges to assist in the guidance of the water at "part gate." (The "*Hercules*" of that date is shown in Fig. 72*a*.) Other makers soon followed with improved designs of their wheels, there being thus produced the "*Victor*," "*Risdon*," and "*New American*"; all with high efficiencies and large capacity for a given diameter. Mr. Webber gives a table showing the progressive increase in capacity (for a given diameter) from the Boyden-Fourneyron design, with which, in the case of a 36-in. wheel under 26 ft. head, 22.95 cub. ft. of water was used per second, up to the more recent "*Samson*," "*Hercules*," and "*Victor*" wheels, each of 36 in. diameter and using 109 cub. ft. of water per second under the same head, 26 ft., and with even greater efficiency. The "*Hercule-Progrès*," made in France on an American type, has the same general appearance as the "Hercules" shown in Fig. 72*a*. (See Prasil's Report on Turbines at the Paris Exposition of 1900; Schweizerische Bauzeitung, vols. 36, 37.)

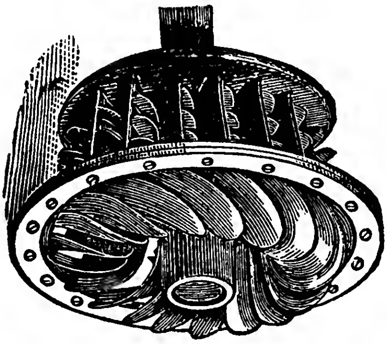

FIG. 72*a*.

88. Choice of Hydraulic Motor for Different Heads.—A valuable article by Mr. John Wolf Thurso on "Modern Turbine Practice and the Development of Water-powers" was published in the Engineering News of Dec. 4, 1902, p. 46, and Jan. 8, 1903, p. 26. The following recommendations are quoted from that article:

"The type (of hydraulic motor) to be employed in each individual case should be in accordance with the height of the head to be utilized, as follows:

1. "*Low heads*, say up to 40 ft.: American type of turbine (i.e., of the "inward and downward" variety) on horizontal or vertical shaft, in open flume or case, nearly always with draft-tube.

2. "*Medium heads*, say from 40 to 300 or 400 ft.: Radial inward-flow reaction or Francis turbine, with horizontal

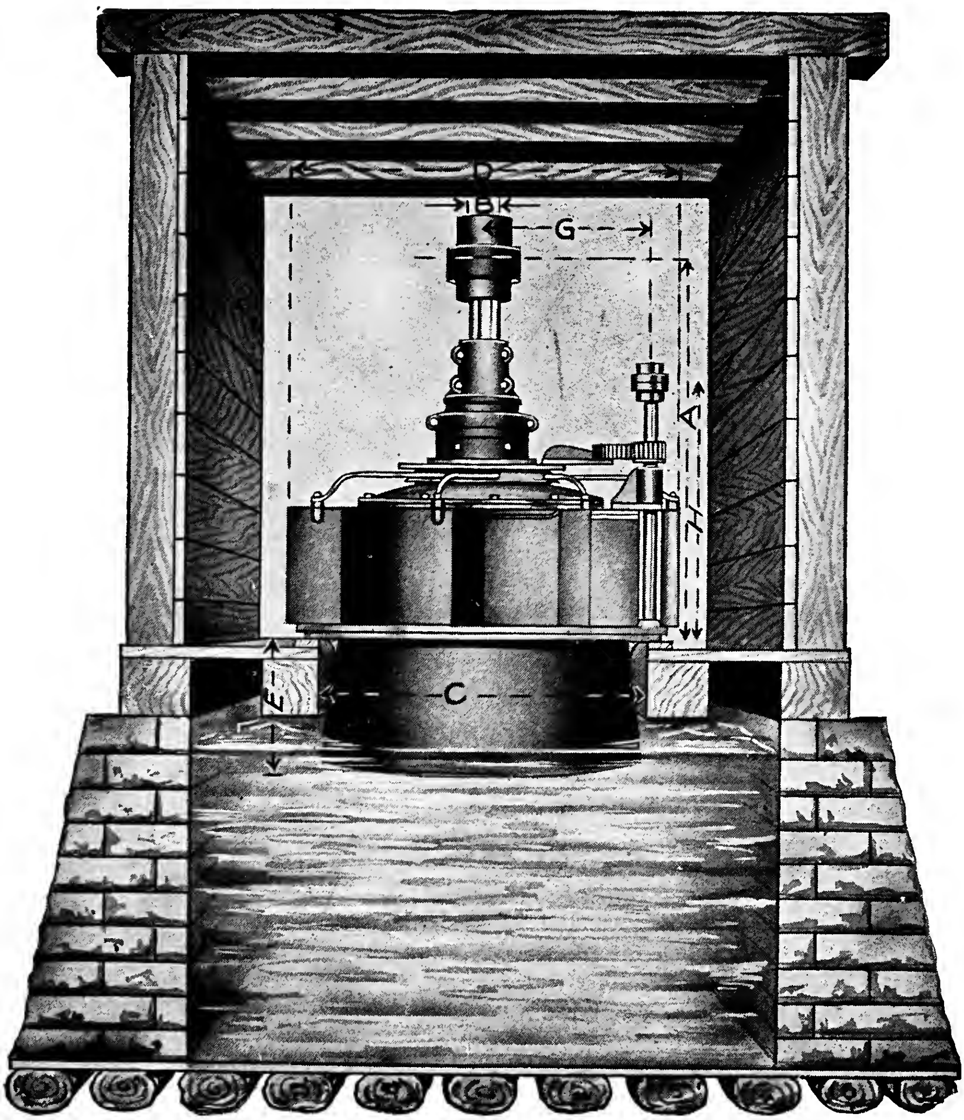

FIG. 73. New American Turbine in Wood Flume.

shaft and concentric or spiral cast-iron case with draft-tube.

3. "*High heads,* say above 300 or 400 ft.: Pelton wheel; or radial outward-flow, segmental-feed, free-deviation turbine (i.e., a Girard impulse wheel); or a combination of both, on horizontal shaft and cast- or wrought-iron case; often with draft-tube."

89. General Theory of Reaction Turbines.—The theory of the mixed-flow turbine will now be presented and finally generalized so as to be applicable to any type of reaction turbine. Fig. 75 represents a single passageway, $1 \ldots N$, of a

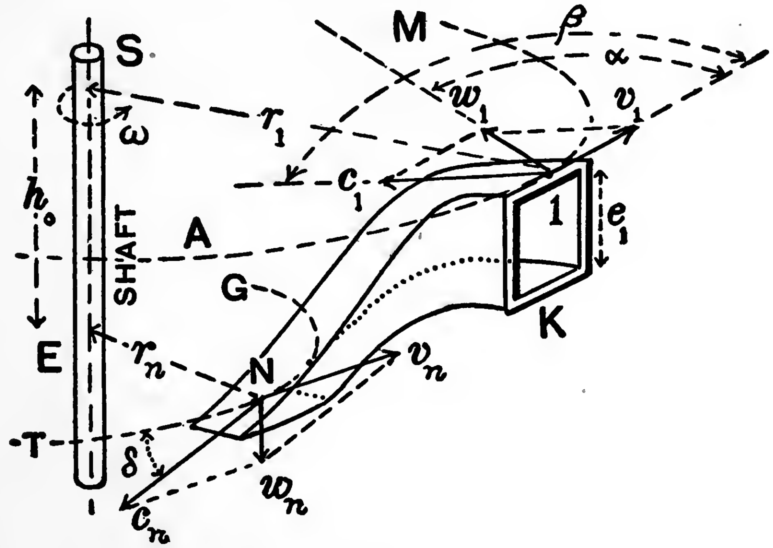

FIG. 75.

mixed-flow turbine having its shaft, S, vertical. The entrance-point, 1, is describing a horizontal circle $A \ldots 1 \ldots M$ with a velocity, v_1, of proper value for best effect, the corresponding velocity of the exit-point N being v_n, in horizontal circle $T \ldots N \ldots G$, this circle being a vertical distance, h_0, below position 1, which is itself h_1 ft. below the surface of the head-water. The wheel is supposed to be in an open flume or wheel-pit, so that there is no loss of head in a penstock; in fact all friction in guide-passages and wheel-channels will be disregarded, at first. There is no diffuser, so that the fluid pressure of the water at the point of exit N is taken as $p_n = (b + h_n)\gamma$,

b being the height of the water-barometer and h_n the vertical distance of the point N *below* the surface of the tail-water. (In case N is above the tail-water, h_n is negative.) The parallelogram of velocities at entrance is in a horizontal plane, w_1 being the absolute velocity of the water at that point making an angle α with wheel-rim tangent, and c_1 the relative velocity It will be assumed. as before, that the angle β is eventually to have such a value that there will be no "shock," or impact, at entrance. At the exit-point N the parallelogram of velocities is in a vertical plane, the absolute velocity w_n being the diagonal formed on the relative velocity c_n, as one side, and the wheel-rim velocity v_n, as the other side, of a parallelogram. Of course the relative velocity c_n follows the tangent to the walls of the turbine passage at N and makes a (small) angle δ with the wheel-rim tangent Nv_n. The two radii are r_1 and r_n, as shown, r_n being the radius of the mean position N, or point half-way out radially along the discharging edge (shown better in the next figure, Fig. 76). Let us denote by F the aggregate sectional area of the exit passages of the turbine, that of each passage being taken at right angles to the relative velocity c_n; and by F_0 the aggregate sectional area of the guide-passages at the entrance-point, 1. For example, in Fig. 76, if there are m_n turbine channels and m_0 guide-passages, then $F_n = m_n a_n e_n$ and $F_0 = m_0 a e_0$ sq. ft.

We now have the following relations (losses of head in guide-passages and wheel-channels being neglected):

From trigonometry,

$$c_1^2 = w_1^2 + v_1^2 - 2w_1 v_1 \cos\alpha, \quad . \quad . \quad . \quad . \quad (1)$$

and

$$w_n^2 = c_n^2 + v_n^2 - 2c_n v_n \cos\delta. \quad . \quad . \quad . \quad . \quad (2)$$

From Bernoulli's Theorem applied to the steady flow of the water between rigid stationary walls from head-water surface to outlet of guides,

$$\frac{p_1}{\gamma} + \frac{w_1^2}{2g} = b + h_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(p_1 being the internal fluid pressure at point 1).

From Bernoulli's Theorem for a steady flow in a rigid pipe rotating uniformly about a vertical axis (see eq. (13), § 41) between entrance-point 1 and exit-point N of a turbine channel (adding in the gravity head h_0),

$$\frac{c_n^2}{2g}+(b+h_n)=\frac{c_1^2}{2g}+\frac{p_1}{\gamma}+h_0+\frac{v_n^2}{2g}-\frac{v_1^2}{2g}. \quad . \quad . \quad . \quad (4)$$

For a minimum residual kinetic energy we may write, as in previous investigations, the relative velocity c_n = wheel-rim velocity v_n at exit, i.e.,

$$c_n=v_n \text{ (see § 53)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Equation of continuity: $F_0 w_1 = F_n c_n$; . . . . (6)

The proportion: $v_1 : v_n :: r_1 : r_n$. . . . . (7)

The volume of water used per second:

$$Q=F_0 w_1, =F_n c_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Theoretic power of the wheel, in case there is no diffuser and all fluid friction between head-water surface and point of exit N (also axle-friction) be neglected, is

$$L, =R'v', =Q\gamma h-\frac{Q\gamma}{g}\cdot\frac{w_n^2}{2}; \text{ \{ft.-lbs. per sec.\}}, \quad . \quad . \quad (9)$$

R' being the resistance, lbs. (overcome by the turbine in steady running), tangent to the circumference of a pulley where the linear velocity is v' ft. per sec. (N.B. If by means of a diffuser all loss of head between exit-point N and the surface of the tail-water could be considered to be obviated, we should have $R'v'=Q\gamma h$ as in eq. (4), § 38.)

Now solve (3) for $p_1 \div \gamma$ and substitute in (4), from which, after inserting the value of c_1^2 from (1) and noting that $h_1+h_0-h_n=h$, the total head of the mill-site (that is, the vertical distance from the surface of the head-water to that of the tail-water, and also writing $c_n=v_n$ [from (5)], we have

$$w_1 v_1 \cos\alpha = gh. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

But, from (6) and (7), $w_1=\frac{F_n c_n}{F_0}, =\frac{F_n c_n}{F_0}$; and $v_1=\frac{r_1 v_n}{r_n}$; substituting which in (10), and solving, we have as the "best

value" of the exit wheel-rim velocity for best effect when fluid friction is disregarded (with the exception of that between point N and surface of tail-water, there being no diffuser)

$$v_n = \sqrt{\frac{F_0}{F_n} \cdot \frac{r_n}{r_1} \cdot \frac{gh}{\cos \alpha}}. \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This is now in such a form as to hold good for *any* reaction-turbine, the subscripts 1 and n referring to entrance and exit, respectively, of the turbine channels; and a fair allowance for fluid friction in the guide-passages and turbine channels may be made (as due to a study of numerous numerical examples and actual tests) by deducting eight per cent. of this value from itself; that is, by writing

$$v_n = 0.92\left[\sqrt{\frac{F_0}{F_n} \cdot \frac{r_n}{r_1} \cdot \frac{gh}{\cos \alpha}}\right]. \quad . \quad . \quad . \quad . \quad (12)$$

In the case of a parallel-flow, or "axial," turbine, $r_1 = r_n$ $(=r)$, being measured to the middle point of the radial dimension of the ring containing the wheel-vanes; see Fig. 62.

90. Turbines. General Theory with Friction.—If in the analysis of the last paragraph we introduce a loss of head $\frac{\zeta_0 w_1^2}{2g}$ between head-water and entrance-point 1, and a loss of head $\zeta_n \frac{c_n^2}{2g}$ in the turbine channels themselves, with $c_n = v_n$ as before, for best effect, the outcome is found to be

$$v_n = \sqrt{\frac{1}{1 + \frac{\zeta_1}{2} \cdot \frac{F_n}{F_0 r_1} \cdot \frac{r_n}{\cos \alpha} + \frac{\zeta_n}{2} \cdot \frac{F_0 r_n}{F_n r_1 \cos \alpha}}} \sqrt{\frac{F_0}{F_n} \cdot \frac{r_n}{r_1} \cdot \frac{gh}{\cos \alpha}}. \quad (13)$$

For ordinary values of the ratios of the radii and sectional areas concerned, and with ζ_0 and ζ_n each equal to about 0.10, as mentioned in § 71, the value of the first radical in eq. (13) above would be found to be not far from the 0.92 of eq. (12).

It is to be noted that the relation $w_1 v_1 \cos \alpha = gh$, in eq. (10), may also be derived in a much more direct manner by the analysis already given in § 67, which applies *to any turbine*

whatever since the parallelograms of velocities at entrance and exit may lie in any position relatively to the shaft of the turbine without vitiating any of the steps taken in the analysis of that paragraph. See § 68.

Another point to be noted is that eqs. (1), (2), (5), (6), (7), and (8) apply even when fluid friction in guides and channels is considered, and will therefore be used in subsequent operations where it is desired to take account of that friction.

91. Sectional Areas F_0 and F_n. Thickness and Number of Guide-blades and Turbine-vanes.—Let us consider the sectional area of the cross-section of a guide-passage at point 1 to be rectangular. It is perpendicular to the absolute velocity w_1, has a width a ($=\overline{md_0}$ in Fig. 47), and a length e_0. See Fig. 76, in which is also shown the rectangular section of a turbine channel at exit; likewise considered rectangular, with width a_n and length e_n; SS is the shaft of turbine. Now let s_0 denote the "*pitch*" of a guide-blade; that is, the length of that portion of the circumference, of radius r_1, which corresponds to one guide; also let m_0 be the number of guides (so that $m_0s_0=2\pi r_1$) and t_0 the thickness of the guide-blade. Similarly, at the exit-point, N, of a turbine channel, let s_n be the pitch of a turbine-vane, m_n the number of vanes, and t_n the thickness of a vane. Then $a=s_0 \sin\alpha - t_0$ (very nearly), and therefore, since $F_0=m_0ae_0$, we have

$$F_0=e_0(m_0s_0 \sin\alpha - m_0t_0)=e_0(2\pi r_1 \sin\alpha - m_0t_0); \quad . \quad (14)$$

and similarly

$$F_n=e_n(2\pi r_n \sin\delta - m_nt_n). \quad . \quad . \quad . \quad . \quad (15)$$

For approximate purposes, however, we may write the *ratio*

$$\frac{F_0}{F_n}=\frac{e_0}{e_n}\cdot\frac{\sin\alpha}{\sin\delta}\cdot\frac{r_1}{r_n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The above is readily understood for radial turbines. As for parallel-flow wheels (or "*axial* wheels") we write $r_1=r_n$ (call it now r) and measure it to a point half-way between the inner and outer rings between which the turbine-vanes are placed. The pitch of the guides and vanes is measured along this inter-

mediate circle of radius r, and the dimensions e_0 and e_n are radial (and horizontal, if the shaft is vertical; see Figs. 62 and 64).

In a mixed-flow turbine (see Fig. 76), the entrance rectangle (that is, for a guide-exit), of area $= ae_0$, is vertical (provided the shaft of turbine is vertical), while at the turbine-exit, N, the right section between vanes has two horizontal edges of length $= e_n$ and an area $= a_n e_n$; where r_n is the radius of the point on edge of wheel-vane at N, half-way (say) between the extreme points of that horizontal edge. e_n is horizontal and may be greater than e_0 if desired. (Of course, in the case of turbine-vanes which have scoop-shaped forms at the exit-point N, the dimensions of this equivalent rectangle, of sides e_n and a_n, are difficult to estimate.)

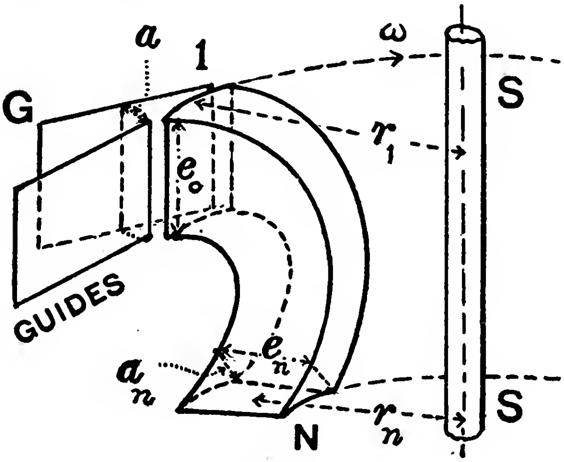

Fig. 76.

92. Empirical Relations for Turbine Design.—In the design of a particular turbine which is to work under a given head and utilize a given quantity, Q, of water per second, there are many dimensions and quantities which have to be determined and finally so adjusted to each other as to produce the best results. Theory cannot determine all these quantities. If the guide-blades and wheel-vanes are too numerous, a disproportionate amount of rubbing surface is offered to the water, with consequent loss of power from fluid friction; whereas if they are too few, the water is not so well guided and leaves the wheel with too great absolute velocity. Durability and ease of construction are also to be regarded. Experience and experiment, therefore, must be relied upon to a large extent as governing certain elements of design. In America the evolution of the "inward and downward" turbine has been very largely a matter of "cut and try"; but very good results have finally been attained for moderate heads (up to about 40 ft.).

In Europe, however, it is more generally the custom to design each proposed turbine for its special site and work.

In the present book space cannot be given to special details of design and construction. For these the reader is referred to the works of Bodmer and Thurso, in English; and to those of Meissner, Mueller, Herrmann, and Zeuner, in German.

It will suffice to give a few empirical relations and assumptions condensed mainly from Bodmer and Mueller.

If we distinguish the following three cases, viz.:

Case I. When $Q \div w_1$ is >16 sq. ft. (large Q and small h);

Case II. When $Q \div w_1$ is >2 and <16 sq. ft. (medium Q and h);

Case III. When $Q \div w_1$ is <2 sq. ft. (small Q and high h); then the angle α may be assumed from 20° to 25° for (I); 15° to 20° for (II); and 15° to 17° for (III); for *axial turbines.*

Angle δ for the three cases, for *axial turbines:*

20° to 25°, 15° to 17°, and 12° to 16°, respectively.

For *radial inward-flow* and *mixed-flow turbines* take α from 10° to 24°; also δ from 16° to 24°.

For *radial outward-flow turbines* assume α from 15° to 24°, and δ from 10° to 20°.

The ratio $\dfrac{F_0}{F_n}$ for *axial turbines* may be taken as 0.5 to 1.5, usually = 1.0.

For *radial inward-flow* and for *mixed-flow wheels* take r_1 from $0.75\sqrt{F_0}$ to $1.75\sqrt{F_0}$; with $r_n = 0.65$ to 0.85 of r_1.

For *radial outward-flow turbines* assume r_1 from $1.50\sqrt{F_0}$ to $2.0\sqrt{F_0}$; with $r_n = 1.25r_1$ to $1.50r_1$.

For *axial wheels,* referring again to the above three cases, I, II, and III, the radius r may be taken from $\sqrt{F_0}$ to $1.25\sqrt{F_0}$ for Case I; $1.25\sqrt{F_0}$ to $1.5\sqrt{F_0}$ for II; and $1.5\sqrt{F_0}$ to $2\sqrt{F_0}$ for III; also, as to the pitch, take $s_0 = 10$ to 12 inches for Case I; from $r \div 3.75$ to $r \div 4.5$ for II; and 4.5 to 6.0 inches for III. p. 141

For *radial inward-flow* and for *mixed-flow turbines* the *pitch* may be taken from 4.5 to 12 inches; and for *radial outward-flow* wheels, from $r \div 4.5$ to $r \div 6$.

As regards the value of e_n with *axial wheels*, Mr. Bodmer recommends a value of from $r \div 1.25$ to $r \div 2$ in Case I (above); from $r \div 2$ to $r \div 2.5$ for Case II; and from $r \div 2.5$ to $r \div 3$ in Case III. Also, for the axial depth of wheel in the case of an axial turbine, from $r \div 5$ to $r \div 3$.

The number of turbine-vanes, viz., m_n $(=2\pi r_n \div s_n)$, should be greater by 1 or 2 than the number, m_0, of guide-blades. The thickness of both guide-blades and turbine-vanes should be taken at from $\frac{1}{2}$ to $\frac{5}{8}$ inch for cast iron, and from $\frac{1}{4}$ to $\frac{3}{8}$ inch for wrought iron or steel; these dimensions for parts near the ends, which should be beveled or sharpened off if possible. (It may sometimes be advantageous to make the wheel vanes much thicker than their middles to give better form to the passageways between them: see Fig. 55.)

93. Computations for a Proposed Turbine. Order of Procedure.—It is supposed that the rate at which water may be used in steady flow, viz., Q cub. ft. per second, is given and also the head h of the mill-site, and that a turbine of some special type is to be designed for the given site and duty. We should first assume values for the two ratios $F_0 \div F_n$ and $r_n \div r_1$, and for the angle α. These assumptions may need to be revised, however, after a certain progress has been made with the computations. For example, with a radial turbine, to have the crown-plates parallel implies equal values for e_0 and e_n, in which case the assumption of the three values, $F_0 \div F_n$, $r_n \div r_1$, and α would determine a value for the angle δ; which value might not be suitable, thus requiring a change in some of the original assumptions. In such a case, therefore, (radial turbine with parallel crowns,) it would be better to assume the three values α, δ, and $r_n \div r_1$; from which the ratio $F_0 \div F_n$ could then be computed (at least approximately) from eq. (16) of § 91.

With these three values, then, viz., for $F_0 \div F_n$, $r_n \div r_1$, and α,—h being already given,—we find the best speed for the exit wheel-rim, v_n, from eq. (12), § 89, viz.,

$$v_n = 0.92\left[\sqrt{\frac{F_0}{F_n} \cdot \frac{r_n}{r_1} \cdot \frac{gh}{\cos\alpha}}\right] \quad . \quad . \quad . \quad . \quad (12)$$

(in deriving which fluid friction has been taken into account).

With v_n known we now apply the relations $v_n = c_n$ for best effect and $F_n c_n = F_0 w_1$ (both of which hold even when friction is considered), and solve for w_1, the absolute velocity at entrance.

We are now in a position to determine the quotient $Q \div w_1$, upon which the choice of angles α and δ partly depends for axial turbines. If necessary, a new choice may now e made and the computation revised.

With w_1 and v_1 known [since $v_1 = (r_1 \div r_n)v_n$], the parallelogram of velocities at the point of entrance of the turbine channel can be solved, trigonometrically or graphically, and thus the value of the angle β becomes known, upon which depends the position of the relative velocity c_1 and of the tangent to wheel-vane at this entrance-point (see Figs. 48, 58, and 62). The tangent to the wheel-vane at 1 must have this position in order that, at the speed of wheel just previously found (v_1), there may, at no "shock" or impact at point 1. In Mueller's recent work* the statement is made that recent practice in designing Francis turbines favors the relations $w_1 = \sqrt{gh}$ and $\beta =$ about 90°.

From the now known value of F_0, $= Q \div w_1$, we pass on to the computation of the value of the radius r_1 from the empirical rules of § 92; from which follows that of r_n, since the ratio of the radii was assumed at the outset. The pitch of the guide-blades is then fixed upon from the rules given in the preceding paragraph and the number of guide-blades computed, i.e., m_0. The value of e_0 is now found from

$$F_0 = C[2\pi r_1 \sin \alpha - m_0 t_0]e_0, \quad . \quad . \quad . \quad . \quad (17)$$

in which, according to Mr. Bodmer, the value of C is to be taken as 1.0 for axial wheels, and as 0.91 for radial and mixed-flow wheels.

94. Excess Pressure at Entrance.—In German treatises on turbines some stress is laid on the desirability of adopting finally such dimensions and angles thae the fluid pressure at point 1 (entrance) shall not greatly exceed that in the tail-

* "Die Francis-Turbinen und die Entwicklung des Modernen Turbinenbaues," by W. Mueller. Hannover, 1901.

water or draft-tube on the other side of the joint or crevice fo med between the edges of the turbine crowns and the opposite, adjoining, stationary edges of the guide floor, or gate. If the difference is great, the leakage through the crevice may be of importance. The amount of the (unit) pressure, p_1, may be found from eq. (3), § 89, after w_1 has been determined.

94a. Numerical Example. Jonval Turbine. Fig. 62.—Compute proper values for the arrangement of a parallel-flow turbine (Jonval) which is to work under a head of $h=13.5$ ft. and to use $Q=210$ cub. ft. per sec. of water.

Here let us assume the angles α and δ to be 20° and 15° respectively, and that $e_0=e_n$ (nearly; for present purposes, in order to compute the ratio $F_0 \div F_n$). Since $r_1=r_n, =r$, the ratio $r_n \div r_1$ is unity; therefore, from eq. (16), § 91, $\dfrac{F_0}{F_n}=\dfrac{\sin\alpha}{\sin\delta}=\dfrac{0.342}{0.259}=1.32$; and, from eq. (12),

$$v_n=0.92\sqrt{1.32\times 1\times\frac{32.2\times 13.5}{0.940}}=22.72 \text{ ft. per sec.}$$

Next, $$w_1=\frac{F_n c_n}{F_0}=\frac{F_n v_n}{F_0}=\frac{22.72}{1.32}=17.2 \text{ ft. per sec.}$$

To find the angle β of Fig. 62, note that in Fig. 77, showing the parallelogram of velocities at entrance, if the perpen-

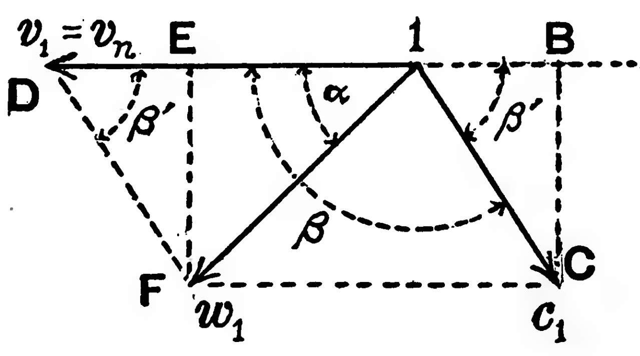

FIG. 77.

dicular FE be dropped from F upon $1..D$, we have $\overline{DE}$, or $\overline{1..B}$, $=\overline{D1}-\overline{E1}$; i.e., $\overline{DE}=v_1-w_1\cos\alpha$. Also, $\overline{FE}=w_1\sin\alpha$. Now the ratio $\overline{FE}\div\overline{DE}=\tan\beta'$, β' being the supplement of the

desired angle β; hence (remembering that $v_1=v_n$ for this type of wheel) we have

$$\tan\beta'=\frac{w_1\sin\alpha}{v_1-w_1\cos\alpha}=\frac{17.2\times0.342}{22.72-17.2\times0.940}=0.954.$$

Therefore $\beta'=43°\,40'$ and $\beta=136°\,20'$. $F_0=\frac{Q}{w_1}=\frac{210}{17.2}=12.2$ sq. ft. and hence, assuming $r=1.25\sqrt{F_0}$ (see § 92), $r=1.25\sqrt{12.2}=$ 4.36 ft.; i.e., the mean diameter of the turbine should be $2r=8.72$ ft. Adopting 36 as the number of guide-blades and 38 wheel-vanes, with the thickness $t_0=t_n=0.5$ in. near extremities, we have for the length of opening of a wheel-channel at entrance [radial in position; see Fig. 62, and eq. (17), § 93]

$$e_0=\frac{F_0}{2\pi r\sin\alpha-m_0t_0}=\frac{12.2}{2\pi\times4.36\times0.342-36\times\frac{1}{24}}$$

$$=12.2\div7.86=1.55 \text{ ft.}, =1 \text{ ft. } 6.6 \text{ in.}$$

If the thickness of blades and vanes were zero, e_0 and e_n would be equal, since the ratio $F_0\div F_n$ has been taken equal to $\sin\alpha\div\sin\delta$. But, taking into account the thickness (0.5 in.), we find for e_n, from eq. (15), § 91,

$$e_n=\frac{F_n}{2\pi r\sin\delta-m_nt_n}=\frac{12.2\div1.32}{2\pi\times4.36(0.259)-38\times\frac{1}{24}},$$

$=1.63$ ft.; which is so little different from e_0 that the work of determining the best speed v_n (which assumed $e_0=e_n$) need not be recast. (Or, the thickness of vanes, t_n, at exit, might be made a little smaller than that, t_0, of guides near entrance of wheel, in the proportion of $\sin\delta$ to $\sin\alpha$. In that case e_n would be more nearly equal to e_0.)

The final dimensions of the turbine, then, are:

Outer diameter $=2r+e_0=10.27$ ft.
Inner diameter $=2r-e_0=7.17$ ft.

The depth of the wheel ($=\overline{ED}$ in Fig. 62) may be taken as $r\div5$; that is, as 0.87 ft.

The probable efficiency to be expected is some 80 per cent. or over (full gate), this being about the figure reached (83 per

cent.) in the test of a turbine, closely resembling that of the present example, at Goeggingen, Germany (see Bodmer's Hydraulic Motors, p. 365), at its best speed of 45.5 revs. per min. (and full gate).

In the present example, from the value of the best linear speed, $v_n = 22.72$ ft. per second, of the extremity of the mean radius, we compute the angular speed as follows, if n denote the revs. per unit time:

$$(2\pi r)n = v_n; \quad \therefore \; n = v_n \div 2\pi r; \quad \text{or,}$$

$$n = 22.72 \div (2\pi \times 4.36) = 0.83 \text{ revs. per sec.,} \quad = 49.8 \text{ revs. per min.}$$

CHAPTER VI.

Testing and Regulation of Turbines.

95. The Prony Friction-brake.—In case a turbine is used to run an electric generator on the same shaft, its power at different speeds may be tested by electrical measurements applied to the electric current produced; but ordinarily, if the turbine is not too large, use is made of a *Prony friction-brake* (see p. 158, M. of E.) applied to the rim of a pulley secured on the shaft of the motor. By the tightening of the break more or less friction is produced on the pulley-rim, and the value of this friction becomes known by the weights necessary to hold the brake in equilibrium; that is, to prevent the brake from being turned by the pulley which moves within it. If the pulley has a horizontal axle, the weights are suspended from the extremity of a lever projecting from the brake and forming a rigid part thereof; but in case the shaft of the pulley is vertical, the rod, or scale-pan, on which the weights are suspended is connected with the rim of the brake by a bell-crank lever.

The friction thus provided, and measured, becomes for the time being the only resistance R' (lbs.) besides the axle friction of the shaft itself The linear velocity, v', of the rim of the pulley becomes known from a measurement of the rate of rotation (revs. per minute) and the radius of the pulley-rim. In any one experiment, then, the power $R'v'$ (ft.-lbs. per second) is the power developed by the turbine over and above that ($R''v''$) spent on axle friction. If the brake is tightened sufficiently, all motion on the part of the turbine may be prevented

and the power is zero, since v' is zero. On the other hand, if no resistance of friction or otherwise (except axle friction) is provided at the pulley-rim or elsewhere, then, although a high rate of rotation may be maintained by the turbine when thus run "unloaded." the useful power developed is again zero; since R' is zero. Roughly speaking, it may be said that the speed of steady motion assumed by a turbine when thus run "unloaded" is about double that to which it adjusts itself in steady motion when a resistance R' is applied of such value that the product $R'v'$, or useful power, is a maximum.

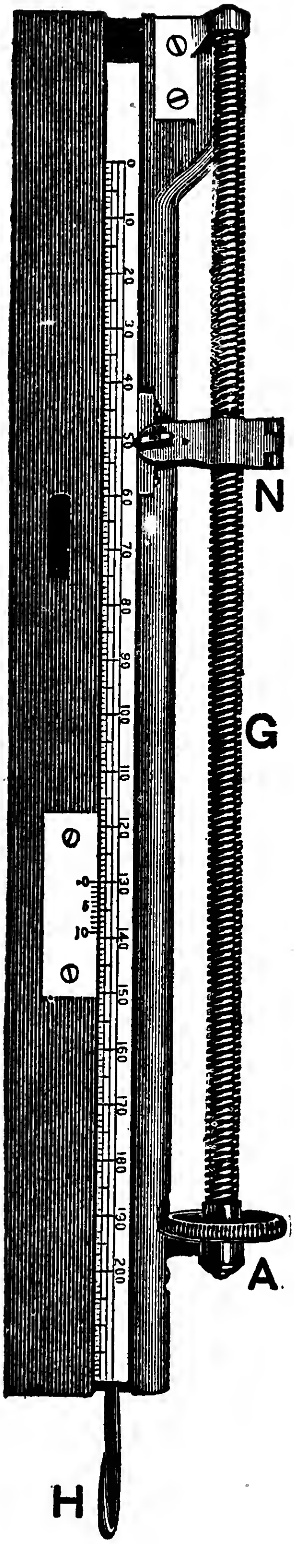

Fig. 79.

96. The Hook Gauge.—A useful instrument employed for the determination of the position, or change of position, of the horizontal surface of a body of still water, is the "*hook gauge.*" If a vertical rod of metal have its lower end turned upward in the form of a hook, with a fine sharp point, as in Fig. 78, the point of the hook can be adjusted to the level of the water surface with great precision. The instant when, in its upward motion, the point of the hook is just about to break through the surface can be noted with

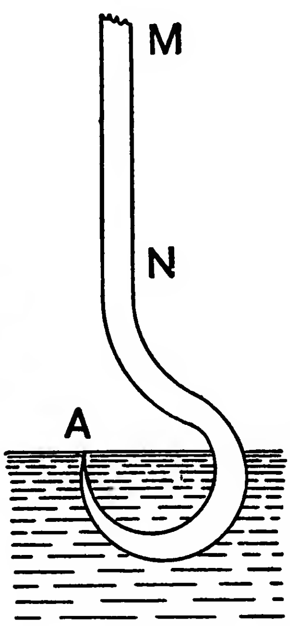

Fig. 78.

great exactness by the eye of the observer, if the water is quiet; and the upward motion of the stem MN may then be arrested. Fig. 79 shows such a hook, H, attached to a graduated rod (like a leveling-rod) supported between two fixed vertical guides carrying a vernier reading to thousandths of a foot. A nut, N, is attached to, and projects from, the rod; and both nut and rod are caused to travel vertically when the milled head A, and with it the screw G, is turned. If the vertical distance of the zero of the vernier above the sill of a weir (for instance) is known, and also that of the zero of the scale above the point of the hook, the vertical height of the point of hook above the sill of weir at any time is easily computed from the observed reading of the vernier on the scale. The instrument in Fig. 79 is one of those used by Mr. James Emerson in connection with his work of turbine-testing when in charge of the testing-flume at Holyoke (to be described in the next paragraph). The engraving in Fig. 79 is reproduced from Mr. Emerson's book "Hydrodynamics" (1881), which gives records of his numerous tests, with related matter.

97. The Holyoke Testing-flume.—At Holyoke, Mass., where the Connecticut River furnishes a large water-power, falling some 60 feet, the Holyoke Water-power Co. controls the water rights and leases power to the many mill-operators of that city. The mill-owners pay a certain price per annum per "mill-power," which in that locality is the right to use 38 cub. ft. of water per second under a head of 20 ft., either for continnous use (a 24-hour day) or for a definite fraction of each day.

The fall of 60 ft. in the river is divided into three parts or steps, two intermediate canals having been constructed at proper levels, in such a way that the tail-water for the highest, or next highest, series of mills forms the head-water of the next lower series; while the water from the third, and lowest, series is discharged into the lower river. In order that the rate at which any mill turbine uses water at any stage or position of its gate or regulating apparatus may become known by simply observing the position of the gate, each turbine,

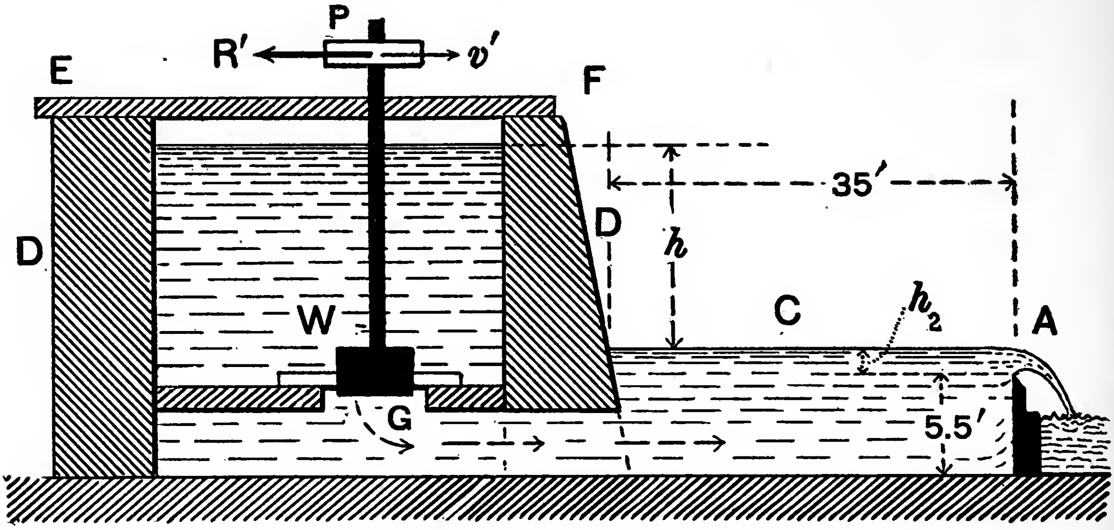

Fig. 80

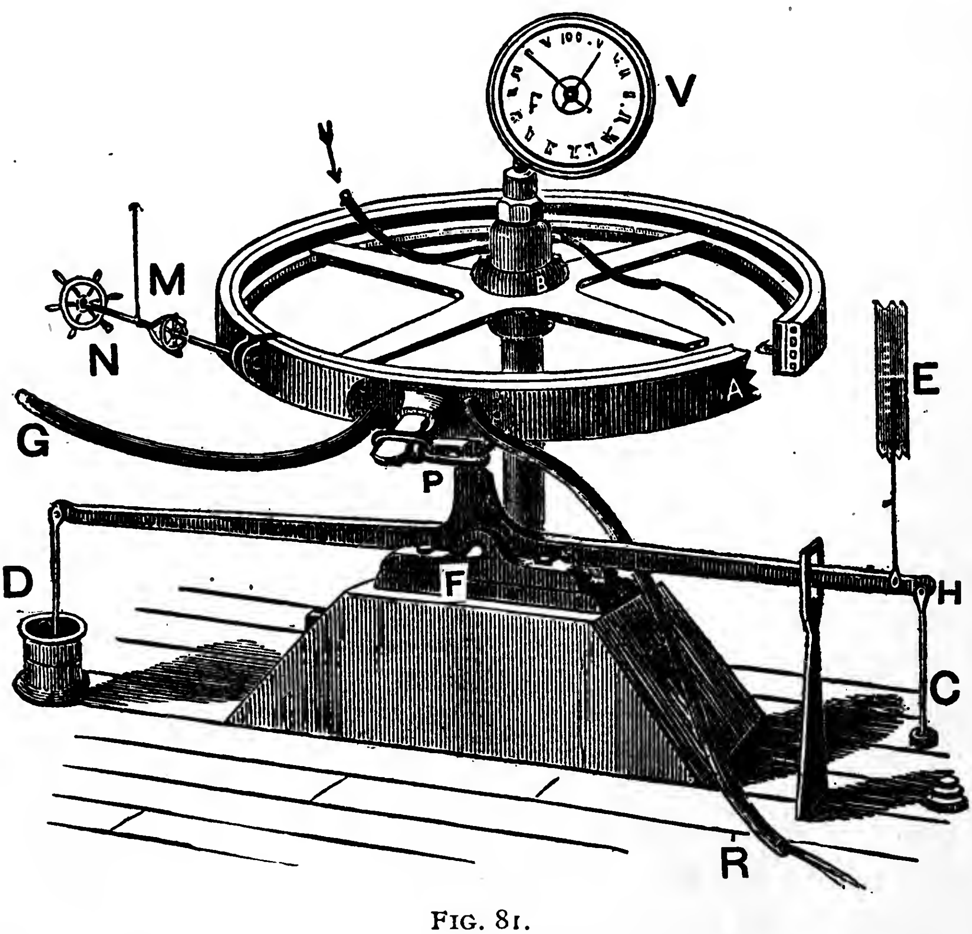

Fig. 81.

before being installed in the mill where it is to work, is tested at the "testing-flume" of the company and thus becomes a water-meter; whose indications, when the motor is in final place, are noted from day to day by an inspector of the company. In the same test its power, best speed, and efficiency are also determined.

The testing-flume occupies the lower part of a substantial building, and its main features are shown in vertical section in Fig. 80. The walls of the wheel-pit *DD*, which is 20 ft. square, are built of stone masonry and lined with brick laid in cement. The water is admitted to it from the head canal through a trunk, or penstock, and vestibule, which are not shown in the figure. Over an opening in the floor of the wheel-pit the wheel to be tested, *W*, is set in place, the water discharged from it finding its way through a large opening into the tail-race *C*, 35 ft. long and 20 ft. wide, and finally over a sharp-crested weir, at *A*, into the lower canal. The whole head *h* available for testing may be from 4 to 18 ft. for the smaller wheels, and from 11 to 14 ft. for large wheels, up to 300 H.P. The measuring capacity of the weir, which may be used to its full length, 20 ft. (and then would have no end-contractions), is about 230 cub. ft. per second. The head *h* becomes known in any test by observations on the water level in two glass tubes communicating with the respective bodies of water *W* and *C*. The water in channel *C*, which is a "channel of approach" for the weir *A*, communicates (at a point some distance back of the weir) by a lateral pipe with the interior of a vessel open to the air, in a side chamber. Water ri es in this vessel and finally remains stationary at the same level as that of the surface n the channel of approach. A hook gauge being used in connection with this vessel, observations and readings are taken from which the value of h_2, or "head on the weir," may be computed; for use in the proper weir formula for the discharge, Q.

Fig. 80 shows a turbine, in position for testing, with a vertical shaft—the more ordinary case. Upon the upper end of the shaft is secured a cast-iron pulley, *P*, to the rim of which the Prony brake is fitted for purposes of test.

98. The Prony Brake and its Use.—The style of Prony friction-brake used by Mr. Emerson in 1880, and for some twenty years afterwards by his successors, at the Holyoke Testing-flume is shown in Fig. 81. Upon the rim of the cast-iron pulley, *B*, keyed upon the shaft of the turbine is fitted the hollow brass band *A* (shown also in section at *A*), the friction of which upon the pulley can be varied by the tightening or loosening of the screw at *M*, this screw being turne by a hand-wheel *N*. Both the rim of the pulley and the friction band are hollow, water being circulated through them by the use of the flexible hose *G* and *R*. The pulley revolves clockwise, as seen from above, and the brake in its tendency to revolve wi h the pulley exerts a horizontal pull toward the left (through the projecting arm shown) upon the upper end *P* of the vertical arm of the bell-crank lever *PFH* (fulcrum at *F*). A sufficient weight hung at *C* holds the bell-crank in equilibrium and a pointer playing along a scale at *E* indicates when the lever is in its median position. At *D* is attached to the lever a vertical rod carrying at its lower end a piston fitting loosely in a fixed vertical cylinder containing oil or water. This is called a "*dash-pot*," its object being to prevent sudden motions of the lever, since while the resistance of the oil to the motion of the piston is practically nothing for a slow motion, it is very great for a sudden movement. In this way oscillations are controlled. At *V* is a counter from which the number of revolutions made by the wheel in a given time becomes known.

The procedure of testing was about as follows: The brake being carefully balanced and adjusted beforehand, a light weight was placed on the scale-pan, and the wheel started at full gate; sufficient friction was then produced to balance the weight, and the speed of wheel noted. "The load was then increased at intervals of two or three minutes, by 25 lbs. at a time, until the speed of the wheel had fallen below that of maximum efficiency for the head; the weights were then reduced again and the velocity of the wheel allowed to increase until the maximum was again passed. The same process was then repeated within a smaller range of speed and with smaller

variations of load, until the speed of best work had been more exactly ascertained, and the performance of the turbine at maximum efficiency, under full head and at full gate, had been very precisely determined. This was repeated at each of the part gates, usually down to one half maximum discharge." *

A letter from Mr. A. F. Sickman, present engineer in charge (1905) of the Holyoke Testing-flume, states that up to April 11, 1905, 1542 wheels have been tested in the flume; and adds: "We use the Emerson brass brake but seldom now, having a full set of Prony brakes—home-made, cast-iron pulleys with wooden jackets, giving very satisfactory work."

99. Test of the Tremont Turbine.—The test of the "Tremont Turbine," a 160-H.P. turbine of the radial outward-flow type (Fourneyron) made at Lowell, Mass., in 1855 by Mr. J. B. Francis, was an event of special interest in the history of hydraulic science and has become classic. Though the test is by no means recent, it was carried out so thoroughly as to make its details highly instructive to the student of hydraulics. The main features of this test will now be presented and commented on.†

The inner and outer radii of the turbine, whose shaft was vertical and whose general arrangement was like that of Fig. 45, were 3.37 and 4.14 ft. respectively; height between crowns, 0.937 ft. at entrance and 0.931 at exit. There were 33 guide-blades and 44 turbine-vanes. As to angles, $\alpha=28°$, $\beta=90°$, $\delta=22°$ (see Figs. 45, 47, and 48). The areas F_0 and F_n were 6.54 and 7.69 sq. ft., respectively; and the head, h, on the wheel about 13 ft. (see details, later). The gate was a thin cylinder, movable vertically, between the guides and the wheel. There were no horizontal partitions dividing up the wheel-channels; in fact, no special device for preventing the loss of head usually arising at part gate with this kind of regulating apparatus.

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* Quoted from Prof. Thurston's paper on the "Systematic Testing of Turbine Water-wheels in the U. S.," in the Transac. Am. Soc. Mech. Engrs., for 1887.

† Full particulars may be found in Mr. Francis' book, "Lowell Hydraulic Experiments," New York, 1880.

TEST OF THE TREMONT TURBINE.

(Selected Experiments.)

1	2	3	4	5	6	7
No. of Exper.	h feet.	n' revs. per sec.	Q cub. ft. per sec.	$R'v'$ ft.-lbs. per sec.	η effic.	H.P.
			FULL GATE.			
1	12.80	0.00	135.6	0	0.00	
2	12.95	0.45	133.4	73,160	.68	
3	12.97	0.53	133.7	78,490	. 2	
4	12.97	0.60	134.8	82,110	.75	
5	12.94	0.64	135.1	83,960	.77	
6	12.90	**0.85**	**138.2**	88,210	**.794**	**160.3**
7	12.90	0.88	139.0	88,190	.788	
8	12.90	0.90	139.6	88,076	.784	
9	12.85	1.00	141.9	86,310	.75	
10	12.85	1.06	142.5	83,970	.73	
11	12.80	1.18	144.8	77,150	.67	
12	12.70	1.31	147.3	66,840	.57	
13	12.65	1.46	152.3	51,680	.43	
14	12.55	1.60	156.6	33,350	.27	
15	12.54	1.79	162.3	0	0.00	
			PART GATE.			
16	13.51	0.00	60.3	0	0.00	
17	13.55	0.46	67.8	24,460	.43	
18	13.48	**0.67**	**71.8**	27,980	**.46**	**50.9**
19	13.39	0.96	76.6	21,250	.33	
20	13.34	1.25	80.4	0	0.00	

A large and strong friction brake was used for the test, with arcs of wood rubbing on the cast-iron pulley which was keyed to the turbine shaft, and was arranged with a bell-crank lever as in Fig. 81, and also a "dash-pot." The various lever-arms concerned were of such values that, with G denoting the necessary weight at C for the equilibrium of the brake in any experiment, the corresponding value of the friction at the rim of the pulley was $R' = 3.938G$ lbs.

The rate of flow, or discharge, Q cub. ft. per second, was measured by two weirs at the end of the tail-race, somewhat as in Fig. 80, use being made of the "Francis Formula" for weirs (see p. 687, M. of E.); while the useful power, $R'v'$ (ft.-lbs. per second spent on friction at rim of pulley), was computed from the relation

$$R'v' = (3.938G)(2\pi\ 2.75n'),\ \text{ft.-lbs. per sec.}, \quad . \quad . \quad (1)$$

in which G is the weight on scale-pan in lbs. and n' the number of revs. per second of the turbine in any experiment (steady operation). The radius of the friction-pulley was 2.75 ft.

The annexed table gives the principal data and results of Mr. Francis's test of the Tremont Turbine, arranged in the order of the speed of wheel. In Experiments Nos. 1 to 15 (see column 1) the cylindrical gate was fully open ("full gate"), while in experiments 16 to 19 it was in a single fixed position leaving open, at the wheel-entrance, about one quarter of the vertical height between crowns; in other words, the gate was drawn up about one quarter of its full range of height. In this special "part-gate" position, however, the quantity of water passing per second was much greater than one quarter of that passing at "full gate"; as is seen from the values of Q in column 4. For example, in Exper. 17, in which (for this position of the gate) the efficiency was a maximum, the value of Q is about one half of the Q used in Exper. 6, which gives the maximum efficiency at full gate. It would be said, therefore, that in Exper. 17 the wheel was working at about "half gate." The heading of each column of the table shows clearly the nature of the quantity given in that column and the units of measurement involved in its numerical value.

The computations relating to a typical experiment will now be given, Exper. No. 6 being selected. In this experiment the weight placed on the scale-pan was 1524 lbs. Hence, when the brake was tightened sufficiently so that the wheel raised the weight and held it balanced, the friction was $R'=3.938\times 1524=6001$ lbs. The speed of the wheel having adjusted itself in this experiment to a rate of $n'=0.851$ revs. per second, the linear velocity of pulley-rim (its radius being 2.75 ft.) was $v'=2\pi rn'$, $=2\pi\ 2.75\times 0.851$, $=14.70$ ft. per sec. Hence the useful work done per second was $R'v'=6001\times 14.70=88,214$ ft.-lbs. per sec.

As to the value of Q, the combined length of the two sharp-edged weirs in vertical "thin plate" was $b=16.98$ ft., the number of end-contractions was $n=4$ (two weirs), and the head on the weir $h_2=1.87$ ft. (velocity of approach negligible). Hence, from the formula

$$Q=3.33(b-0.1nh_2)h_2^{\frac{3}{2}},\ \text{cub. ft. per sec.,} \quad . \quad . \quad (2)$$

which is the same as eq. (14) of p. 687, M. of E., when 32.2 is put for g (that is, for the foot and second as units), we have

$$Q=3.33[16.98-0.1\times 4\times 1.87]\times(1.87)^{\frac{3}{2}}=138.2\ \text{cub. ft. per sec.}$$

The difference of elevation of head- and tail-waters was 12.90 ft., so that $Q\gamma h$ was $138.2\times 62.5\times 12.90$, $=111,400$ ft.-lbs. per second; and consequently the efficiency, η, $=(R'v')\div(Q\gamma h)$, $=0.794$; or 79.4 per cent.

100. Discussion of the Test of Tremont Turbine.—See table on p. 156.) In the experiments with full gate, Nos. 1 to 14 inclusive, on account of the progressive lessening of the weight G in the scale-pan (the brake friction being regulated each time to correspond) the uniform speed to which the wheel adjusts itself in successive experiments increases progressively from the zero value, or state of rest, of Exper. 1, when the friction was so great as to prevent any motion, up to a maximum rate of 1.79 revs. per sec., attained when no brake friction whatever ("no *load*") was present. In this last experiment, there being no useful work done, all the energy of the

mill-site is wasted, partly in axle friction, but chiefly in fluid friction (eddying and foaming of the water; finally, heat) both in the wheel-passages and also in the tail-race, where the water which has left the wheel with high velocity soon has its velocity extinguished. The same statement is true, also, for Exper. No. 1, except that axle friction is wanting. In both experiments the efficiency is, of course, zero.

The quantity of water discharged per second, Q, is seen to increase slowly (after Exper. 2) from 133.4 to 162.3 cub. ft. per sec., though not differing from the average by more than ten per cent. This may be accounted for in a rude way as an effect of "centrifugal action" (as in a centrifugal pump), since the Tremont turbine is an outward-flow wheel. The reverse is found to be true for inward-flow turbines, notably the Thomson vortex wheel (see § 90), which is therefore to some extent self-regulating in the matter of speed; since a less discharge at a speed higher than the normal diminishes the power and hence the tendency to further increase of speed.

In the succession of experiments Nos. 1 to 15 (all at **full gate** and under practically the same head h) the efficiency is seen to have a zero value both at beginning and end of this series, and to reach its maximum at about the 6th experiment, in which the speed is noted as being about one half that at which the turbine runs when entirely "unloaded" (Exper. 15). This is roughly true in nearly all turbine tests, but a notable feature of considerable practical advantage is that a fairly wide deviation from the best speed affects the efficiency but slightly. For instance, a variation of speed by 25 per cent. either way from the best value (of 0.85 revs. per sec.) causes a diminution in the efficiency of only about four per cent.

It should be remembered, also, in this connection, that since the water used per sec. (i.e., Q) is somewhat different at different speeds (at full gate), the speed of maximum power differs slightly from that of maximum effic ency.

In the five "**part-gate**" experiments, Nos. 16 to 20, the gate remains fixed in a definite position (about one quarter raised; although the discharge is about *one half* that of full

gate) through all these five runs. The head is practically constant. As first the wheel is prevented from turning. The power and efficiency are then, of course, zero; but $Q=60.3$ cub. ft. per sec. As the turbine is permitted to revolve under progressively diminishing friction (R'), the speed of steady motion becomes greater, reaching its maximum (1.25 revs. per sec.) when the wheel runs "unloaded," in Exper. 20; but the power, or product, $R'v'$, reaches a maximum and then diminishes. The same is true of the efficiency, whose maximum (in Exper. 18) is seen to be about 46 per cent., *only*. This forms a striking instance of the disadvantage and wastefulness of a cylindrical gate, unaccompanied by other mitigating features, when in use at part gate. This defect, however, may be largely remedied by the use of horizontal partitions in the wheel-channels, as in Fig. 46, or by employing curved upper crowns, as in the American "inward and downward" turbines.

101. Tremont Turbine Test. Graphic Representation.— Taking the angular speed revs. per sec. as an abscissa, and the efficiency as an ordinate, points on paper may be plotted and the curve thus formed called an "efficiency curve," showing the variation of the efficiency with the speed of the turbine. Such a curve is shown as $OAaB$ in Fig. 82, having been plotted from the fifteen full-gate experiments of the table on p. 156. The point of maximum efficiency occurs at a, and the scale of efficiency is marked on the vertical axis at the left. Similarly, if the values of Q are used as ordinates, with the speeds as abscissæ, the curve CE results, showing very plainly the gradual increase of Q, with the speed, after the second and third experiments. The scale for Q is given along the right-hand edge of the diagram. Similarly, the smaller curves GF and NM show the variation with speed of the efficiency and discharge, respectively, for the five part-gate experiments. In all the curves the point corresponding to each experiment is shown by a small circle.

(Details of many other tests may be found in Mr. Bodmer's book, in Mr. Emerson's Hydrodynamics, and in technical journals.)

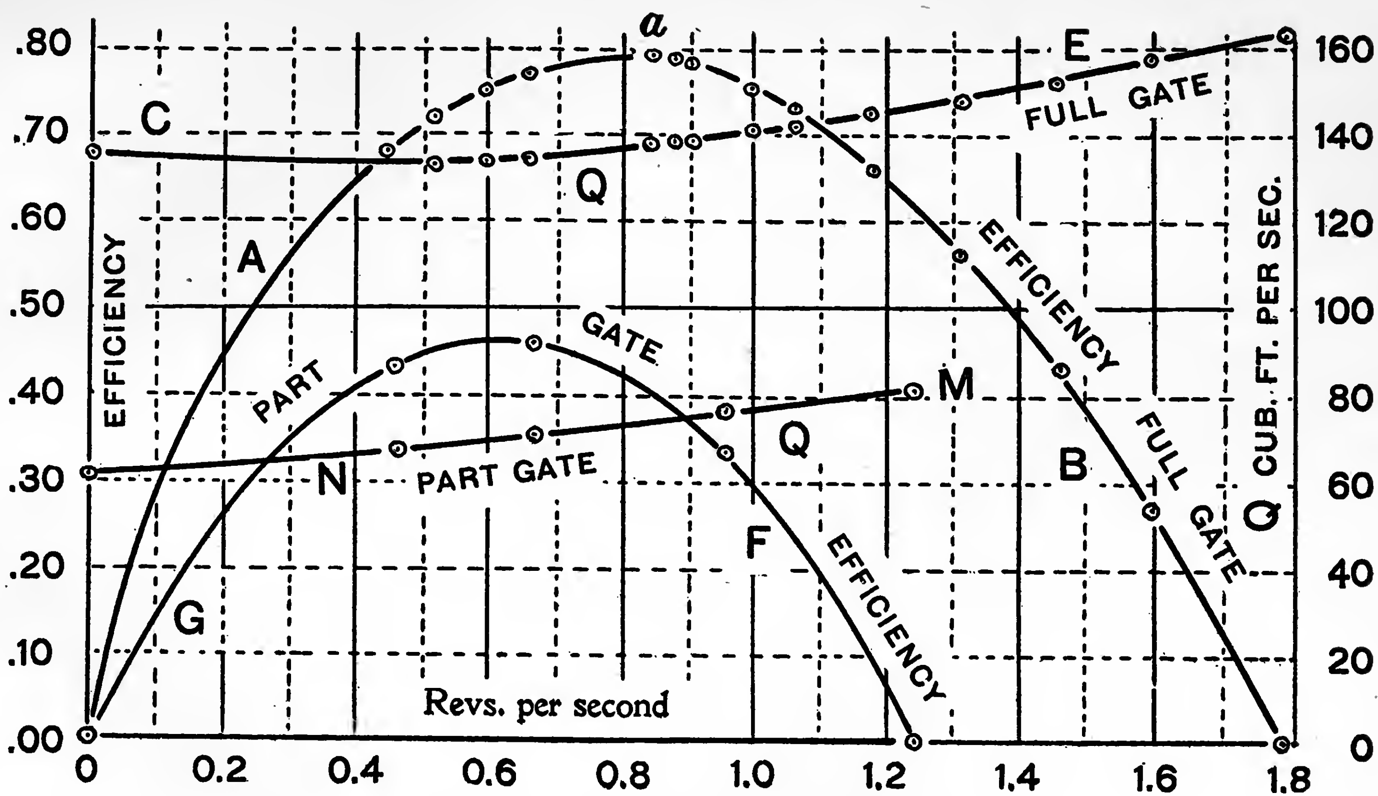


FIG. 82. Tests of the Tremont Turbine.

102. Regulating "Gates" of a Turbine.—When a variable power is demanded of a turbine, as when in a factory the number of machines operated is not constant, or when the amount of electric current generated in a dynamo run by the turbine is required to be variable to suit the varying demands of street-railway work or electric lighting, the average position of the turbine gate is not that of "full gate." Since the speed of the turbine should be fairly constant, especially for electric work (and this has to do with the question of governors treated in § 103), the required variation in power must be provided by varying the amount of water used per second, i.e., Q; and this requires movement of the gate or regulating apparatus. It is therefore of importance, where economy in the use of water is necessary, that a turbine should have a fairly high efficiency at "part gate."

At the outset the statement should be emphasized that perhaps the most wasteful device for varying the discharge of water is the "throttling" of the flow by the use of a gate in the penstock or supply-pipe, or in the draft-tube (see § 51 in this connection); or by the use of a cylindrical gate encircling the lower end of a draft-tube; since these either induce losses of head due to sudden enlargement of waterway, or bring about impact of the water at the turbine entrance, where for the usual speed of wheel the value of the angle β is only suited to a fixed value of the velocity w_1.

The plain cylindrical gate moving axially is open to similar objections, unless, as already stated, the turbine channels are provided with partitions or their equivalents, or have an upper crown which curves downward.

Perhaps the most perfect "gate," from a theoretical point of view, for a radial turbine is the device of Nagel and Kaemp, in which not only are the "roofs" of the guide-passages movable, but also the corresponding crown of the turbine. The crown being always opposite, even with the roof, sudden enlargement at the turbine entrance is prevented in all positions of the regulating apparatus. The turbine thus becomes one of variable

height, e, between crowns. This design is, however, expensive and attended with practical difficulties.

The three kinds of gate often used with American "inward and downward" turbines (viz., the cylinder, register, and wicket gate) have been already mentioned in § 86. See also § 79.

The regulation of the Jonval or parallel-flow ("axial") type of turbine is usually accomplished by sliding plates or swinging flaps for closing of the guide-passages. The entire closing of a number of the guide-passages, instead of the partial closing of all of them, is found to conduce to a higher efficiency; since in the former case the value of the absolute velocity (w_1) at entrance of the turbine remains the same as when all the guide-passages are in use. (See Bodmer for many further details; also Buchetti, and Mueller.) The "Duplex" Jonval wheel, made by R. D. Wood and Co. of Philadelphia, has already been referred to in § 86.

In this connection attention should be called to Mr. Thurso's valuable article, already mentioned in § 88.

103. "Mechanical" Governors for Turbines.—The power to move the turbine gates is usually furnished by the turbine itself; but, more frequently, in large modern plants, by a hydraulic "relay" motor, or hydraulic piston and cylinder actuated by water or oil; pressure-water from the penstock, or oil from a pressure-tank (compressed air above the oil). The "governor" proper consists of revolving centrifugal balls and their connections whose change of relative position, brought about by a slight change in the speed of the turbine (with which the governor is in gear), moves the proper valves or other parts necessary to bring into play the motor or mechanism which moves the turbine gates. Electric governors have been used, but not extensively.

A mechanism which in its general form has been long in use in cases where the turbine itself furnished the power directly for moving the gate, and furnishing an instance of a "mechanical governor," is illustrated in the King water-wheel governor, shown in Fig. 83. The turning of the horizontal shaft 7, caused

by the rotation of the spur-wheel 1, moves the turbine gate. The vertical shaft, S, carrying the centrifugal balls (B, B) rotates at a speed proportional to that of the turbine, being in gear with the latter; and also causes the continuous rotation, in one direction, of the wheel 5, connected by a crank and connecting-rod with arm, or crank, 4. There is thus brought about a continual to-and-fro horizontal motion of the upper end of arm 4, to which are pivoted two "pawls," 2 and 3, either of which, if hanging low enough to do so, would by a succession of direct thrusts against the cogs turn the wheel 1, and thus either open or close the turbine gate, according to which pawl might be in action. When the speed of the turbine is normal neither pawl can turn the wheel 1, since in that case its extremity is held out of contact with the cogs of 1 by a projecting "peg" which slides along the edge of the thin disc 6. At normal speed of turbine the disc 6 is stationary and in its median position; but when that speed changes and the balls consequently change their distances from the axis of the vertical shaft, the vertical spindle 8 is moved either up or down and rotates disc 6 sufficiently to bring one or the other of two depressions (in the edge of the disc) under the "peg" of one of the pawls, thus allowing the pawl to drop and actuate wheel 1, which then moves the gate in the proper direction.

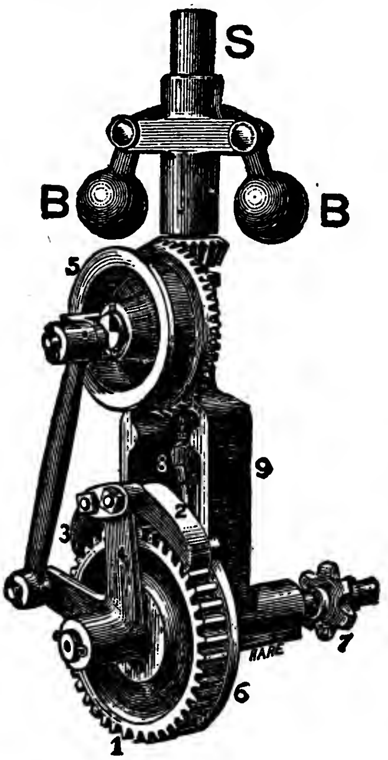


Fig. 83.

The "Snow Water-wheel Governor" has been extensively used both in England and America, using practically the same design of pawls, etc., as in the King governor, and is shown in Fig. 84. The turbine gate is moved by the turning of the vertical shaft PA, which can also, on occasion, be actuated by the hand-wheel at upper end. The two lower

SNOW'S

WATER-WHEEL GOVERNOR

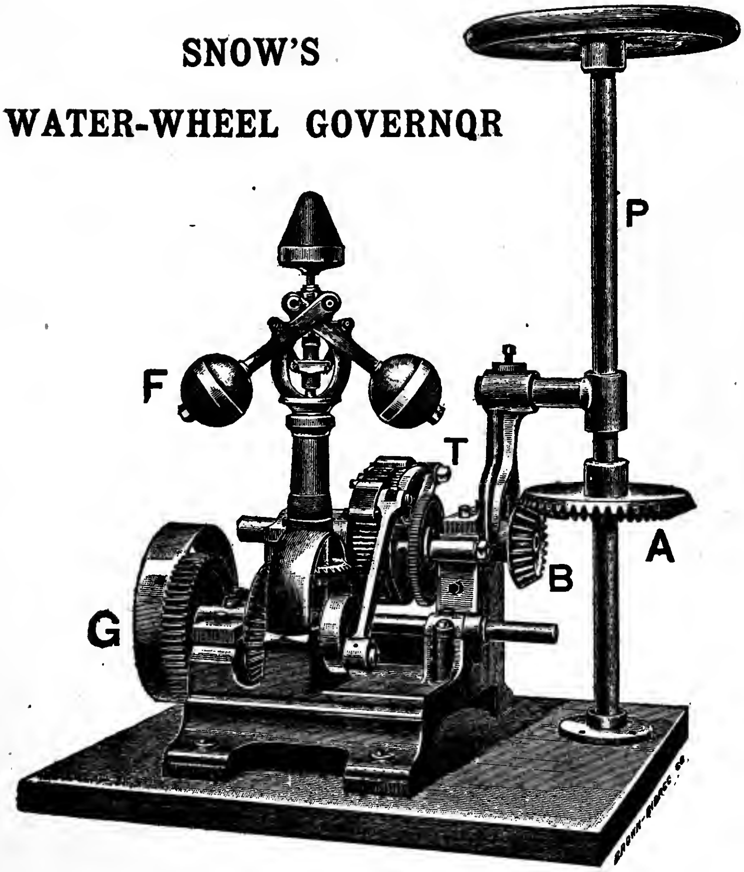


FIG. 84.

horizontal shafts at G turn continuously, being in gear with the turbine, but the third one, B, turns only, and in the proper direction, when the speed of the turbine changes slightly from the normal, and moves the turbine gate by means of the bevel-gear at B.

104. Hydraulic Governors for Turbines.—The foregoing are called "mechanical governors," the power for moving the gate being furnished directly by the turbine itself. A **"hydraulic governor"** made by a prominent American firm, the Lombard Governor Co. of Ashland, Mass., and called "Type N" (among their various designs), is shown in Fig. 85. The vertical hydraulic cylinder, with piston ("main piston"), etc., constituting the "relay motor," occupies the lower half of the mechanism in the figure. To the cross-bar secured to the upper end of the (main) piston-rod are attached two vertical racks by whose motion the horizontal shaft (seen projecting out at the right) is made to turn and actuate the turbine gate. This shaft can also be rotated, if necessary, by the large hand-wheel seen in front. The small pulley near the top (on left) is belted to another, actuated by the turbine shaft, and continual rotary motion of the centrifugal balls results. These balls when rotating at normal speed stand out considerably from the axis of rotation. The mechanism is such that if the balls spread out under the action of an incresae of speed, they depress the top plate into which the flat springs supporting them are inserted; and *vice versa*. This top plate is attached to a rod which passes down through the hollow vertical shaft carrying the balls, and terminates in a small "primary valve," a slight motion of which from its normal position causes admission of oil (under pressure) to actuate the hydraulic plungers of a "relay-valve" device, whose motion causes movement of the main valve. The office of the main valve is to admit oil from a pressure-tank to one side, or the other, of the main piston whose motion, through the vertical racks and gear-wheel, causes motion of the turbine gate. The other side of the main piston is at the same time put into communication with the "vacuum-tank."

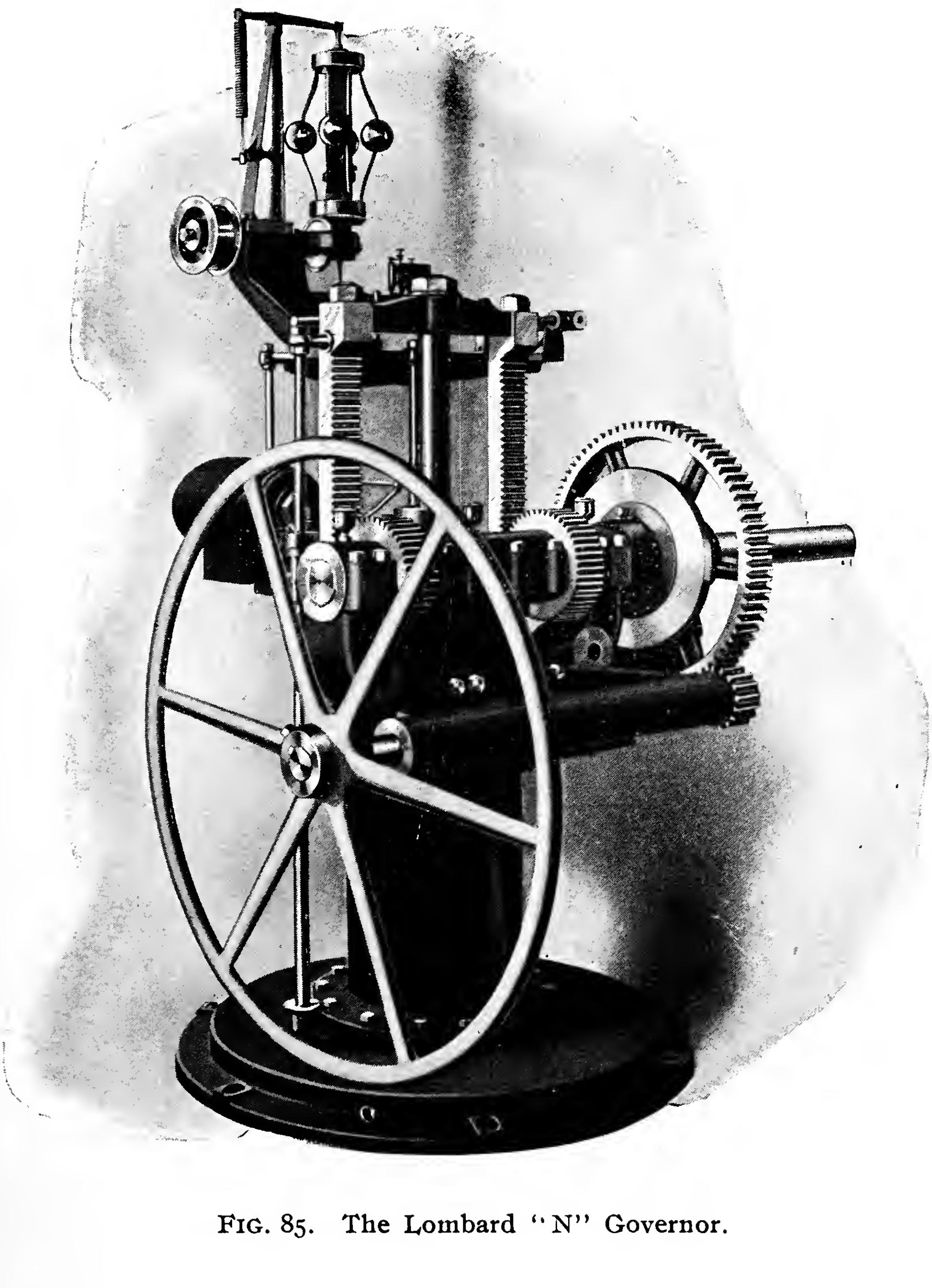


Fig. 85. The Lombard "N" Governor.

A pressure-tank (not shown in the figure) contains compressed air and oil under about 200 lbs. per sq. in. pressure and supplies oil for the main, and relay, hydraulic cylinders. Pumps run by the turbine itself pump the oil back into the pressure-tank from the vacuum-tank as occasion requires. One complete stroke of the main piston entirely opens or closes the water-wheel gates; consequently any motion of this piston less than a complete stroke causes a proportionally smaller motion of the gates.

The Allis-Chalmers Co. of Milwaukee, Wis., manufacture the hydraulic-governor designs of the Swiss firm Escher, Wyss, and Co.

A description of the "Replogle Differential Relay" governor, made by the Replogle Governor Works at Akron, Ohio, may be found in the Engineering News of Nov. 13, 1902, p. 409. This governor has a heavy "inertia wheel" to supplement the action of the ordinary fly-balls when very prompt motion of the turbine gate is called for.

104a. Fly-wheels.—If a fly-wheel is placed upon the shaft of a turbine, the inertia of the mass so added tends to retard a change of speed on the part of the turbine when the "load" changes, thus giving the governor and its accessories more time to act, and enabling the speed to be kept within a smaller range of variation. The revolving part of an electric generator is sometimes made to serve the purpose of a fly-wheel, as occurred with the turbines in Power-house No. 1 of the Niagara Falls Power Co., no other fly-wheel being found necessary.

Mr. Thurso mentions the case of a 1000-H.P. turbine at Jajce, Bosnia, (see reference in § 88,) as using a hydraulic governor which keeps the speed within $1\frac{1}{2}$ per cent. of the normal. A small fly-wheel is employed to assist the governor.

CHAPTER VII.

CENTRIFUGAL AND "TURBINE" PUMPS.

105. Turbine as Centrifugal Pump.—Let us suppose that we have a radial outward-flow turbine in steady operation, as in Fig. 45 (which see), and that suddenly the depth of the tail-water is largely increased so that its surface T is at a *higher* elevation than that, H, of the "head-water" or supply reservoir. To keep up the same flow of water as before, radially outward through the turbine passages, will necessitate the application, to the turbine, of working forces from some external source of power, such as a steam-engine. That is, instead of providing a resistance R' lbs. at the periphery of the upper pulley M on the turbine shaft to prevent acceleration, we must now furnish a working force P lbs. (pointing toward the right on the near side of the pulley M) to prevent retardation. The work done by P each second is Pv ft.-lbs. per sec. and is employed in maintaining the steady flow. Since water is now being raised from a lower to a higher level, the turbine has become a pump; called a "*centrifugal pump*."

In actual centrifugal pumps there are ordinarily no guide-blades at G (Fig. 45) inside the wheel, but of late years (since 1900, about) many such pumps have been built with guide-passages *outside* the wheel (or "impeller," as it is called) with gradually enlarging passageways constituting a "diffuser," to diminish losses of head at that point; with consequent improvement in efficiency. To this more recent variety of centrifugal pump the name "turbine pump" is now frequently applied (1905).

In a centrifugal pump the action of the water on the "im-

peller" is equivalent to a resisting couple, instead of a working couple, and the moment of the working force P about the axis of the shaft is numerically equal to that of this couple (augmented by moment of axle friction); the rotation being uniform and the flow "steady."

From the figure (45), the vanes of the turbine (now pump) being curved backwards as regards the direction of rotation, it is seen that these vanes tend to crowd the water radially outward; but even if the vanes were straight and were *radial*, the same general effect would be produced if the speed were sufficient; since, from its "inertia," a particle of water tends to persist in a straight-line motion and thus incidentally to increase its distance from the axis of the wheel. In a rough general way this outward flow of the water between radial vanes is sometimes said to be due to "centrifugal force," and rude methods of analysis have been based on this idea. It is better, however, to avoid these imperfect notions of "centrifugal force" and to use the relations that have been proved to apply to the steady flow of water in uniformly rotating channels and pipes; as already established in §§ 31–42*a* (see particularly §§ 35*a* and 42*a*). These relations were, of course, based on the fundamental laws of mechanics as applied to a material point.

106. Notation for Centrifugal Pump.—The number of vanes used in the majority of centrifugal pumps is so small (four to ten, perhaps) that the guidance of the water is far from perfect and consequently the theory now to be presented must be considered as giving results that are only roughly approximate; especially as the frictional conditions in these pumps are only imperfectly understood. Certain general indications, however, are clearly brought out by theory.

The pump to be considered is one with a horizontal shaft, and is placed above the source of supply, a suction-tube being therefore necessary. Fig. 87 gives a vertical section through the axis of the shaft; the shaft and the crowns or side plates of the "impeller," or revolving part, being shown in solid black shading. Steady flow of the water, with full pipes and

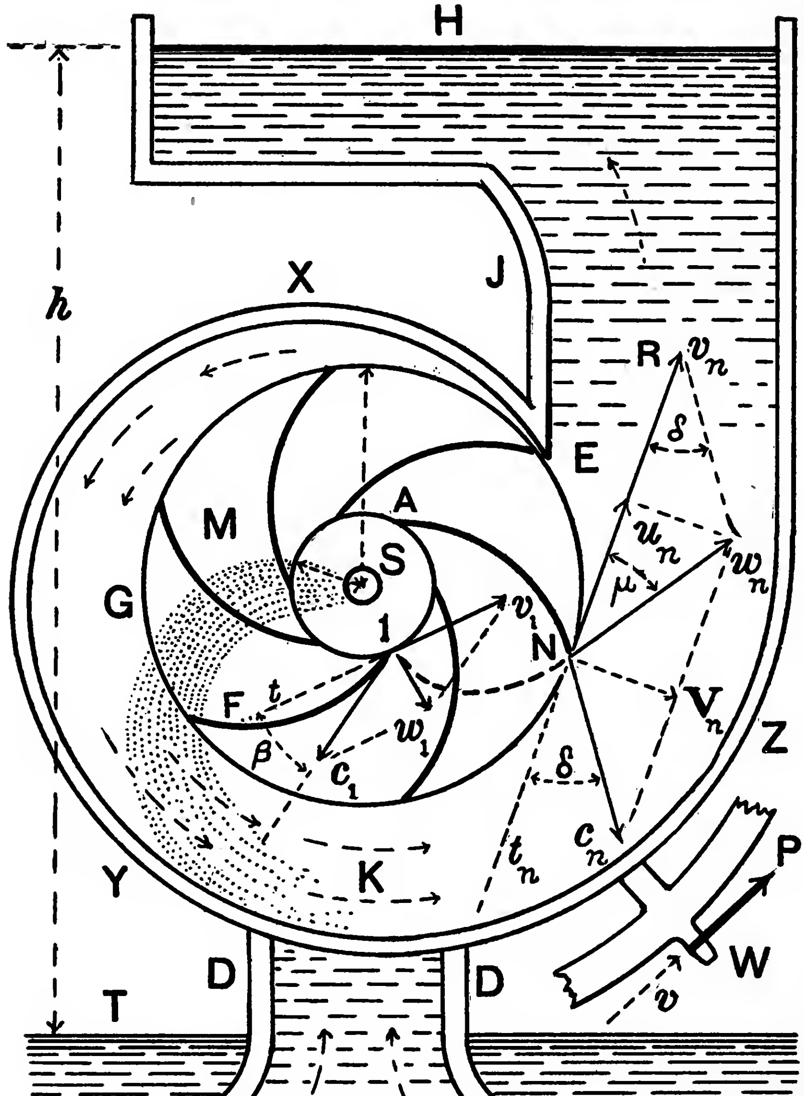


FIG. 86.

passageways, and uniform angular velocity ω of the impeller, are postulated. Fig. 86 gives a vertical section, at right angles to the shaft, showing the (six) impeller-blades or vanes, such as $A..N$, the supply-reservoir T, and receiving-reservoir H; also suction-tube (or supply-pipe) DD, conducting the water from T to the central space, S, of the impeller; and the delivery-pipe EJ. The casing, XYZ, within which the impeller rotates is of the scroll or "volute" form so commonly used; and may

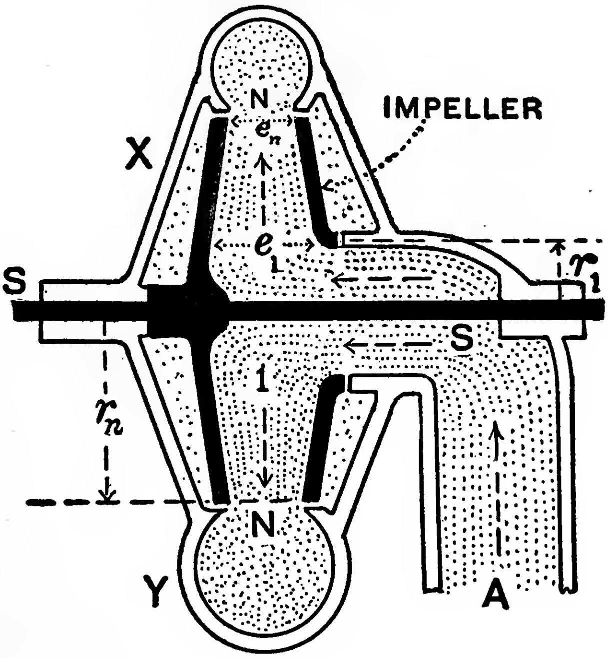


FIG. 87

be looked upon as a single external guide-blade, the average radial width of the volute space increasing from E toward X, G, and K, to provide for the increasing number of water filaments issuing from the outer edges of the impeller-vanes; hence the velocities of these filaments are about equal. All of these filaments have to pass through the horizontal section at E at the base of the delivery-pipe J. Upon the shaft is supposed to be secured a gear-wheel, W, at whose periphery ("pitch-circle") a constant tangential pressure, or working force, P lbs., is assumed to be acting; furnished by a motor of some kind (a steam-engine, say), and of suitable amount to maintain uniform motion of the pump and steady flow of the water.

The linear velocity of the point of application of P being denoted by v ($=\omega r$, if r is the corresponding radius), the power exerted by P is Pv ft.-lbs. per sec. At the entrance, 1, of the impeller channels the absolute velocity w_1 of the water is supposed to be radial, since there are no internal guides. The tangent to the impeller-blade at that point is supposed to be placed at such an angle β with $1..t$, the tangent to wheel-rim, or circle of rotation, at 1, so as to avoid impact. That is, the former tangent should follow the direction of the relative velocity c_1 at point 1. The linear velocity of wheel-rim at 1 is $v_1=\omega r_1$, and the width between crown-plates is e_1 (see Fig. 87). Similarly, v_n, e_n, and r_n refer to the exit wheel-rim, or N.

The absolute path of a particle of water from entrance to exit of wheel is shown by the dotted line $1..N$, the vane along which it moves having passed from position $1..F$ to position $A..N$. The absolute velocity w_n of the particle at N is the diagonal of the parallelogram on the wheel-rim velocity at N, viz., v_n, and the relative velocity c_n which is tangent to the vane curve at N and makes some angle δ with the wheel-rim tangent Nt. The angle between w_n and wheel-rim tangent is μ, so that its component tangent to the wheel-rim is $u_n=w_n \cos \mu$, called the "*velocity of whirl*," and its radial component is $V_n=w_n \sin \mu$, called the *velocity of flow*.

Evidently at the entrance, 1, the velocity of whirl is zero and the velocity of flow is $V_1=w_1$, itself.

Figs. 87*a*, 87*b*, and 87*c* show a section through the shaft, a section transverse to the shaft, and a perspective of the impeller, respectively, of the centrifugal pump made by the De Laval Steam Turbine Co. of Trenton, N. J. The impeller is of the "enclosed" type. (See § 114.)

107. Form of Loss of Head to be Considered.—In the theory now to be given the only loss of head (and corresponding waste of power) that will be considered will be that due to the more or less abrupt change of absolute velocity occurring in the water just after exit from the impeller passages. In the pumps of older design in which the water at exit is discharged into a body of water having a much smaller absolute velocity this loss

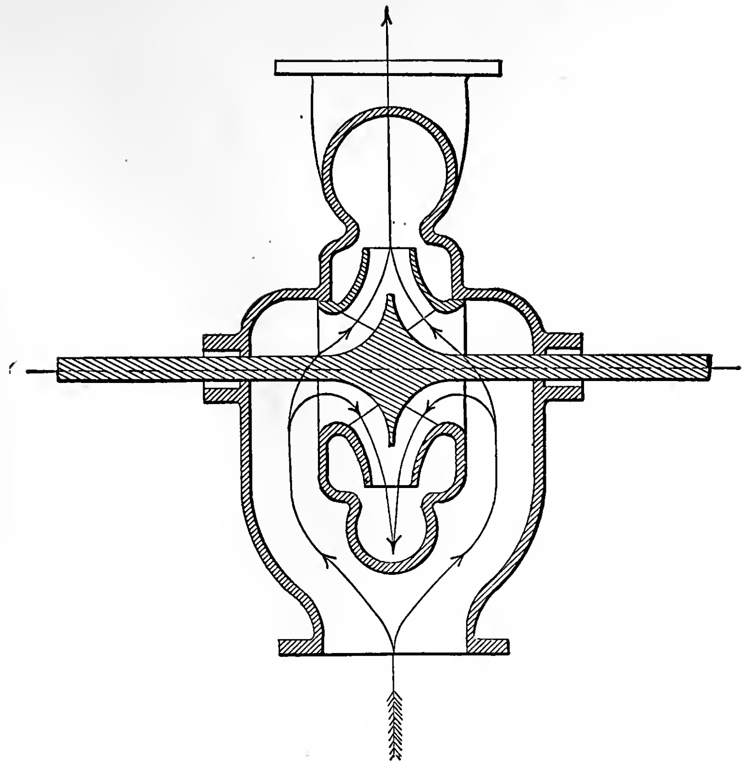


Fig. 87a.

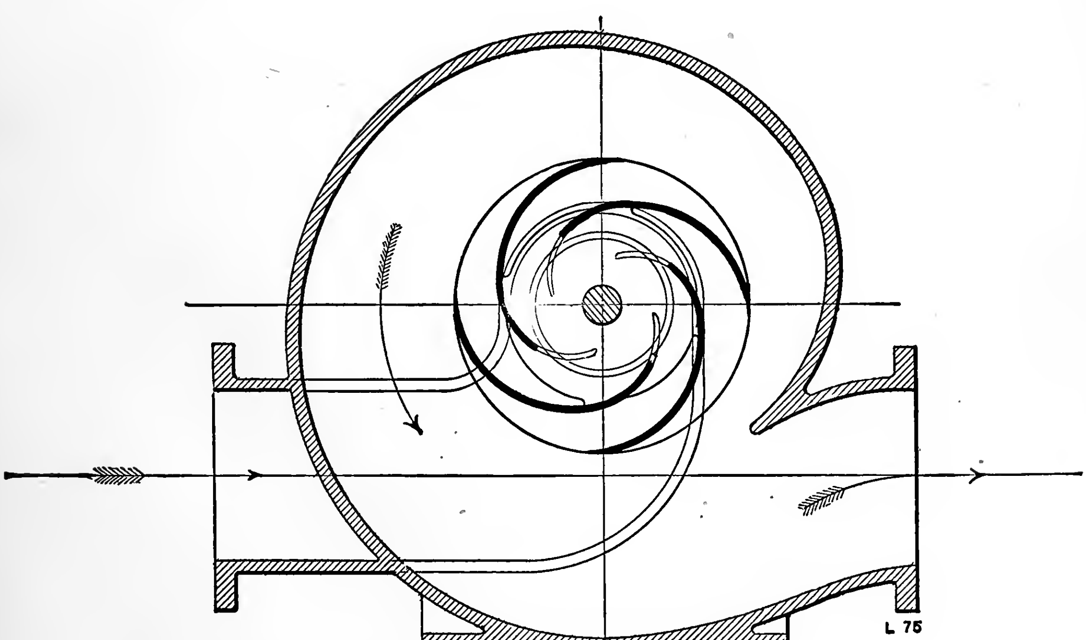


Fig. 87b.

of head is perhaps greater than that due to any other cause. It may be written in the form $\frac{\zeta w_n^2}{2g}$, in which the value of the coefficient ζ would be given by Borda's formula (p. 721, M. of E.). If the velocity finally assumed by the water in the volute space is only one fifth of w_n, or smaller, the value of ζ is practically unity or 1.0. If, however, the change of absolute velocity at exit is made gradual by gently flaring passageways between fixed guide-blades, the value of ζ may be as low as 0.2 or 0.3 (if we may judge from experiments made on the loss of head occurring in the down-stream diverging portion of a Venturi meter; see p. 726, M. of E.). But to offset the fact that the losses of head occurring in the impeller channels themselves will be ignored in the theory now to be developed, it would probably be advisable to take no lower value than 0.5 to 0.6 for ζ, even in the case of a "turbine pump" (that is, one provided with external guide-blades); while for the ordinary pump with the usual abrupt change of section from impeller to volute space ζ may range as high as 1.5 (especially with high heads; over 20 ft.) in order to include losses in impeller channels with the loss after exit due to sudden enlargement.

The neglect of losses of head in both suction-pipe and delivery-pipe implies that they are so wide and short that the skin friction therein is negligible. (In this connection, see §§ 115, etc.)

108. Theory of the Centrifugal Pump. Speed of "Impending Delivery."—If the centrifugal pump itself and both pipes have been originally filled with water, a foot-valve being provided at the base of the suction-pipe to prevent a backward flow before the pump is started, the question arises as to how great the speed of rotation must be before any upward flow at all is brought about. In other words, what must be the velocity, v_n, of the tips of the impeller-blades, such that the only effect is to prevent any downward flow on the part of the water in the delivery-pipe and upper reservoir? When this state of equilibrium occurs the water in both suction- and

FIG. 87c. The DeLaval Impeller.

delivery-pipes will be at rest, and that in the impeller passages will rotate with the impeller without travelling either to or from the axis. Hence the fluid pressure, p_n, between the revolving water and the stationary water in the upper pipe is the hydrostatic pressure due to the depth h_n of point N below surface H, plus atmospheric pressure (let b denote the water-barometer height); that is, $p_n=(h_n+b)\gamma$; also, the fluid pressure p_1 at point 1 is that due to the depth of point 1 below an imaginary water surface 34 ft. (i.e., b ft.) above 1, or $p_1=(b-h_1)\gamma$, where h_1 is the height of point 1 above surface T of lower reservoir. (In Fig. 86 the pump is above the supply-reservoir T; if it were at a lower level, p_1 would be $=(b+h_1)\gamma$, but the final result would be the same.)

In this case we may consider the water in the pump-channels to have a steady flow outwards from 1 to N with relative velocities (c_1 and c_n) = zero, and apply Bernoulli's Theorem for a rotating channel, etc., i.e., eq. (16) of § 42a; in which both c_1 and c_n will be zero, and p_1 and p_n will have the values just given; while the loss of head h'' will be zero since there is no flow; whence we have

$$b+h_n+0=b-h_1+0+\frac{v_n{}^2-v_1{}^2}{2g}. \quad . \quad . \quad . \quad . \quad (1)$$

Solving, we have, after noting that $h_1+h_n=h$, and that $v_1=v_n(r_1\div r_n)$,

$$v_n'=(\sqrt{2gh})\div\left(\sqrt{1-\left(\frac{r_1}{r_n}\right)^2}\right) \quad . \quad . \quad . \quad . \quad (2)$$

as the value for the linear velocity of the tip of the impeller-blades necessary to keep the water from flowing back; or it may be called the "velocity for *impending delivery*," since, if the speed is increased beyond this, a flow will take place up the delivery-pipe.

For example, if in Fig. 86 r_1 is taken as one-third of r_n, and the minute and foot be used as units, we have (very nearly)

$$v_n'=[500\sqrt{h\text{ (in ft.)}}]\text{ feet per minute.} \quad . \quad . \quad . \quad (3)$$

With a very small r_1 (call it zero) we derive 481 instead of

the 500. Experiment shows that frictional conditions and also the shape of the blade have an influence on the value of v_n' for impending delivery. Results quoted on p. 98 of Engineering News for August 1900 are as follows: Instead of the 500 in eq. (3) above, the following numbers were found, in the case of pumps 24 in. in diameter with r_1 = about one-half of r_n:

For blades curved about as Fig. 86 ($\delta=27°$).......	610
" " " " " " " ($\delta=18°$).......	780
Straight radial blades..........................	480
Straight blades leaning backward about 45°........	554
Curved blade somewhat like that in Fig. 86 and with its chord in same position, but concave on the *advancing* side..........................	394

Theoretically, in such a case, since no water is pumped, no power (Pv) is required to maintain the rotation of the pump; that is, if once started it should continue the motion indefinitely; but practically, on account of the friction between the rotating water and the stationary water in the pipes and between the discs or crowns and the surrounding water, together with axle friction some little power is necessary to keep up the motion. After pumping is once started the velocity of the tips may sometimes be allowed to sink below the value of eq. (3) if the pump contains provision for a gradual enlargement of section in the casing at exit from the wheel.

109. Theory of the Centrifugal Pump, with Friction. Best Velocity. Maximum Efficiency.—Returning to Figs. 86 and 87 and assuming a steady flow of water, all passages full, and uniform rotation of pump with angular velocity ω, with other notation of § 106; also Q denoting the rate of discharge, or cub. ft. per sec. of water pumped. At first all the quantities concerned, except h, r_1, r_n, and δ, will be considered variable. Later, special conditions will be imposed.

Applying the equation for power based on "angular momentum," etc. (eq. (10) of § 34); (see also § 35) (work per second done on equivalent couple), and remembering that in this case u_1 is zero and that the couple representing the action of

the water on the wheel is a resisting instead of a motive couple, we have, neglecting axle friction,

$$Pv=\frac{Q\gamma}{g}v_n u_n \text{ (ft.-lbs. per sec.) } \textit{power} \quad . \quad . \quad . \quad (1)$$

required of the working force P for steady motion. But, considering the whole collection of moving "*rigid*" bodies, including each particle of water, but ignoring the power spent on axle friction, and considering all fluid friction to be entirely represented by $Q\gamma\times$loss of head $\frac{\zeta w_n^2}{2g}$ (see § 107), we also have, from eq. (15), § 42a,

$$Pv=Q\gamma\left[h+\frac{\zeta w_n^2}{2g}\right]; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

also, w_1 being radial, $$w_1=V_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From trigonometry,

$$w_n^2=u_n^2+V_n^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$V_n=u_n \tan\mu, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$V_n=(v_n-u_n)\tan\delta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and $$c_n\cos\delta=v_n-V_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The equation of continuity of flow is

$$Q=2\pi r_n e_n V_n=2\pi r_1 e_1 V_1; \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

that is, $$r_n e_n V_n=r_1 e_1 V_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Also, $$v_1\tan\beta=w_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$v_1=\omega r_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and $$v_n=\omega r_n. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Combining (1), (2), and (4), we eliminate P, v, Q, γ, and w_n, obtaining

$$2gh=2u_n v_n-\zeta(V_n^2+u_n^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

in which, if for V_n we substitute its value $(v_n-u_n)\tan\delta$, from (6), there results

$$2gh=2u_n v_n-\zeta\tan^2\delta(v_n^2-2v_n u_n+u_n^2)-\zeta u_n^2. \quad . \quad . \quad (14)$$

Now the efficiency η is equal to the ratio of the portion of power applied to the useful purpose of raising $Q\gamma$ lbs. of water through an elevation of h ft. each second, to the whole power, Pv, exerted by the working force per second; i.e.,

$$\eta=\frac{Q\gamma h}{Pv}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14a)$$

or [see eq. (1)]

$$\eta=\frac{gh}{u_n v_n}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The value of h from (14) may now be substituted in (15), yielding

$$\eta=1-\frac{\zeta}{2}\left[\tan^2\delta\left(\frac{v_n}{u_n}-2+\frac{u_n}{v_n}\right)+\frac{u_n}{v_n}\right]. \quad . \quad . \quad . \quad (16)$$

Evidently (16) gives the efficiency as a function of the ratio $u_n \div v_n$; and if that ratio be denoted by x, that is, if $x=\frac{v'_n}{v_n}$, (16) may be written

$$\eta=1-\frac{\zeta}{2}\left[\tan^2\delta\left(\frac{1}{x}-2+x\right)+x\right], \quad . \quad . \quad . \quad . \quad (17)$$

which is a function of but one variable, x.

By differentiation,

$$\frac{d\eta}{dx}=-\frac{\zeta}{2}\left[\tan^2\delta\left(-\frac{1}{x^2}+1\right)+1\right]; \quad . \quad . \quad . \quad (18)$$

the placing of which equal to zero gives the special value of x (call it x') which makes η a maximum, viz.,

$$x'=\frac{\tan\delta}{\sqrt{1+\tan^2\delta}}, \quad \text{or} \quad x'=\sin\delta; \quad . \quad . \quad . \quad (19)$$

i.e., for a maximum efficiency we must make

$$u_n=v_n\sin\delta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and this relation substituted in eq. (14) gives, after considerable reduction, the value of v_n for best effect, viz.,

$$v_n'=\frac{\sqrt{gh(1+\sin\delta)}}{\sqrt{\sin\delta[1+\sin\delta(1-\zeta)]}}; \quad . \quad . \quad . \quad . \quad (21)$$

while the corresponding maximum efficiency itself becomes [see eq. (17)],

$$\eta' = 1 - \frac{\zeta}{\operatorname{cosec} \delta + 1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

As to the influence of the choice of the exit vane-angle δ upon this expression for the maximum efficiency, we note that the latter is the largest possible, viz., 1.00, when $\delta = 0°$. This supposition, however, would imply a zero discharge, which is inadmissible. It would also make the corresponding $v_n' =$ infinity. But it is evident that δ should be taken as small as practicable, say from 15° to 30°. If δ were as great as 90° (radial tips) and the friction coefficient ζ as large as 1.00 (which would doubtless be justified if the casing surrounding the pump did not provide a gradually enlarging passageway, with guides, for the water leaving the pump), we should find from eq. (22) that η' is only about 0.50. The corresponding value for v_n' from eq. (21) proves to be $v_n' = \sqrt{2gh}$. In fact, Prof. Zeuner states, in his book on "Theorie der Turbinen," that the peripheral speed of most centrifugal pumps in regular service should not greatly exceed this value, $\sqrt{2gh}$.

That a greater efficiency is obtained from impeller-blades curving backward as in Fig. 86, as against that obtained when straight radial blades are used ($\delta = 90°$), was conclusively proved by actual test in 1851 by Mr. Appold, who introduced the curved blade.

110. Numerical Example. Centrifugal Pump. — A centrifugal pump having external guides ("diffusion-guides") providing for a gradual change of absolute velocity for the water as it leaves the impeller-blades (and hence now called a "*turbine pump*") is to be designed for a head of $h = 36$ ft., is to pump $Q = 50$ cub. ft. per sec. in steady operation, and is to work at an angular speed of 300 revs. per min. The angle δ is to be taken $= 30°$ and r_1 as $= \frac{1}{2} r_n$. The suction- and delivery-pipes being short and wide, no loss of head will be considered as occurring in them.

Solution (the ft.-lb.-sec. system of units being used).—Taking

a value of 0.5 for ζ from the favorable conditions provided by the guides at exit, we find the best velocity for the impeller-tips to be, from eq. (21),

$$v_n' = \sqrt{\frac{32.2 \times 36(1+0.5)}{0.5\,[1+0.5\,(1-0.50)]}} = 52.8 \text{ ft. per sec.}$$

To find r_n that the angular speed may be 300 revs. per min., we write

$$2\pi r_n\left(\frac{300}{60}\right) = 52.8; \quad \text{whence} \quad r_n = 1.68 \text{ ft.};$$

and hence $r_1, = \frac{1}{2}r_n, = 0.84$ ft.

As for the distance between crown-discs (or sides of the chamber, if there are no crowns), we have, from eq. (2), $u_n = v_n' \sin\delta$; that is, $u_n = 52.8 \times 0.5 = 26.4$ ft. per sec.; and hence, from eq. (6), for "velocity of flow" at N,

$$V_n = (v_n' - u_n)\tan\delta = (52.8 - 26.4)0.577,$$

$= 15.2$ ft. per sec.; and hence, finally, from eq. (8),

$$e_n = \frac{Q}{2\pi r_n V_n} = \frac{50}{2\pi(1.68)15.2} = 0.311 \text{ ft.},$$

or (say) 0.33 ft. to allow for the thickness of the (six or eight) impeller-blades; i.e., $e_n = 4$ inches.

In order to secure a moderate absolute velocity of flow, V_1, at the entrance of the impeller channels, e_1 may be assumed equal to $3e_n$, i.e., $= 1.00$ ft.; hence, from eq. (9), we have the "velocity of flow" at entrance, $V_1 = \frac{2}{3}V_n = 10.1$ ft. per sec., which also $= w_1$, since the latter is supposed radial. The necessary value for the vane-angle at entrance, i.e., β, follows; viz., $\tan\beta = \frac{w_1}{v_1} = \frac{V_1}{v_1} = \frac{10.1}{\frac{1}{2}v_n'} = \frac{10.1}{26.4} = 0.485$; or β must be taken as (say) 26°. A smooth curve AN (see Fig. 86), having the proper values for β and δ at its extremities, and convex on its *advancing* side (as in that figure), will serve as the form of the (thin) impeller-blade.

As to the efficiency and necessary power to operate the pump, we have from eq. (22), with $\zeta = 0.50$ and angle $\delta = 30°$,

$$\eta' = 1 - \frac{0.50}{2.00+1} = 0.83;$$

which may be called the "hydraulic efficiency," since it leaves out of account the power spent on axle friction of the pump. Deducting 0.05 for this cause we obtain 0.78 as the value of the efficiency from which the necessary power is to be computed. Therefore, placing $\eta' = \frac{Q\gamma h}{Pv}$, we have $Pv = Q\gamma h \div \eta'$; i.e., $Pv = (50 \times 62.5 \times 36) \div 0.78$, $= 144,200$ ft.-lbs. per sec.; or 262 H.P.; since $144,200 \div 550 = 262$.

From the acknowledged imperfection of the theory, these results must be looked upon as only roughly approximate. Much experimentation is still needed to supplement the deductions of theory.

111. Practical Points.—When the pump is situated above the source of supply, T, and a suction-pipe is therefore necessary, its elevation above T is of course restricted (as in the case of the draft-tube of a turbine) to a value considerably less than that of the water-barometer height. In such a case, when the pump is to be started, it is found impossible by the rotation of the pump itself to exhaust the air from the suction-pipe. This must first be done by closing the foot-valve at the base of that pipe and filling up with water; or, after closing a valve in the delivery-pipe, to exhaust the air by the use of a steam-ejector, as is frequently done when a steam-engine is the source of power, the water being thus caused to rise in the suction-pipe by the pressure of the atmosphere on the lower reservoir.

If the suction- and delivery-pipes have considerable length and the respective losses of head thus occasioned, when the flow Q is passing through them, are h' and h''' respectively, and if the water is delivered in a free jet, of w' ft. per sec. velocity, at the point of delivery, then the h of the preceding theory will be replaced by

$$h + h' + h''' + \frac{w'^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

It amounts to the same thing to say that if piezometer tubes are arranged for the two pipes, at points near the pump (see now Fig. 5, in which the flow of water must be conceived to be from K toward A; through the casing M, which is now supposed to contain the pump, in steady operation), then the h of the preceding theory is to be replaced by the h of Fig. 5, augmented by the term $\left[\frac{w_3^2}{2g}-\frac{w_4^2}{2g}\right]$; where w_3 is the (mean) velocity of the water passing at A, and w_4 its velocity. as it passes section KH; see example in § 13.

The surfaces of the impeller-blades should be as smooth as possible, this being conducive to higher efficiency. Experiment has shown this (see Barr's Pumping Machinery, p. 343).

112. Centrifugal Pumps without Gradual Enlargement Beyond Exit.—This older style of pump has been found to give fairly good results only with low heads (say below 30 ft.), the high velocity of impeller and water necessary at high heads causing a large amount of fluid friction, eddying, etc., giving rise to large losses of head. A good example of the ordinary centrifugal pump with volute, etc., but without external guides, is shown in Fig. 88 (the Van Wie pump, made at Syracuse, N. Y.). The suction-pipe is attached at S and the delivery-pipe at R. E is a steam-engine furnishing the power to operate the pump; while F is a fly-wheel. Pumps of this type have given efficiencies, under low heads, as high as 65 per cent., or over.

In the new water-supply system of Rockford, Ill., designed and carried out by Prof. D. W. Mead in 1897, three centrifugal pumps are used, constructed by the Byron Jackson Machine Co. of San Francisco, which gave on test efficiencies of from 70 to 75 per cent. Each pump worked against the same head, 100 ft., of which 26 ft. was "suction-head." The impellers, 3.5 ft. in diam., are of bronze and have carefully smoothed interior walls. They are of the enclosed type (see § 114), with blades curving backward (δ=about 30°), and have "dead-spaces" toward the outer rim between water-

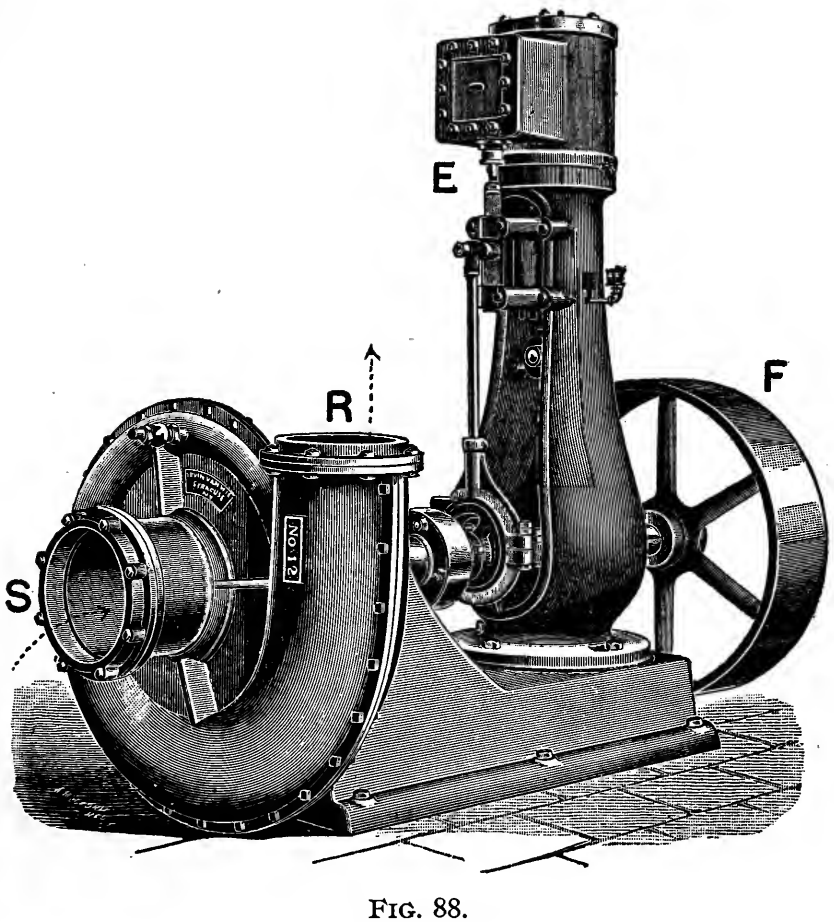


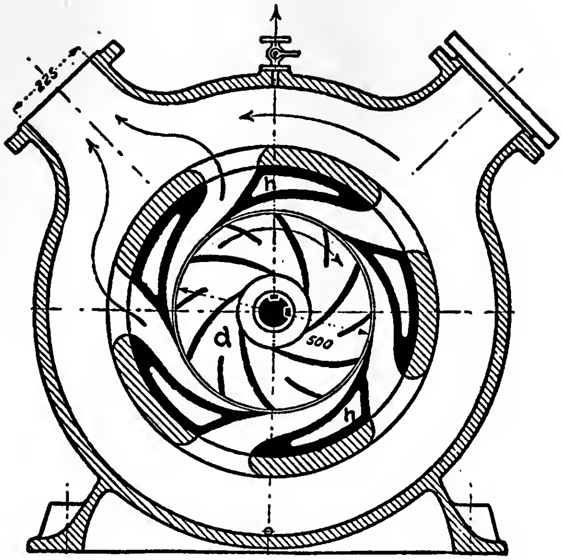
FIG. 88.

channels (see pp. 302 and 607 of Turneaure and Russell's *Public Water-supplies;* also p. 18 of Engineering News for July 13, 1899). A valuable article by Mr. Richards may be found in vol. xxxviii of the Engineering News, pp. 75 and 91.

113. Turbine Pumps. Multi-stage Pumps.—Within a few years* centrifugal pumps have been constructed in Europe, and more recently in America, attaining a high efficiency under high heads by the use of gradually enlarging guide-passages receiving the water immediately on exit from the impeller channels, thus enabling its velocity to be gradually reduced from the value, w_n, at the exit-point of impeller to much slower velocity of the delivery-pipe or other passage provided. These are called "*turbine pumps.*" A "*multi-stage*" pump consists of a series of two or more impellers on the same shaft, each pumping water into the central space of the next adjoining (except that the last one pumps into final delivery-pipe), the peripheral pressure of one being therefore nearly equal to the central, or receiving, pressure of the next. The intervening stationary guide-passages are so designed as to produce only gradual changes in the absolute velocity of the water, and comparatively high efficiencies are thus attained. By this device a high head (say 1000 ft.) can be broken up into steps, as it were, each impeller having to deal with a difference of pressure corresponding to the fraction of the whole head which corresponds to the number of impellers.

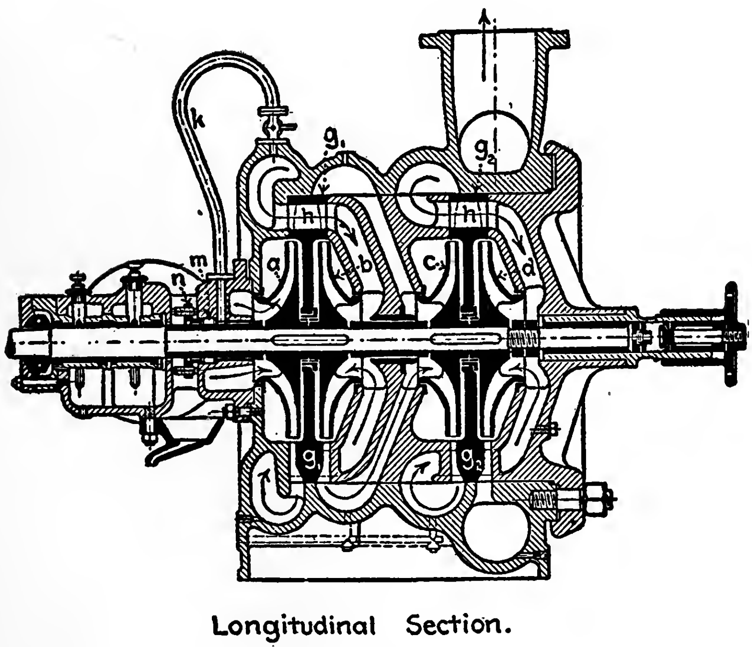
A good example of a multi-stage turbine pump is shown in Fig. 89 (which gives a section through the axis of rotation) and Fig. 90 (showing a section, at right angles to the shaft, through one of the four impellers). In the latter figure the walls of the external flaring guide-passages are shown in solid black shading. The impeller is seen to have six long blades and six intervening short ones, all curving backward with respect to the motion of rotation (with β and δ each = about 45°). By proper passageways the water is conducted from the space outside of an impeller to the central space of the next

* See Mr. Webber's article in Cassier's Magazine for June 1905, p. 154.



Transverse Section.

Fig. 89.



Longitudinal Section.

Fig. 90.

one of the series and finally into the delivery-pipe (for detailed explanation, see Engineering News, Jan. 1902, p. 66). The diameter of each impeller is 20 in. and (on test) water was pumped at the rate of $Q=2.47$ cub. ft. per second against a head of 425 ft., the pump rotating at 890 revs. per min. Each impeller therefore had to pump against a difference of pressure corresponding to a head of 106 ft. In the same test the efficiency was found to be 76 per cent. This pump was designed and constructed by the firm of Sulzer Bros. of Winterthur, Switzerland.

On p. 324 of Engineering News for April 7, 1904, may be found an illustrated article describing several "High-pressure Multi-stage Turbine Pumps" built by the Byron Jackson Machine Works of San Francisco, California. (Quoting from this article:) "Pumps of this design are built for heads of from 100 to 2000 ft., the number of separate impellers or 'stages' being properly proportioned to the head. About 100 to 250 ft. head per stage appears to be allowed." A two-stage pump built for the water-works of the city of Stockton, Cal., delivers 1500 gallons per minute against a head of 140 ft. at 690 revs. per min. The pump was guaranteed to have an efficiency of at least 75 per cent., but developed 82 per cent. at the official test

Since the water is usually admitted to the central impeller space from one side only, an end thrust of the shaft against its bearings is thereby created unless prevented by special device . In Fig. 91 is shown a section (through axis of shaft) of a 6-in., six-stage, "spherical," "compound pump" (i.e., multi-stage pump) built by the Lawrence Machine Co. of Lawrence, Mass., and so constructed, by the arrangement of the impellers in pairs and by the position of the intervening guide-passages, that the resultant end thrust is zero. To quote from the printed circular: "These pumps, like all of this type, are provided with diffusion-vanes directly at the periphery of the impellers, and, unlike others of their type, the liquid is not forced through short tortuous passages immediately after passing through the diffusion-vanes, before enter-

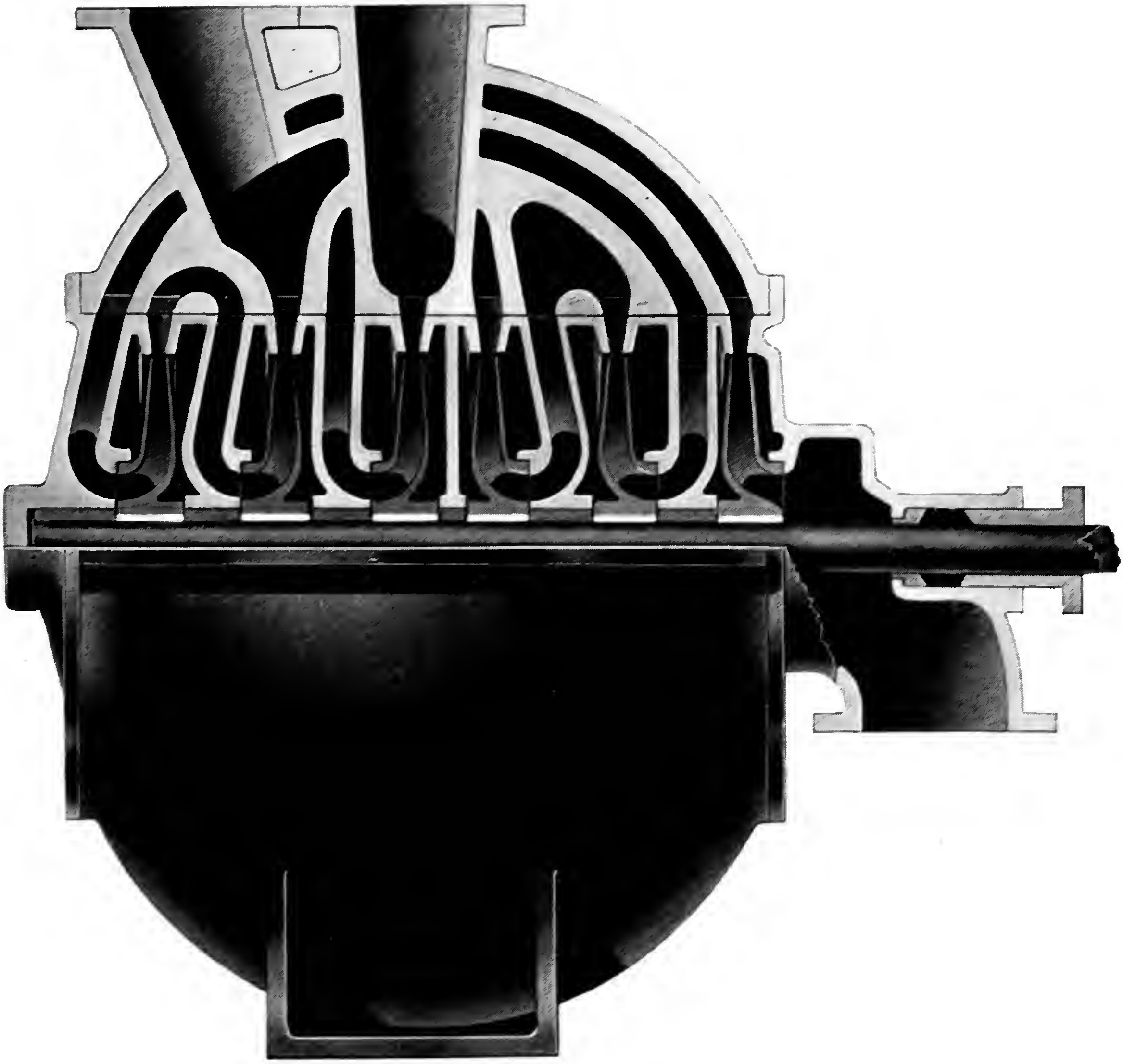


FIG. 91. Six Stage "Spherical" Compound Pump.
Made by the Lawrence Machine Co.

ing the next successive impellers, but instead through long easy passages of uniform cross-section and easy curves." The term "spherical" is due to the outside appearance of the pump-case.

114. Practical Notes.—Since there are no valves or other moving parts in a centrifugal pump, except the impeller itself, this type of pump is admirably adapted for the pumping of viscous and stringy liquids, or liquids containing sand or silt in suspension, or even carrying chips, bark, and gravel.

The large hydraulic dredges used by the Mississippi River Commission pump the river-water, charged with silt or sand by previous stirring of the bottom, through long pipes to a "spoil-bank" at some distance, thereby deepening the channel for purposes of navigation. (See reference in § 13; also Engineering News, Oct. 1898, p. 236.)

Another advantage is that, since the centrifugal pump is a body rotating continuously in one direction, the shaf may be coupled directly to that of an electric motor. A pump having crowns or discs forming part of the impeller is said to be of the "enclosed" type. If it consists merely of the impeller-blades fastened to and projecting from a spindle or shaft, it is called "unenclosed." In this latter case the stationary sides of the pump-case serve as crowns, the edges of the impeller-blades revolving almost in contact with them.

CHAPTER VIII.

PIPES, WEIRS, AND OPEN CHANNELS.

(**Note.**—This chapter contains matter supplementary to Chapters VI and VII of the writer's Mechanics of Engineering. Bernoulli's Theorem for steady flow of water in (rigid, stationary) pipes and stream-lines is already proved in §§ 492 and 512 of that work.)

115. Friction-head in Long Pipes.—Since long pipes and penstocks are frequently used to convey water to hydraulic motors, the loss of head so occasioned is an important consideration. For a steady flow of water in a stationary rigid pipe of cylindrical form the loss of head due to fluid friction (see eq. (4), p. 700, M. of E.) is conveniently expressed in the form

$$h_F=\frac{4fl}{d}\cdot\frac{v^2}{2g}, \qquad \ldots \ldots \ldots \quad (1)$$

where l is the length and d the internal diameter of the pipe, v the mean velocity (component parallel to axis of pipe) of the particles of water passing through any given cross-section (generally about 83 per cent. of the velocity of particles near the center of the section) and f a "coefficient of fluid friction," or abstract number, to be determined by experiment.

The volume of water flowing per second is, of course, $Q=Fv$, i.e., $Q=\frac{\pi d^2}{4}v$.

For new and clean cast-iron pipes, and for such small sizes of wrought-iron pipe as involve no riveting, Mr. Fanning's tables of values for the coefficient give fairly trustworthy results; but much time may be saved by the use of diagrams

which enable the friction-head itself to be found with great directness. Of course in such a case it makes no difference whether the formula upon which the diagram is based is simple or complicated. The diagrams prepared by the present writer for pipes of above description, founded on Mr. Fanning's values for f, have been placed in the Appendix of this work. Results obtained from these diagrams will be found to differ but slightly from those based on Mr. Metcalfe's "Diagram D," published in the Engineering Record of June 20, 1903. This diagram is stated by Mr. Metcalfe to be "for general use with new cast-iron pipes" and is based on the Hazen-Williams formula,

$$\text{(mean velocity)} \quad v = 71.6 d^{\frac{63}{100}} s^{\frac{54}{100}}; \quad . \quad . \quad . \quad (2)$$

in which s denotes the ratio $\frac{h_F}{l}$ and the *foot and second* are to be used as units. (For an account of the Hazen-Williams hydraulic slide-rule, see the Engineering Record for March 28, 1903.)

With increasing age of service cast-iron pipes are liable to become corroded and tuberculated (if originally tar-coated this action may be much retarded), which diminishes the discharge under the same head (both from increased roughness and diminished sectional area).

Mr. E. B. Weston recommends that for pipes of cast-iron the friction-head for a given Q be taken as 16 per cent. greater than when the pipe is new and clean, for *each five years of age.* For example, for an age of 15 years take as the friction-head for a given flow Q, and per 1000 ft. of length, the value obtained by multiplying the result given by the diagram by 1.48.

According to the recommendations of Mr. Metcalfe (see above article), we may find the friction-head h_F for old and tuberculated pipe for a given mean velocity by taking $\frac{163}{100}$ of that given by the diagram for clean cast-iron pipes for the same velocity; or, to put it another way, for a given friction-head the velocity obtained from the diagram for clean cast iron pipe must be multiplied by $\frac{10}{13}$ to give the velocity for the tuber-

culated pipe. However, considerations of friction-head in old and tuberculated pipes involve much uncertainty.

Similarly, according to Mr. Metcalfe's article, in the case of riveted iron and steel pipe, the coefficient f is so increased (on account of the projecting rivet-heads, etc.) that a value of the friction-head taken from a diagram for clean cast-iron pipe must be multiplied by $\frac{136}{100}$ to give that for the riveted pipe for the same diameter and velocity; or, conversely, if the value of the mean velocity has been taken from the diagram for clean cast-iron pipe for a given diameter and friction-head, it must be multiplied by $\frac{12}{13}$ to give the proper velocity for the riveted pipe.

116. Conversion Scales.—From the fact that great nicety is useless in computations for hydraulic problems involving friction-heads, it is sufficiently accurate in most cases to use values taken from diagrams; to expedite the work (and, incidentally, to avoid gross errors).

In the Appendix to this work will be found a page of "con version scales," by the use of which the velocity-head, h_v, corresponding to a given velocity, v, may be found, and *vice versa;* the hydrostatic pressure, p, in lbs. per sq. in., due to a "pressure-head," or static head, $h=\frac{p}{\gamma}$, of water, in feet; or the pressure-head, $\frac{p}{\gamma}$, in feet, corresponding to a given pressure, p, in lbs. per sq. in., etc.; and scales for converting a discharge, Q, in cub. ft. per second, into gallons per minute; etc., etc.

The quantities involved in any two adjoining scales are directly proportional to each other except in the case of the *velocity-scale,* where the velocity-head h_v is proportional to the *square* of the velocity v. In the use of the velocity-scale, therefore, this relation must be borne in mind in dealing with values that extend beyond the limits of the scales. For example, if the velocity-head h_v for a velocity of $v=120$ ft. per sec. is desired, find the h_v for one half of 120 (i.e., for 60) or 56 ft., and multiply by 4, which gives 224 ft.; and, again, if we wish the velocity corresponding to an $h_v=180$ ft., we first

find the v (or 35.8) for 20 ft., which is one-ninth of 180, and multiply by 3, obtaining 107.4 ft. per second (or find $v\,(=53.8)$ for one-quarter of h_v and multiply by 2).

117. The Hydraulic Grade-line.—This has been defined (see p. 715, M. of E.) as the line containing the summits of the stationary water columns in the open piezometers that may be imagined to be placed at various points along a pipe in which water is flowing in "*steady flow.*" Along a straight pipe of uniform diameter this line is straight and slopes downward for points farther and farther down-stream (the slope of the pipe itself is immaterial). The reason for this inclined position is the friction-head along the pipe; if this were zero, the grade-line would be horizontal. But if a portion of pipe has a increasing sectional area (going down-stream) (e.g., a conically converging pipe), the grade-line drops more rapidly on account of the increase in velocity-head in successive cross-sections; and, conversely, along a portion of the pipe which is conically divergent the grade-line rises (unless the divergence is so slight that the rise due to decrease in velocity-head is offset by the drop due to friction-head). All of these statements are easily proved by the application of Bernoulli's Theorem to the two extremities of any given portion of the pipe. A few numerical examples will now be worked out in illustration of the conception of the hydraulic grade-line and also of the use of the friction-head diagrams (Appendix).

118. Numerical Problems. (I) Single Pipe; without Nozzle.—Fig. 92. A steady flow of water is taking place through the horizontal cylindrical pipe (clean cast-iron pipe), whose length is 80 ft. and diameter 4 in., from the large reservoir R. The entrance of the pipe at E is not rounded. The head $h=9.3$ ft. There is no nozzle at the end m of the pipe, so that at that point the jet entering the atmosphere has the same sectional area as the pipe and a mean velocity v_m equal to that, v, in the pipe. At any point, such as S, (not nearer than 12 inches to the side of reservoir,) if the length $ES=x$, we find, by applying Bernoulli's Theorem between the point S of flow and the surface of (still) water in R, that the height of the

open piezometer at S is

$$\overline{PS}, =y, =h-\frac{v^2}{2g}-\zeta_E\frac{v^2}{2g}-h_x; \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where h_x is the loss of head due to skin friction along length ES of pipe, and equals the vertical drop from B to P; and $\zeta_E\frac{v^2}{2g}$ is loss of head at entrance E (see pp. 706 and 711, M. of E.).

Now h_x is proportional to x and becomes $=h_F$ or vertical drop from B to m when $x=$ whole length l, h_F being the friction-

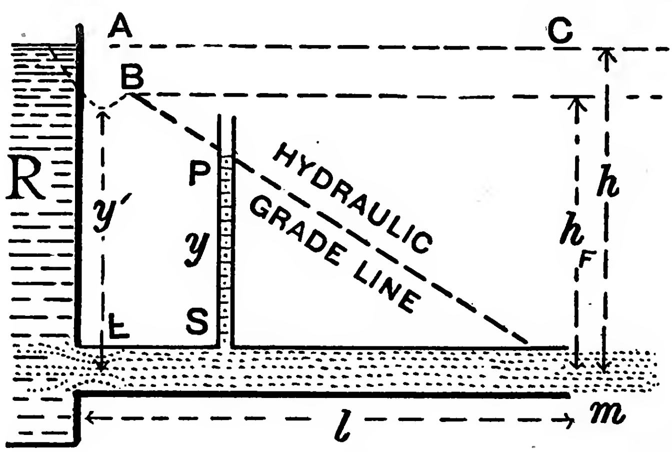


Fig. 92.

head for whole length of pipe. Here (no nozzle) the velocity $v=v_m$, and is to be determined. Therefore $y=$ zero at m;

and
$$h=\zeta_E\frac{v^2}{2g}+h_F+\frac{v_m{}^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The whole h is seen to be made up of three parts, of which, in this case h_F (there being no nozzle) will probably be nearly equal to h itself. We shall now solve by trial, using the friction-head diagrams in Appendix.

Assume as a first trial value that $h_F=7$ ft. This is at the rate of $(7\div 0.080=)$ 87.5 ft. friction-head per 1000 ft. of pipe length. Turning now to the diagram for the smaller pipes ($\frac{1}{2}$ to 8 in. in diameter), we find the vertical line corresponding to 87.5 among the figures along the upper edge and note its

intersection with the oblique line marked "4-in. pipe." Among the other oblique lines (velocity lines) this intersection-point corresponds to a velocity of 9.1 ft. per sec., for which (see highest scale on page of "Conversion Scales," Appendix) the velocity-head, or $v^2 \div 2g$, $=1.3$ ft. Since $\zeta_E = 0.50$, or $\frac{1}{2}$, the loss of head at E is $\frac{1}{2}(1.3) = 0.65$ ft. Hence the sum

$$0.65 + 7 + 1.3 = 8.95 \text{ ft.}$$

But this lacks 0.35 ft. of what it should be, viz., 9.30 ft. For the next trial it will probably occasion no great error if the whole of this 0.35 be added to the original 7 ft. That is, assume $h_F = 7.35$ ft., which is at the rate of $(7.35 \div 0.080 =)$ 92 ft. friction-head per 1000 ft. of pipe. The diagram now gives $v = 9.4$ ft. per sec. in a 4-in. pipe, and the velocity-head $= 1.4$ ft. and $\frac{1}{2}$ of $1.4 = 0.7$ ft. Adding, we have $0.7 + 7.35 + 1.4 = 9.45$ ft., which is so near to the required 9.3 ft., or h, that this second trial may be considered final. The corresponding discharge is $Q = 0.81$ cub. ft. per sec. (found by following a horizontal line, through the intersection of the vertical 92 and the 4-in. pipe line, to the scale on right-hand edge of diagram).

In Fig. 92 the vertical distance AB is the sum of the entrance loss of head and the velocity-head $v^2 \div 2g$. In the contracted vein at E the pressure-head is less (velocity being much greater) than for the point under B, which is about three diameters or 12 inches from the side of reservoir. Most of the entry loss of head occurs between the neck of the contracted vein and a point under B, but it is less than the difference of velocity-heads at that point and B.

Note.—If the jet discharges under water, the results are the same provided the surface of the water in the receiving-reservoir is 9.3 ft. below that in the supply-reservoir R (both reservoirs large).

It is immaterial whether the pipe is horizontal or not, if $l = 80$ ft. and $h = 9.3$ ft.

119. Numerical Problems. (II) Single Pipe; with a Nozzle. (Fig. 93.)—Clean cast-iron pipe of 6 in. diameter, 1600 ft. long; with a gradually tapering nozzle (or "play-pipe," for a

fire-stream). At the tip of the nozzle the water forms (in the atmosphere) a jet with parallel filaments (no contraction) and a diameter $d_m=2$ inches. The head h is 90 ft. and the loss of head in the nozzle may be taken as 0.05 (or 1/20) of $v_m^2\div 2g$, where v_m is the velocity of the jet;* while the entry loss of head at E (corners not rounded) is $\frac{1}{2}$ of $v^2\div 2g$, v being the velocity of the water in the 6-in. pipe. From the equation of continuity, the pipe running full, and the flow having become steady, we have $v_m=9v$. It is required to find the two velocities v and v_m and the discharge Q; use being made of the friction-head diagrams (Appendix).

Solution.—Bernoulli's Theorem applied between reservoir surface A (R is a large reservoir, so that velocity at A is taken as zero), note being made that the water-barometer height, b, cancels out (occurring in the expression for pressure head both at A and at m), gives

$$h=\frac{1}{2}\cdot\frac{v^2}{2g}+h_F+\frac{1}{20}\cdot\frac{v_m^2}{2g}+\frac{v_m^2}{2g}, \quad . \quad . \quad . \quad . \quad (5)$$

h_F denoting the loss of head in the 6-in. pipe.

We here note that the whole head h is made up of four items, viz., three losses of head and the velocity-head in the free jet (the student will note the corresponding vertical heights in Fig. 93). In this case h_F is not necessarily a large portion of h, since there must be considerable pressure-head $(=\overline{DE'}+b)$ at E', the base of the nozzle, to account for the great change of velocity between E' and m. We now solve, by the use of the proper friction-head diagram (containing the 6-in. size of pipe) by successive assumptions for the smaller velocity, v.

First assume $v=5$ ft. per sec., for which from the diagram (for 6-in. pipe) we find the friction-head would be at the rate of 18 ft. per 1000 ft. of length, and hence h_F would be $\frac{1600}{1000}$ of 18, $=28.8$ ft. From velocity-head scale (above), since $v=5$ and $v_m=9\times5=45$, we obtain $v^2/2g=0.40$ ft. and $v_m^2/2g=$ 31.4 ft. Hence the two small losses of head would be one half

* See foot-note on p. 706, M. of. E.

of 0.4, $=0.2$; and 1/20 of $31.4=1.57$ ft. The sum of these is $0.2+28.8+1.57+31.4$, $=61.97$ ft. (but it should be 90 ft.).

Second Trial.—Take $v=6$ ft. per sec. v_m would be 54 ft. per sec., and the two velocity-heads would be 0.56 and 45 ft.; hence the two small losses of head are 0.28 and $\frac{1}{20}$ of 45, $=2.25$ ft. Now 6 ft. per sec. in a 6-in. pipe implies a friction-head at rate of 25 ft. per 1000 ft. of length and hence h_F would be $\frac{1600}{1000}$ of 25, $=40$ ft. Forming the sum, we have $0.28+40+2.25+45$, $=87.53$ ft. the difference between which and

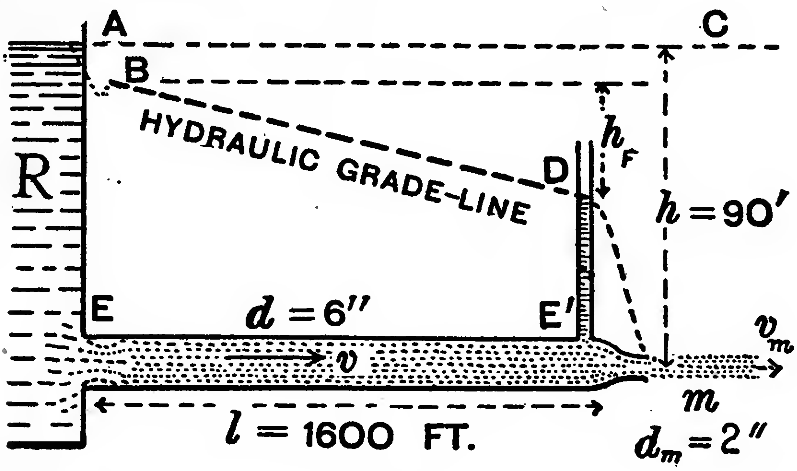


FIG. 93.

90 ft. is so small that a value of 6.1 ft. per sec. may be considered as a final solution for v; from which follow the values 55 ft. per sec. for v_m and (see diagram) 1.2 cub. ft. per sec. for the discharge, Q; *Ans.*

With increasing age the discharge and (v) would of course gradually diminish unless the pipe were kept clean. If the entrance E were rounded, a slight increase of Q would result.

These values of the jet velocity v_m and discharge Q are the same as if the nozzle or play-pipe issued from the vertical side of a large tank containing water the height of whose upper surface above the point E' is $\overline{DE'}+\frac{v^2}{2g}$; (proved by applying Bernoulli's Theorem to the base of nozzle as up-stream position and m as down-stream position). In the present case $v^2 \div 2g$ is small, only 0.60 ft. The height DE' of piezometer at E' is

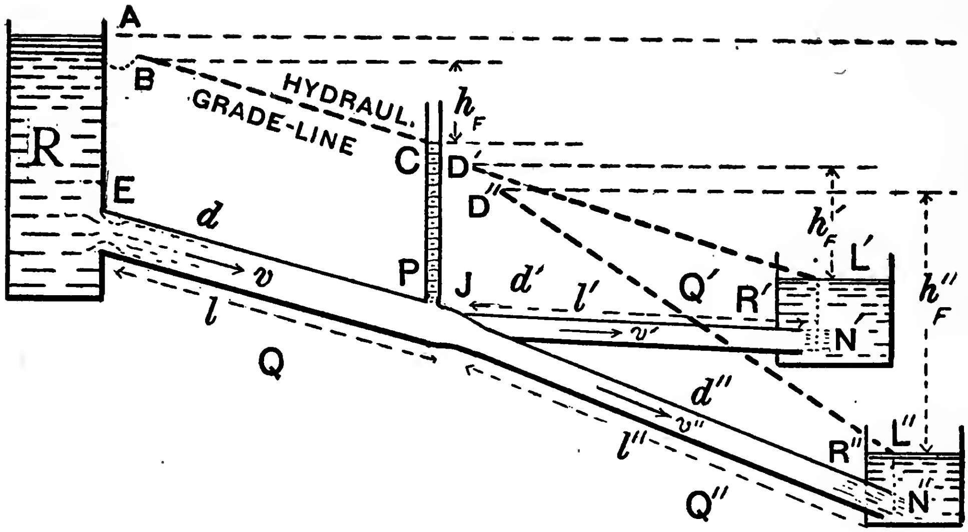

A
B
HYDRAUL. GRADE-LINE
R
E
C
D′
D″
h_F
h'_F
h''_F
d
v
l
P
J
d′
l′
Q′
R′
L′
N′
v′
Q
d″
v″
l″
R″
L″
N″
Q″

FIG. 94.

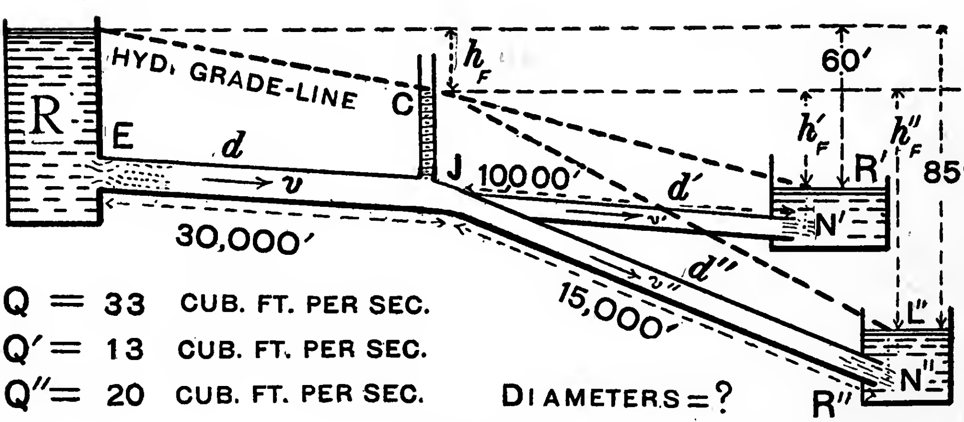

HYD. GRADE-LINE
R
E
C
h_F
60′
h'_F
h''_F
85′
d
v
J
10000′
d′
R′
N′
v′
30,000′
d″
v″
15,000′
L″
N″
R″
Q = 33 CUB. FT. PER SEC.
Q′ = 13 CUB. FT. PER SEC.
Q″ = 20 CUB. FT. PER SEC.
DIAMETERS = ?

FIG. 95.

easily found to be 48.7 ft.; and the pressure at base of nozzle is therefore 21 lbs. per sq. in. (see *Conversion Scales*, Appendix).

120. Variation in Last Problem. (Fig. 93.)—Instead of the diameter of the pipe being given, let us inquire what should be its value in order that 80 ft. of the total 90 ft. head may be available to produce the jet velocity v_m; that is, that only 10 ft. of the 90 ft. may be lost in friction-head and the two entrance losses of head; the remainder, 80 ft., being $=v_m^2/2g$. In this case v_m itself would be 71.8 ft. per sec.

In the nozzle the loss of head would be 1/20 of 80 ft.; i.e., 4 ft.; while that at E may be neglected. This leaves $10-4$, $=6$ ft., for h_F, which is at the rate of $(6/1.6=)$ 3.75 ft. per 1000 ft. of length. Now a jet of 71.8 ft. per sec. velocity and of 2 in. diameter is discharging $Q=1.56$ cub. ft. per sec. [obtained by multiplying 71.8 by the area (sq. ft.) of a 2-in. circle; or, more simply, by the friction-head diagram (one quarter of 71.8 is 18 (say), which is within the limits of diagram and for a 2-in. area gives $Q=0.39$, which multiplied by $4=1.56$)].

With the 3.75 and $Q=1.56$, we find from diagram that a diameter of 9.9 inches (say 10 in.) must be given to the pipe in Fig. 4. *Ans.*

This change of design calls for a greater consumption of water (1.56 instead of 1.20 cub. ft. per sec.), but the "*kinetic power*" of the "free jet" at m (that is, the kinetic energy of the mass flowing per sec. in jet), viz., $\frac{Q\gamma}{g}\cdot\frac{v_m^2}{2}$, will be more than doubled. It will be 7800 ft.-lbs. per sec. instead of 3513; i.e., 14.2 H.P., instead of 6.4.

As another variation (for the student to work out): Given Q, h, d, and l, determine necessary values for v_m and d_m to realize this discharge. Also find the H.P. of jet and the power to be expected from a Pelton wheel of 80 per cent. efficiency.

121. Main Pipe and Two Branches. (Fig. 94.)—A steady flow of water is to take place from reservoir R to two lower reservoirs, R' and R'', through a main pipe EP and two branch pipes, JN' and JN'', each of which discharges under water (at N' and N'' respectively). No nozzles are provided, so that the ve-

locity of each submerged jet is equal to that in the branch pipe itself, and the hydraulic grade-line for each branch is a straight line from the junction J to points L' and L'' in the receiving-reservoirs vertically over the discharging ends of the pipes. The flow having adjusted itself to a "steady" condition, the flow in EP of Q cub. ft. per sec. will be equal to the sum of those, Q' and Q'', in the two branches. If a piezometer were inserted just above the junction, J, the summit of the stationary water column therein would be at some point C in the tube; and the straight line BC is the hydraulic grade-line for EP. Similarly $D'L'$ would be the (straight), hydraulic grade-line for pipe JR', D' being vertically over a point in the pipe where the loss of head due to skin friction proper begins; there being a local loss (like that for an elbow) at the junction, and also a change of velocity, for those stream-lines which enter this branch. A corresponding statement may be made for the other branch.

Let v, v', and v'' be the velocities of steady flow in the three pipes, respectively; and their lengths and diameters, and the elevations of reservoirs, be as indicated in Fig. 94. The friction-head h_F for pipe EP is the vertical projection of its hydraulic grade-line. Similarly h_F' nad h_F'' are the friction-heads of the branch pipes. As to the other vertical "drops" between A and L', and A and L'', we have (from Bernoulli's Theorem)

$$A \text{ to } B = \zeta_E \frac{v^2}{2g} + \frac{v^2}{2g}; \quad C \text{ to } D' = \zeta' \frac{v'^2}{2g} + \left[\frac{v'^2}{2g} - \frac{v^2}{2g}\right];$$

and
$$C \text{ to } D'' = \zeta'' \frac{v''^2}{2g} + \left[\frac{v''^2}{2g} - \frac{v'^2}{2g}\right];$$

in which $\zeta' \frac{v'^2}{2g}$ and $\zeta'' \frac{v''^2}{2g}$ are losses of head due to change of section (if abrupt) or elbow resistance.

Now in most cases in practice the velocities in the pipes of a system are rarely over 10 ft. per second, and the pipes are very long (as in next paragraph); so that in treating a problem like the present (one where the Q's are required if the diameters

are given, or *vice versa*), it is sufficiently accurate to neglect the small "drops" AB, CD', and CD'' in the hydraulic grade-lines and consider that the whole drop from surface of water in R to that in R' is equal to the sum of the two friction-heads h_F and h_F'; and similarly that the drop from R to $R''=h_F+h_F''$. (However, this would not be justified if there were nozzles at N' and N''; see Fig. 93.)

Problems of this kind are best solved by trial, use being made of friction-head diagrams. Other modes of solution are very tedious and intricate.

122. Numerical Problems. (III) Main Pipe and Two Branches.—For the system of pipes in Fig. 95 (same as in Fig. 94, but with numerical data), such diameters are to be determined for the three pipes respectively that the discharge shall be $Q=33$ cub. ft. per sec. through the main pipe, of which $(Q'=)$ 13 is to pass to reservoir R' and $(Q''=)$ 20 to R''. Elevations and lengths are as printed in Fig. 95. (Clean cast-iron pipes.)

We are at liberty to assume one of the diameters; or the friction-head, h_F, of the main pipe; say the latter. Take $h_F=40$ ft. A steady flow is to take place in pipe EJ of 33 cub. ft. per sec. and the friction-head is to be at rate of $(40 \div 30 =)$ 1.33 ft. per 1000 ft. of length. In the diagram of friction-heads for large pipe (see Appendix) and note that the vertical line for 1.33 (interpolating) intersects the horiz. line for $Q=33$ in a point corresponding to a diameter of 38 in. (among the lines sloping up to the right), while among the other inclined lines (sloping *down* to the right) we find that with this discharge the velocity of the water in this 38-in. pipe would be 4.1 ft. per sec. (which is not extreme). Deducting the assumed h_F (40 ft.) from the altitude 60 ft., we find the corresponding value of h_F' to be 20 ft.; i.e., at the rate of $(20 \div 10 =)$ 2 ft. per 1000 ft. length of pipe. From same diagram we note that the intersection of the vertical 2 with the horizontal for $Q=13$ is a point calling for a 25-in. pipe; in which with this value of Q (13) the velocity of the water would be 3.8 ft. per sec. (a permissible value).

Similarly, deducting the h_F (40 ft.) from the 85 ft. altitude we obtain for the h_F'' of the other branch pipe 45 ft.; which is

at the rate of ($45 \div 15 =$) 3 ft. friction-head per 1000 ft. of length; for which, with $Q = 20$, the diagram gives a diameter of 27.5 in. for pipe JN'' with a velocity $= 5$ ft. per sec.

If h_F had been assumed somewhat > than 40 ft., a smaller diameter would have resulted for the main pipe, EJ, with a higher velocity in it than before; but larger diameters and smaller velocities in the two branch pipes. Results should be sought involving the *least cost*, with sufficient velocities (above 2 ft. per sec.) to prevent the deposit of silt.

123. Variation from Foregoing Problem.—In the above example the diameters were the quantities sought; but if the diameters were given and the rates of flow that would occur in the respective pipes were to be determined, proceed thus: Assume a trial value for Q and find from diagram the friction-head per 1000 ft. length of pipe of given diameter d, thence the value of h_F for actual leng h of EJ. Values of h_F' and h_F'' corresponding to h_F are now noted and corresponding values of Q' and Q'' found from the diagram for respective diameters d' and d''. The sum $Q' + Q''$ should be equal to Q. If such is not the case as a result of the first trial, assume a new value for Q; and so on, until the necessary equality is obtained.

In the above it is supposed that water flows *into R' and R''*, and *out of R*; but if R' is at a sufficient elevation, or if pipe EJ is small in diameter, water may flow *out of R'*, as well as out of R. In such a case the summit C would be lower than the surface in R', and $Q + Q' = Q''$.

Similar principles and methods apply to any system or network of pipes.

124. Numerical Problems. (IV) Supply-pipe for Turbine. Loss of Head.—In previous problems of this chapter examples have been treated in which the water reaches the atmosphere at the lower level without having given up energy for any useful purpose, some or all of its energy having been expended in fluid friction. Let us now consider the case of a turbine supplied with water through a supply-pipe of riveted steel, 2000 ft. in length. See Fig. 96. The suction-head (for the short draft-tube) is 10 ft.; whole head, 80 ft. The consump-

tion of water in steady flow is limited to 20 cub. ft. per sec. How much of the total head of 80 ft. will be lost in the supply-pipe, and correspondingly how much power lost in fluid friction?

Solution.—We find from the friction-head diagram (in Appendix) that a flow at rate of 20 cub. ft. per sec. in a pipe of 24 in. diameter implies a mean velocity, v, of 6.4 ft. per sec.; and also, if the pipe is of clean cast iron, a friction-head of 5.8 ft. per 1000 ft. length; that is, of 11.6 ft. for 2000 ft. length. Multiplying this 11.6 by $\frac{136}{100}$ for riveted steel pipe (see § 115), we obtain 15.8 ft. as the friction-head from E to K. This 15.8 ft. is the "drop," FD, in the hydraulic grade-line, while $\overline{CF}$, $=(v^2 \div 2g)(1+0.5)$, $=1.02$ ft. Hence the open piezometer height $\overline{DK}$, at K (taking $\overline{CK}$ as 70 ft.), is $70 - (15.8+1.02)$, $=53.18$ ft.; and the vertical distance from summit D to tail-surface T is 63.18 ft. In computing the efficiency η, of the turbine, (in a test,) from the expression $\eta = R'v' \div Q\gamma h$, we should

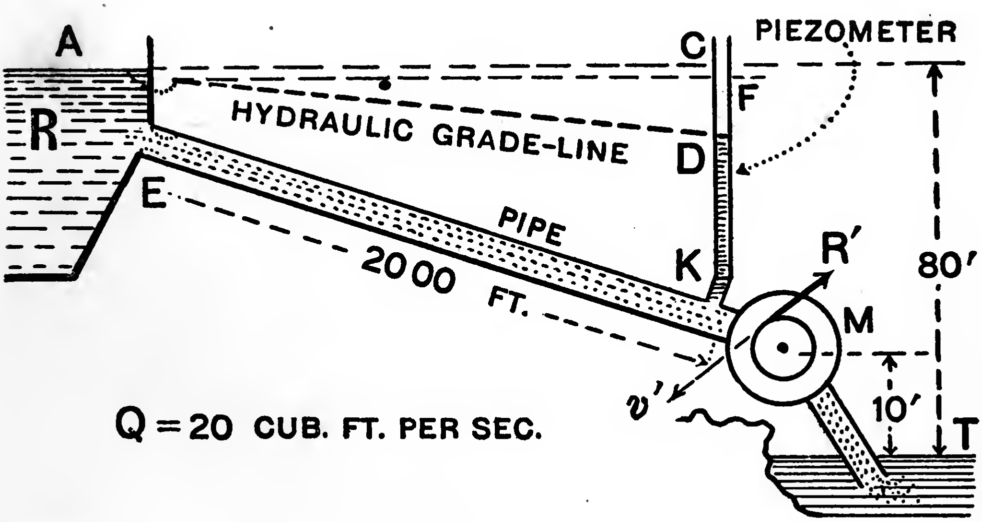


FIG. 96.

write for h the value $63.18+(v^2 \div 2g)$; i.e., 63.86 ft., and not 80 ft.; since the 24-in. pipe is not a part of the turbine. Again, referring to Fig. 45, the h_1 of that figure would be represented by $\overline{DK}+(v^2 \div 2g)$, i.e., by 53.86 ft., in the present case; and h_n by -10 ft.; that is, the h, $=h_1-h_n$, of Fig. 45 will be (as

already stated) $53.86-(-10)$, $=63.86$ ft., for the purposes of the present problem.

If, then, a diameter of 24 in. be adopted for the 2000 ft. supply-pipe, the loss of head thereby occasioned is about 16 ft. $(=h_2)$ and the loss of power is $Q\gamma h_2$, $=20\times 62.5\times 16$, $=20,000$ ft.-lbs. per sec.; or 36.4 H.P.

As the loss of head of 16 ft., in the supply-pipe of 24 in. diameter, is about one-fifth of the total head (80 ft.) of the mill-site, it will be instructive to note the great reduction in this loss of head as due to an increase in the diameter of the supply-pipe from 2 ft. to 3 ft., Q remaining as before (20 cub. ft. per sec.). For a 36-in. pipe, from the friction-head diagram for clean pipes we find $h_2=0.73$ ft. for 1000 ft. length, and hence $(0.73\times 2=)$ 1.46 ft. for the actual 2000 ft. length. If 1.46 be multiplied by $\frac{136}{100}$, as before (for riveted steel pipe), the result is a loss of head of only 2 ft.; instead of the 16 ft. when the diameter was 24 inches. However, in an actual case in practice, the annual interest on the extra cost of the 36-in. pipe might be greater than the annual income from sale of power due to the head so saved (14 ft.). Commerical considerations of this nature are of great importance in situations where long supply-pipes are needed to develop a water-power.

124a. Power Lost in a Supply-pipe.—In general, in this connection, it is to be noted that if in the expression for the friction-head in a long pipe [eq. (1), § 115], viz., $h_F=\frac{4fl}{d}\cdot\frac{v^2}{2g}$, there be substituted for v its equivalent $Q\div\left(\frac{\pi d^2}{4}\right)$, we have

$$h_F=\frac{32fl}{\pi^2 g}\cdot\frac{Q^2}{d^5}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

from which it is seen that if the coefficient f be considered constant (as a rough approximation), the friction-head is inversely proportional to the fifth power of the diameter d, for a constant Q. Evidently, then, an increase in the diameter produces a relatively large decrease in the friction-head, as has just been illustrated.

Again, as to the power lost in a supply-pipe, L_F ft.-lbs. per sec., we have

$$L_F = Q\gamma h_F = \frac{32fl}{\pi^2 g}\gamma \cdot \frac{Q^3}{d^5}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

on which the statement may be based, as approximately true, that the power lost in a supply-pipe is directly proportional to the cube of the volume of flow (Q cub. ft. per sec.) and inversely to the fifth power of the diameter (d) of pipe. For instance, doubling the discharge, without change in length or diameter, would involve about *eight times* as much loss of power in the supply-pipe.

124b. Note.—If M, in Fig. 96, were a centrifugal pump (instead of a turbine) requiring a power Pv' to drive it, pumping 20 cub. ft. of water per sec. from T to A, the summit D' of the piezometer column at K would stand at a height $D'K$ above K equal to $\overline{CK}+h_F$; or for a 24-in. pipe $70+15.8=85.8$ ft.; and therefore 15.8 ft. *above* C. See §§ 12 and 13. The hydraulic grade-line would then be a straight line from D' to a point in A vertically above E.

125. Water-hammer in Pipes. Unsteady Flow.—When the water supplying a turbine is conducted through a very long pipe, flowing with some velocity v, a more or less sudden closing of the wheel-gates may cause high bursting pressures within the pipe, unless relief-valves are provided, or a stand-pipe communicating with the supply-pipe just up-stream from the wheel-gates. Without such provision the arresting of the motion of the large mass of water in the pipe creates a great increase of pressure of the water against the walls of the pipe, sufficient in some cases to rupture it. The most extreme instance of this kind would be occasioned by the instantaneous closing of a valve-gate in a pipe in which water is flowing. This will now be investigated. If the pipe does not move lengthwise, the original kinetic energy of the water will exhaust itself in compressing the water itself and in distending the walls of the pipe. In our first treatment the walls of the pipe will be considered as inextensible; that is, their disten-

sion will be neglected. The maximum (unit) fluid pressure to be determined, as due to the arrest of the motion, will be that over and above the pressure already existing before the interruption of the condition of steady flow, and may be called the "excess-pressure."

126. Water-hammer in a Pipe. Distension of Pipe Neglected.—We shall at first disregard the distension of the pipe walls due to increase of internal pressure. As regards the compressibility of water it is known from physics that water has only one kind of modulus of elasticity, viz., that or change of volume (or "Bulk-modulus"), which may be called E. If a mass of water, of original volume V, is by compression from all sides reduced in volume by an amount ΔV, the fluid pressure so far induced being p lbs. per sq. in., then E is defined as the quotient $p \div$ relative change of volume, i.e.,

$$E = \frac{p}{\Delta V \div V} = \frac{pV}{\Delta V}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For pressures below $p = 1000$ lbs. per sq. in. (and at ordinary temperatures) E may be taken as 294,000 lbs. per sq. in. (For very high pressures, see Engineering News, Oct. 4th, 1900, p. 236.)

In Fig. 97 we have a horizontal pipe of indefinite extent in which at first water is flowing (from left to right) with a constant velocity of c ft. per second, the valve-gate G being open. The pipe is non-distensible. If now the gate G is instantaneously closed, passing into position GC', the vertical laminæ of water on the left of the gate crowd up against it, and at the end of a short time, dt seconds, all the laminæ up to some position BB', a distance ds' from C, have come to rest, with reduced volume and under some pressure p (excess pressure) whose value we wish to determine. At the beginning of this short time dt there were certain laminæ in the position AA' which at the end of the time dt have just reached position BB', having traveled a distance AB, $= ds$, without reduction of volume and with unchecked velocity c; so that $c = ds \div dt$. That is, a "wave of compression" travels from C to B in time

dt, and hence the velocity of the "wave front," or of "wave propagation," is $ds' \div dt$, which may be called C, or the velocity of sound in water.

Therefore, in a time dt the prism of water $AA'C'C$, whose original volume was $V = F(ds + ds')$, (where F is the sectional area of the pipe,) has had its velocity changed (different laminæ

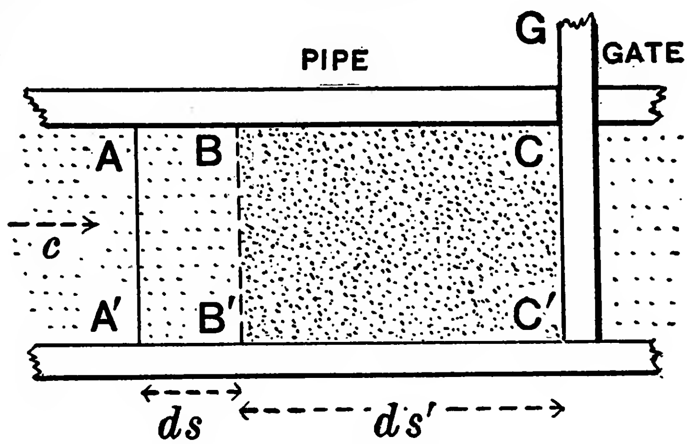


Fig. 97.

successively) from c to zero and has undergone a change of volume of $\Delta V = F ds$. Each of the vertical laminæ composing this prism has encountered a retarding force increasing regularly from zero up to its final maximum, $P = pF$, and we may for simplicity assume that the value of this final maximum pressure is the same as if the prism in question had remained rigid; that is, had remained of unaltered length AC while describing the distance ds in being brought to rest; its retardation being brought about by an imponderable spring (say), the compressive force in which increases progressively in proportion to the amount of shortening of the spring, from zero to P.

Now for a *uniformly* retarded motion we have from eq. (3), p. 54, of M. of E., when the initial velocity is c and the final is zero, $0^2 - c^2 = 2 \times \text{distance} \times \text{acceleration}$. The motion of the prism in the present case is *not* uniformly retarded; that is, the (negative) acceleration is not constant; but we may use the relation just quoted if we substitute the *average* acceleration, which is one-half of its final value, viz., $-\frac{1}{2}(pF \div \text{mass})$,

$F = Ma$

Q

$=-\frac{1}{2}pF \div [F(ds+ds')\gamma \div g]$. The result of such substitution is

$G = Q\gamma$ $\quad M = \frac{G}{g}$

$$c^2 = pg \cdot ds \div (ds+ds')\gamma; \quad . \quad . \quad . \quad . \quad (8a)$$

But since $c = ds \div dt$, this may be written

$$p \cdot g \cdot dt = (ds+ds')\gamma c. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Also, from definition of E (see eq. (8)),

$$E = \frac{pF(ds+ds')}{Fds}, \quad \text{or} \quad E = \frac{p(ds+ds')}{ds}. \quad . \quad . \quad (10)$$

Dividing (9) by (10) we have
$$p^2 = \frac{E\gamma c^2}{g}, \quad . \quad . \quad . \quad . \quad (11)$$

or
$$p = c\sqrt{\frac{E\gamma}{g}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for the value of the "excess pressure." It is seen to be proportional to the original velocity, c, of the water in the pipe.

Incidentally, we may now determine the velocity of sound in water, C; viz., by multiplying eq. (9) by (10), whence

$$Egds \cdot dt = (ds+ds')^2\gamma c. \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Now ds is usually so small compared with ds' that we may neglect it when added to the latter, and thus obtain $Egds \cdot dt = (ds')^2\gamma c$. But $ds \div dt = c$, and $ds' \div dt = C$; therefore, finally,

$$C = \sqrt{\frac{Eg}{\gamma}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

With $E =$ 294,000 lbs. per sq. in., $g =$ 32.2 (ft. and sec.), and $\gamma = 62.5$ lbs. per cub. ft., this gives $C = 4670$ ft. per sec.

Eq. (12) may be written in this form (taking $E = 294{,}000$ lbs. per sq. in. and $\gamma = 62.5$ lbs. per cub. ft.):

$$p \text{ (in lbs. per sq. in.)} = 63 \times [c \text{ in ft. per sec.}]. \quad . \quad (14a)$$

√

127. Water-hammer in a Pipe, Distension of Pipe Considered. (See Fig. 98.)—In this case, the water in the pipe being originally in motion in steady flow from left to right with velocity c, let the gate G be suddenly closed, into position GH'; and let BB' be the position of the "wave front" at the end of dt seconds

after the closure. The compressed prism of water, which originally occupied the position and space $AA'C'C$, its volume being then $V=F(ds+ds')$, is now found to occupy the space $BB'H'H$ (dotted sides), the pipe having been distended, and its radius having increased from a value r to a new value,

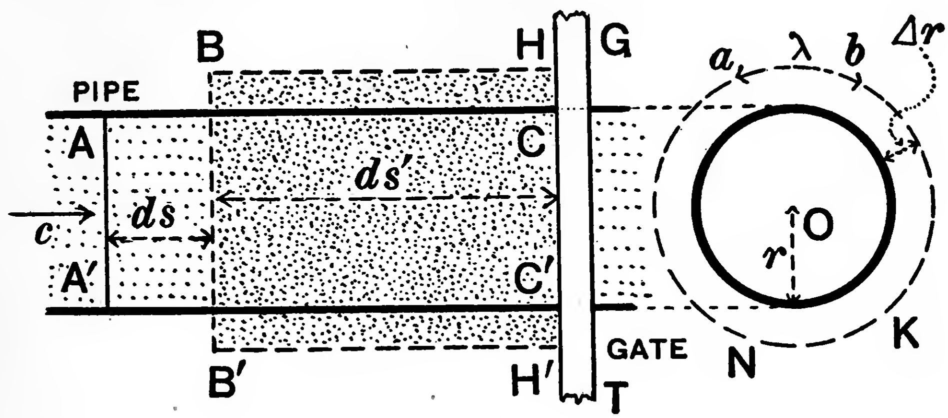


FIG. 98.

$r+\Delta r$, (see the end-view on the right, where the thick outline shows the original size of the pipe.) The change (decrease) of volume of this prism is evidently $\Delta V=F\cdot ds-2\pi r\cdot\Delta r\cdot ds'$, where F is sectional area of pipe, $=\pi r^2$, and hence [see eq. (8)],

$$E=\frac{pF(ds+ds')}{Fds-2\pi r\cdot\Delta r\cdot ds'}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

By the same reasoning as in the previous paragraph we may repeat eq. (9), viz.,

$$p\cdot g\cdot dt=(ds+ds')\gamma c. \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The unit pressure being p at this instant, acting also as a bursting pressure radially outward on the inner surface of the pipe-wall, between B and H, the simultaneous tensile stress (or "hoop-tension") in the pipe-wall, p' lbs. per sq. in., will have a value of $p'=rp\div t'$, where t' is the thickness of the pipe-wall [see p. 537, M. of E., eq. (2)]. Now if E' is the modulus of elasticity (linear; Young's modulus) of the metal of which the pipe is made, and λ is the increase of length of the circumference of the pipe due to stress p', we have (see p. 203, M.

of E.), by definition,

$$E' = p' \div (\lambda \div 2\pi r), \quad \text{or} \quad E' = (rp2\pi r) \div (\lambda t').$$

But, from proportion, $\lambda : 2\pi r :: \varDelta r : r$, or $\lambda = 2\pi \cdot \varDelta r$; hence

$$\varDelta r = \frac{pr^2}{t'E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

If this be substituted in (15) and F replaced by πr^2, we finally obtain [see also (16)], the relation

$$\frac{ds}{dt} - 2r \cdot \frac{ds'}{dt} \cdot \frac{p}{t\,E'} = \frac{p^2 g}{Ec\gamma}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

But if in (16) we neglect ds when added to ds', writing C for $ds' \div dt$, we obtain

$$p = \frac{cC\gamma}{g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

which may be substituted in (18) and a solution made for C (note being made that $ds \div dt = c$ and that $ds' \div dt = C$), whence

$$C = \sqrt{\frac{g}{\gamma} \cdot \frac{EE't'}{(t'E' + 2rE)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

as the (diminished) velocity of sound * along the water in the pipe now that the distension of the latter is brought into play; and therefore [see (19)]

$$p = c\left(\sqrt{\frac{\gamma EE't'}{g(t'E' + 2rE)}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

is the "excess pressure" tending to burst the pipe.

(N.B. These same results could also be obtained by putting the original kinetic energy of the prism $AA'C'C$ equal to the work of compressing itself and of distending the pipe-wall; see § 196, M. of E.)

128. Joukovsky's Experiments on Water-hammer. — That formulæ (20) and (21) are practically true has been demonstrated by Prof. Joukovsky in experiments conducted at

* First proved by Korteweg in 1878. See also Mr. J. P. Frizell's book on "Water Power," New York, J. Wiley & Sons, 1901.

Moscow, Russia, in 1897–98. These experiments * were made with horizontal pipes of cast iron of four different lengths, viz., 2494, 1050, 1066, and 7007 ft.; their diameters being 2, 4, 6, and 24 inches, respectively.

It was found that so long as the time of closing the valve was less than that required for the wave of compression, or sound wave, to make a "round trip" from the valve to the reservoir from which the pipe issued and back to the valve, the effect was practically the same as if the closure had been instantanoeus. The wave being reflected down the pipe from the water in the reservoir, the time for the "round trip," if l denote the length of the pipe, is $t_r = 2l/C$. It was found that when the time, t'', of closure was longer than t_r, the excess pressure produced, p'', was less, and in the same proportion as t_r was less than t''; that is, that $p'':p::t_r:t''$.

On account of the elasticity of the water its condition of compression is only temporary, being followed, during the "recoil," as it may be called, by a period of "rarefaction" or of pressure below the original or normal pressure; thus there occur at the gate successive pulsations of pressure a complete cycle of which is equal to the time of two "round trips." These pulsations of pressure diminish gradually in intensity through friction.

In the case of a pipe of smaller diameter connected with the main pipe and terminating in a "dead end" or valve permanently closed, a much greater excess pressure is produced in the smaller pipe—about double that in the main pipe.

Some practical conclusions reached as the result of these experiments are quoted (see foot-note below): "The simplest method of protecting water-pipes from water-hammer is found in the use of slow-closing gates. The duration of closure should be proportional to the length of the pipe-line. Air-chambers of adequate size placed near the valves and gates eliminate almost entirely the hydraulic shock, and do not allow the pressure wave to pass through them; but they must

* A good *résumé* of these experiments was published in the Proceedings for 1904 of the Amer. Water-works Assoc., p. 335.

be very large and it is difficult to keep them supplied with air. Safety-valves allow to pass through them pressure waves of only such intensity as corresponds to the elasticity of the springs of the safety-valves."

129. Time of Closure Longer than t_r.—When the time of closure is very much longer than that, t_r, for the "round trip," the rate at which the opening of the valve-gate is closed up would seem to have an important bearing on the rise of pressure produced. Theoretical investigations along this line have been made by Mr. B. F. Latting, C.E., and the present writer; and a few experiments were also made by Mr. Latting, the results of which were fairly confirmatory of theory. See the Engineering Record for Feb. 25, 1905, p. 214, or Engineering of March 17, 1905, p. 363; also Transac. Assoc. Civ. Engineers of Cornell University, for 1898, p. 31.

130. Water-hammer. Numerical Examples.—(I) If the original velocity of the water in a 2-in. pipe is 4 ft. per sec. and a valve-gate is closed instantaneously, what excess pressure is produced?

This pipe being small in diameter, eq. (14*a*) may be used, from which we have $p=63\times4=252$ lbs. per sq. inch.

If the length of the 2-in. pipe is 1000 ft. the same pressure would be produced so long as the time, t', of closing the valve was less than $t_r=2\times1000\div4670,=0.428$ sec. If the time of closing were longer than 0.428 sec., the excess pressure (p'') would be less in accordance, with the relation $p''=(t_r\div t'')p$.

$\frac{2l}{c} = t_r$

If the 2-in. pipe were only 200 ft. long the full water-hammer of $p=63\times4$, or 252 lbs. per sq. in., would not be produced, unless the time t'' were less than 0.085 sec.

(II) A riveted steel pipe is 5 ft. in diameter, the thickness of pipe-wall being $\frac{1}{4}$ inch. The water within it has originally a velocity of 4 ft. per sec. What is the full excess pressure of water-hammer if E' be taken as 30,000,000 lbs. per sq. in.?

We now substitute in eq. (21) and obtain $p=137$ lbs. per sq. in. Also from eq. (19) we have for the velocity of the

compression wave

$$C = \frac{137 \times 144 \times 32.2}{4 \times 62.5} = 2552 \text{ ft. per sec.}$$

In case the length of the pipe is 7000 ft. the full value of p, $=137$ lbs. per sq. in., would not be produced unless the time of closing were less than t_r, which $= 2 \times 7000 \div 2552 = 5.48$ sec.; and similarly for other values of the length.

The "hoop-tension" in the wall of the pipe, due to the excess pressure p, would be $p'' = rp \div$ (thickness), i.e.

$$p = 30 \times 137 \div \tfrac{1}{4} = 16{,}440 \text{ lbs. per sq. in.}$$

To this would have to be added the hoop-stress due to original fluid pressure; and the weakening of plates due to riveting would have to be considered. Evidently the total hoop-stress would be too great for safety.

131. Prevention of Water-hammer with Turbines.—The prevention of much increase of pressure at the turbine end of a long penstock is not only desirable for the safety of the penstock itself, but also in some cases absolutely necessary for the proper regulation of the motor.

For instance, when the resistance or "load" on the turbine diminishes, and when consequently by the action of the governing apparatus the wheel-gates begin to close, in order that by the diminution of the rate Q (cub. ft. per sec.) of water used by the wheel the working force exerted on the wheel may be reduced, so great a rise of pressure might be produced just outside the gates as to bring about an *increase*, instead of a decrease, in the working force acting on the wheel; and thus produce an effect just the contrary of that intended. Provision therefore is often made for the escape of some of the water through a side outlet or "by-pass" leading to the atmosphere; which is only opened, and that automatically, whenever the pressure increases slightly above its normal value. The valve closing this outlet is called a "relief valve." (See p. 422 of the Engineering News of Nov. 1904, where a valve disc 23 in. in diameter

is described, with its appurtenances; made by the Lombard Governor Co.).

Another method of preventing any material increase of pressure in the penstock when the turbine gate is being lowered is by the use of a stand-pipe of large diameter communicating with a side opening in the penstock near the wheel. When the consumption of water is normal and the flow steady the water in this pipe is at rest and stands at a height reaching to the hydraulic grade-line (see *DK* in Fig. 96). When the wheel-gate closes more or less, a part of the flow from the penstock passes into the stand-pipe and spills over its upper edge; and the rise of pressure near the wheel-gate is not excessive. Conversely, when the wheel-gates open beyond the normal position the extra flow desired is at first furnished by the water in the stand-pipe and the pressure just above the wheel-gates does not fall to too low a value while the water in the penstock is adjusting itself to a new and greater velocity of steady flow. In Fig. 99 is shown the terminal arrangement of a long penstock in Fall Creek gorge at Ithaca, N. Y. This penstock, of some 6 ft. diameter and about 1000 ft. long, supplied two pair of 30-in. "New American" turbines on horizontal shafts (see also Fig. 63), working under 90 ft. head, with draft-tubes as shown. In the upper part of the figure is seen the lower part (only) of a stand-pipe or "relief-pipe" 42 in. in diameter and 47 ft. high. Two air-chambers are also provided, one in each branch of the penstock, just above each wheel-case (containing a pair of turbines, as shown in the figure).

The use of a stand-pipe is considered the best method of obviating water-hammer, etc., in the case of a turbine supplied by a long penstock when the head is not too great and freezing can be prevented. With impulse-wheels supplied through a long penstock the rate at which water is used by the wheel (Pelton, for instance) is sometimes varied by the use of a "deflecting-nozzle" through whose lateral or downward movement, controlled by the governor, more or less of the jet passes on without acting on the buckets. In the Cassell impulse-wheel (see Engineering News, Dec. 1900, p. 442) the two lobes or halves of

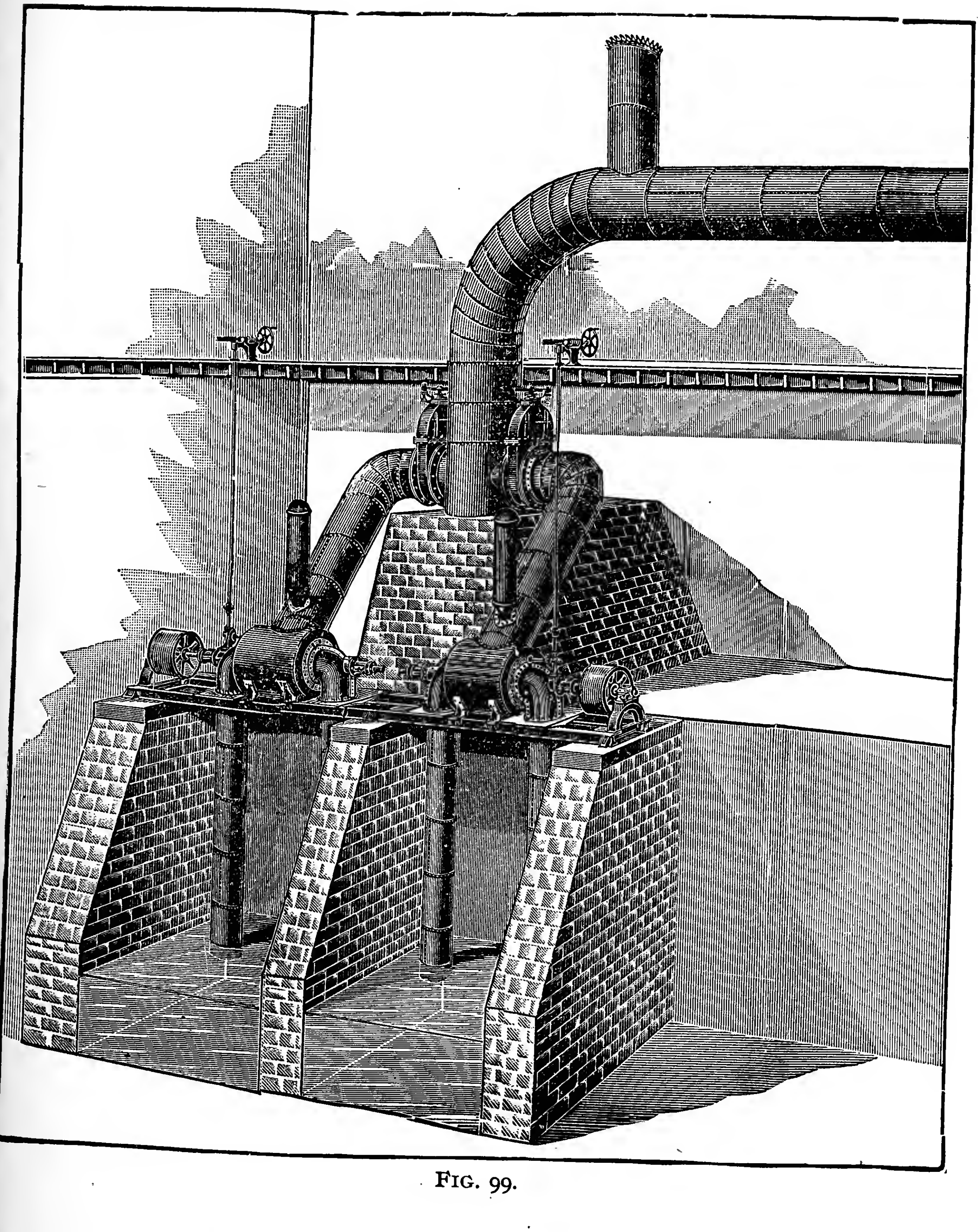


FIG. 99.

each bucket are caused to separate more or less by the action of the governor, and the same object is thus accomplished; a portion of the jet passing between the two parts of the bucket and without action on it. In this way there is no checking of the velocity of the water in the supply-pipe and water-hammer is completely avoided; but of course such a device is not economical of water.

132. Open Channels, or Canals.—Since these are frequently used to conduct water from a reservoir to a wheel-pit or to the inlet of a pipe or penstock, for supplying a hydraulic motor, a few pages will be given to their consideration in the present work in addition to what is already presented in the author's Mechanics o Engineering.

The situation usually presented is that of "uniform motion" in steady flow. By this it is implied that the body of water in motion is of indefinite length and has the form of a geometric prism, i.e., the surfaces of the bed, banks, and of the water itself are parallel, the mean velocity of the water in any section is equal to that in any other and does not change with lapse of time (see p. 756, M. of E.). The flow will not be of this character, however, unless the quantities concerned bear a certain relation to each other. These quantities (as concerned in the most widely used formula, Kutter's Formula, for uniform motion) are the ratio called the "*slope*" $s=\frac{h}{l}$, where h is the fall of the surface (and also that of the bed) in a length l along the channel; the "*hydraulic radius,*" or "hydraulic mean depth," R, =area of cross-section, F, divided by the wetted perimeter; the *mean velocity*, v, of flow (about 0.83 of the surface-velocity in mid-stream); and a "*coefficient of roughness*," n, dependent on the character of the surface of bed and banks. For uniform motion, then, to subsist, the relation which must hold between these quantities, as expressed in Kutter's Formula (which is fairly well supported by a wide range of experiments; though considerable uncertainty must generally prevail in matters of this kind), is (for the English foot and second as units)

$$v=\left[\frac{41.6+\dfrac{1.811}{n}+\dfrac{0.00281}{s}}{1+\left(41.6+\dfrac{0.00281}{s}\right)\dfrac{n}{\sqrt{R}}}\right]\sqrt{Rs}; \quad . \quad . \quad . \qquad (1)$$

or, for brevity, $\qquad v=A\sqrt{Rs}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$

where A stands for "Kutter's coefficient" in the bracket in (1).

The ordinary scheme of values for n is here appended, viz.:

$n=.009$ for well-planed timber evenly laid.

.010; plaster in pure cement; glazed surfaces in good order.

.011; plaster in cement with one-third sand; iron and cement pipes in good order and well laid.

.012; unplaned timber, evenly laid and continuous.

.013; ashlar masonry and well laid brickwork; also the above categories when not in good condition nor well laid.

.015; "canvas lining on frames"; brickwork of rough surface; foul iron pipes; badly jointed cement pipes.

.017; rubble in plaster or cement in good order; inferior brickwork; tuberculated iron pipes; very fine and rammed gravel.

.020; canals in very firm gravel; rubble in inferior condition; earth of even surface.

.025; canals and rivers in perfect order and regimen and perfectly free from stones and weeds.

.030; canals and rivers in earth in moderately good order and regimen, having stones and weeds occasionally.

.035; canals and rivers in bad order and regimen, overgrown with vegetation, and strewn with stones and detritus.

The value of the coefficient A is most readily found from the diagrams in the Appendix of this book. A separate diagram has been constructed for each of the above values of n (and also for $n=.040$). For example, for a hydraulic radius of 2 ft. and a slope of $s=0.0002$ which is 0.2 ft. per thousand we find that when $n=.012$ (unplaned timber) A is equal to 139; to be used with the English foot and second in eq. (2). The value for A

for a slope of 10 ft. per thousand will also hold for all higher slopes with sufficient accuracy (as is also evident from the diagrams).*

The student should guard against the error of supposing that eq. (1) or (2) would hold for measurements made at a single cross-section of a body of water flowing with steady flow in an open channel. The depth, area, and shape of cross-section, and character of surface, etc., must be the *same*, respectively, at all sections of a *fairly long reach* of the channel, to constitute a case of uniform motion to which eq. (1) and (2) apply. Problems involving non-uniform, or variable, motion (with steady flow) where the surface is *not* parallel to the bed (in longitudinal profile) will be considered later.

132a. Coefficient of Fluid Friction for Open Channels.—If we go back to the theoretical basis of the form of the relation in eq. (2) (see pp. 757 and 758, M. of E.), we find the formula for uniform motion to be

$$v=\sqrt{\frac{2g}{f}}\sqrt{Rs}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

involving f, the "*coefficient of fluid friction,*" corresponding to that for flow in pipes. In other words, Kutter's coefficient, A, may be written as $A=\sqrt{2g \div f}$, or

$$f=\frac{2g}{A^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Of course, while f is an abstract number, the same in value whatever units of measurement and time are selected, A is not. Since problems are to be treated in which the flow is not "uniform" (although "steady"), we shall need the quantity f; and this may conveniently be found by first finding A from a diagram, as if the case were one of uniform motion, and then determining f from eq. (4). Or, *vice versa*, if preferable, we may replace the f of a formula applying to a non-uniform steady flow (depths different along the length at different points, e.g.) by its

* A book of *Diagrams of Mean Velocity of Water in Open Channels; Uniform Motion*, by the present writer (New York, J. Wiley & Sons, 1902), obviates the necessity of numerical substitution in the use of Kutter's formula (eq. (1) above) for all practical purposes.

equivalent in terms of A; thus $f = 2g \div A^2$; but if values of A are used from the diagrams (Appendix), the foot and second must be used throughout the whole formula in which A appears.

133. Forms of Section. Open Channels.—The forms of section most generally employed for open channels, for water-power, or for irrigation are the semicircle, or other segment of a circle; the rectangle; and the trapezoid with horizontal base; occasionally the horseshoe, if the channel is roofed over or is in tunnel.

For the semicircular section running full, or for the lower half of any regular polygon, also running full, the hydraulic radius R is equal to half the radius of the inscribed circle. It is also worth noting that any such half regular polygon has a minimum wetted perimeter for a given area and consequently is of the most advantageous form from a theoretical point of view; i.e., to deliver a maximum quantity of water per sec., Q, for a given slope of bed, given area of water prism, and given number of sides for the polygon.

It is also to be noted that of all trapezoidal sections running full and having a common side slope, or angle θ, (see Fig. 100,) of the bank, that one is of the most advantageous form whose three sides forming the wetted perimeter are tangent to the semicircle having a radius CE equal to the depth h and with its diameter in the surface of the water; and its hydraulic radius, R, is equal to the half-depth.

According to Prof. Bovey (Hydraulics, 2nd ed., p. 231) the angle θ should not be made greater than the values given below for different characters of bank, respectively:

with retaining walls 63° 36

with stiff earthen sides, faced.....45°

with stiff earthen sides, unfaced...33° 41′

with sides in light or sandy soil....26° 34′

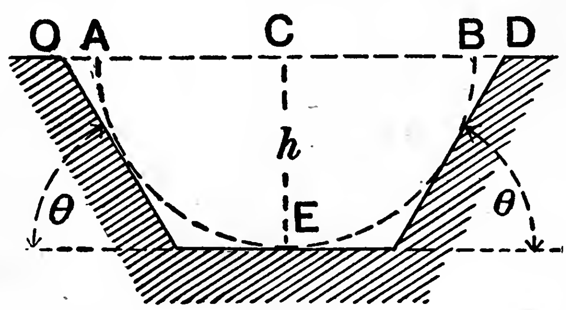


Fig. 100.

To avoid erosion velocities in some soils may have to be limited

to 2 ft. per second or under; with timber or rock for bottom and sides, however, v may be allowed to reach values of 6 to 10 ft. per second.

134. Numerical Example. Open Channel Supplying Wheel-pit.—An open channel with bottom and sides of "average rubble" masonry and whose depth h is to be one-half of its width is to conduct a water-supply of $Q = 120$ cub. ft. per sec. with "uniform motion," with a fall of only 3 ft. in its length of 1200 ft. Compute a proper value for the depth h. See Fig. 101. A is the reservoir and C the wheel-pit.

Solution.—The sectional area being $2h^2$ and the wetted perimeter $4h$, the hydraulic radius is $R = 2h^2 \div 4h = h^2 \div 2$. The coefficient of roughness, n, is 0.017 (for "average rubble")

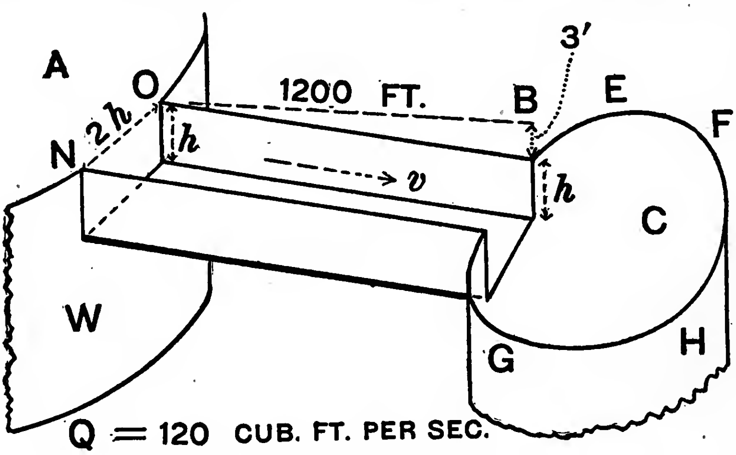


FIG. 101.

(see § 132); and the slope is $3' \div 1200' = 0.0025$, i.e., 2.5 ft. per thousand; but as the value of R is as yet unknown, it is impossible to use the diagram directly for finding the value of Kutter's coefficient A.

Since the values of A, however, range between 65 and 150 for ordinary cases, a value of $A = 100$ may be assumed for a first trial, and a first approximate value of h determined, as follows: With $A = 100$ (for the foot and sec.) and $v = Q \div F = 120 \div 2h^2$, we have from eq. (2), § 132,

$$\frac{120}{2h^2} = 100\sqrt{\frac{h \times 0.0025}{2}}, \quad \text{or} \quad h^5 = 288;$$

i.e., $h = 3.10$ ft., as a first approximation.

The corresponding R is $h \div 2 = 1.55'$ for which and the given slope of 2.5 per thousand we find in the diagram for $n = 0.017$ a value of 96 for A. With this more exact value of A we again use eq. (2), obtaining

$$\frac{120}{2h^2} = 96\sqrt{\frac{h \times 0.0025}{2}}, \quad \text{or} \quad h = 3.155 \text{ ft.},$$

as a second approximation; for which R would be 1.57 ft. Similarly, from this R and the diagram we find $A = 94$, with which (for a third approximation) the value of $h = 3.2$ ft. is found; which is sufficiently close.

Owing to the uncertainty generally involved in the choice of a "coefficient of roughness," n, in problems of this class, results obtained must be looked upon as only approximate. They may be as much as five per cent. in error.

(The solution of this problem would be much shortened by the use of the diagrams mentioned in the foot-note on p. 216. These diagrams deal with v, R, and s; and not directly with the coefficient A.)

A practical matter to be noted in the problem now treated is the fact that where the water passes from the reservoir A into the entrance of an open channel, a drop of the surface will occur of an amount equal to $v^2 \div 2g$; which in the present case is not small.

Since $v = 120 \div 2h^2 = 5.86$ ft. per sec., we have for $v^2 \div 2g$, or corresponding velocity-head, about 0.54 ft.; (see page Conversion Scales, in the Appendix). This drop should be allowed for in arranging the position of the bottom of the channel, and in consequence of it the bottom of the channel at NO should be placed $3'.2 + 0'.54 = 3.74$ ft. below the surface of the (still) water in reservoir A; while the bottom at B should be 3 ft. lower yet, or 6.74 ft. below the surface of the water in A.

135. Height and Amplitude of Backwater.*—If an obstruction such as a weir or dam, for water-power purposes or otherwise, is thrown across the bed of a stream or channel of indefinite

* See Engineering Record, July 1892, p. 91.

length and of regular form, in which originally there was a "steady flow" with "uniform motion"; when the flow again becomes steady, over the weir, the depth of water just above the weir is greater than before and the increase of depth at that point is called the *height of backwater.* Also, the longitudinal profile of the water surface above the weir is more or less curved, the depth being found in general to be less and less as we proceed up-stream. The greatest distance up-stream from the weir at which any increase of depth is perceptible is called the "*amplitude of backwater.*" A knowledge of this distance in the case of a proposed weir and also of the increase of depth at any distance, occasioned by the building of the weir, is often of much importance; since if another weir were built up-stream from the one proposed, its available head of water for power purposes might be affected by the backwater of the first.

After a weir has been built and a steady flow resumed, the conditions of flow of the stream *below* the weir are of course unchanged.

135. Height of Backwater for a Weir not Submerged.—Fig. 102 represents a vertical section of an overfall weir (see pp. 674 and 683, M. of E.) having a sharp-edged sill or crest, A, *higher* than the surface of the tail-water or original surface of the stream and with its up-stream face vertical. We suppose that the whole discharge Q, cub. ft. per sec., of the stream is passing over the weir and that the air has free access under the sheet; and that there are no "end-contractions"; that is, that the crest terminates in two vertical faces parallel to the axis of the stream (see p. 686, M. of E.) forming a "channel of approach." These conditions justify the use of

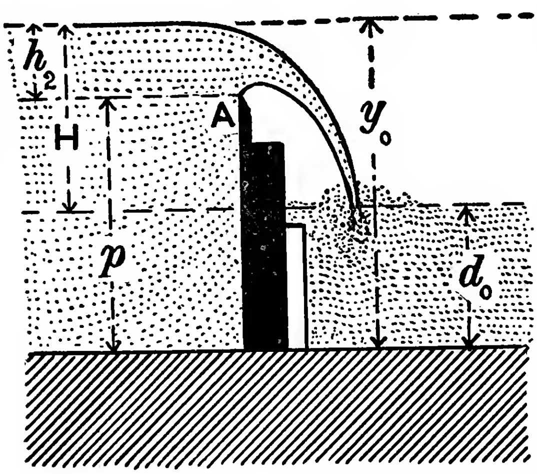


Fig. 102.

Bazin's formula * (p. 688, M. of E.), b being the length of crest of the weir (and also the width of the channel of approach) and p the height of the weir above the horizontal bottom of the channel of approach; see Fig. 102. (The stream itself may, however, be wider than the weir.) The formula is

$$Q=\tfrac{2}{3}\mu'\left[1+\frac{0.55h_2}{p+h_2}\right]bh_2\sqrt{2gh_2}; \quad . \quad . \quad . \quad . \quad (1)$$

in which μ' has the value

$$\mu'=0.6075+0.0148\div(h_2 \text{ in feet}). \quad . \quad . \quad . \quad . \quad (2)$$

Problem.—Required the height p of weir to produce a given height of backwater H, b and Q being both known, as also d_0, the original depth of the stream and (still) the depth of the water below the weir. Evidently we have $d_0+H=y_0$ (see Fig. 102) and thus y_0 (the total new depth at weir) becomes known. For the determination of p, therefore, we have eq. (1) above and the relation

$$p+h_2=y_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The solution is best effected by writing (1) in this form:

$$1+\frac{0.55h_2}{y_0}=\frac{3Q}{2\mu'b\sqrt{2g}\cdot h^{\frac{3}{2}}}; \quad . \quad . \quad . \quad . \quad . \quad (4)$$

to be solved by trial for h_2 (or "head on the weir").

Example—Let the channel be rectangular in section with a width equal to that, b, of the weir (which is of the form just described and indicated in Fig. 102); with $b=30$ ft. and $Q=310$ cub. ft. per second; while the original depth is $d_0=3$ ft. It is required to find such a value for the height p of the weir as to make the increase of depth or height of backwater, H, equal to 4.5 ft.; or the total depth just above the weir, y_0, $=7.5$ ft.

First assuming $h_2=3$ ft., with 0.60 as a first approximation for μ', the right-hand member of eq. (4) $=0.619$; while the left-hand member becomes equal to. . . . . . . 1.22.

Trying $h_2=2.5$ ft. with μ' still equal to 0.60, we find

---

* In Bazin's experiments p ranged from 0.2 to 2 metres; h_2 from 0.05 to 0.6 metres; and b from 0.5 to 2 metres.

the right-hand member of (4) = 0.819, and
" left-hand " " " = 1.18

Again, with h_2 assumed = 2.0 ft., and hence $\mu' = .6075 + \frac{.0148}{2} =$ 0.6149, the right-hand member of (4) = 1.113
" left-hand " " " = 1.1467

We may therefore conclude without further trial that a value of $h_2 = 2.1$ ft. will serve the purpose.

In case the channel of approach is considerably wider than the length of the weir crest, b, there will be end-contractions and we may use the formula of Francis as given on p. 687, M. of E.

137. Special Forms of Weir. Mr. Rafter's Experiments.— In 1899 experiments were made at the Hydraulic Laboratory of the College of Civil Engineering at Cornell University by Mr. G. W. Rafter for the United States Board of Engineers on Deep Waterways (see vol. xliv of the Transac. Amer. Soc. Civil Engineers, p. 220) on special forms of weirs; some of which involved a sloping face on the up-stream or down-stream side, or both; some with flat tops. Results for a number of these forms will now be quoted. Air was given free access under the sheet, or "nappe," of water on the down-stream face in each case.

The crest of the weir was in each case 6.56 ft. long and end-contractions were suppressed, i.e., the channel of approach had the same width as each weir and the depth of water (h_2) above the crest of the weir in some of these experiments was in some cases as great as 5 ft. The channel of approach had the same width as each weir and was rectangular in section; and extended back some 40 ft. from the weir.

Fig. 103 gives a general idea of the form of some of these weirs and of the quantities involved. The "head on the weir," h, was observed, and the height of weir p was measured and recorded in each case, as also the data fixing the form of the top and two faces of the weir. The rate of flow Q became known in each case from the observed head and known dimensions of a standard Bazin weir (sharp-edged with vertical faces,

etc.) over which the water flowed on its way to the experimental weir.

The formula used by Mr. Rafter in expressing the rate of

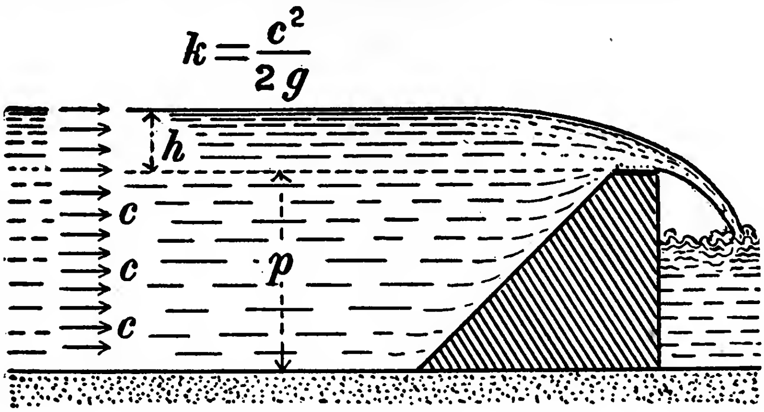


FIG. 103.

flow over any of these experimental weirs may be put into the form

$$Q=mb\sqrt{2g}\cdot(h+k)^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where b is the length of the weir, m a coefficient corresponding to the $\frac{2}{3}\mu$ of former equations, and h the observed head on the

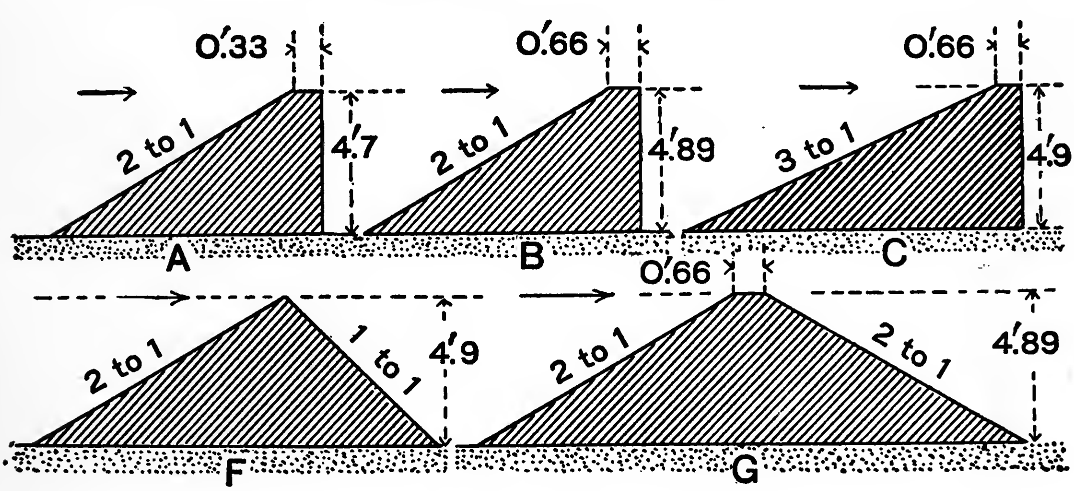


FIG. 104.

weir (see Fig. 103); while k stands for $c^2 \div 2g$, or height due to the "velocity of approach," c, this velocity being equal to the $Q \div$ area of cross-section of channel of approach.

It is seen that k depends on the discharge Q itself; but it is

generally so small that a value for it obtained from an approximate value of Q, based on a zero value for k in eq. (5), is sufficiently close for substitution in a second use of eq. (5); from which a second and closer value of Q is secured.

The values of the coefficient m will now be given as obtained by Mr. Rafter for nine weirs of different shapes, to be designated as A, B, etc. The dimensions and form of vertical section of seven of these weirs are shown in Figs. 104 and 105, the direction of flow being indicated by the arrow. The weirs called C and D in the following table differed from B only in the slope of the up-stream side; which was 3 to 1 for C, and 4 to 1 for D.

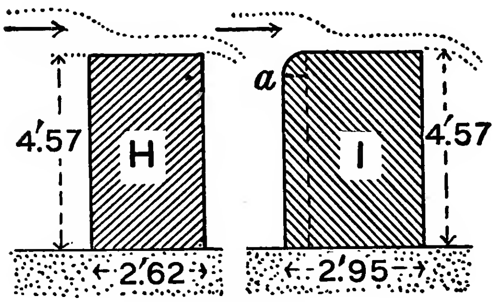


Fig. 105.

It will be noticed that form I differs from H (see Fig. 106) in being about one-eighth wider and in having the up-stream corner rounded in a quadrant of radius = 0.33 ft., or 4 inches; and it will also be noted, from the table below, that the discharge is thereby increased by more than ten per cent., for the lower heads.

This rectangular weir of broad crest with rounded up-stream shoulder as shown in form I is also capable of theoretic treatment for the determination of discharge. Such treatment has been applied by Prof. Unwin in his article Hydromechanics in the Encyclopædia Britannica (p. 472 of that article). Prof. Unwin's result gives a value of 0.385 for the coefficient m of eq. (5). With a slight deduction to allow for friction, which has been neglected in Prof. Unwin's treatment, this agrees well with Mr. Rafter's values for m for form I with the lower heads (under 2 ft.); and it is for the lower heads that the theory is more reasonable.

The following table gives values of the coefficient m (to be used in eq. (5)) for six different values of the head h in feet, for the different forms of weir, A, B, etc., as mentioned above.

If a weir is very long, as often occurs with mill-dams, it

$h=$	0.5 ft.	1.00	1.5	2	4	6
A	$m=0.418$	0.459	0.476	0.470	0.461	0.462
B	$m=$ .401	.428	.447	.456	.461	.462
C	$m=$ .454	.476	.478	.460	.442	.442
D	$m=$	.429	.432	.434	.434	.434
E	$m=$ .412	.415	.416	.418	.422	.422
F	$m=$ .525	.529	.505	.486	.461	.453
G	$m=$ .391	.426	.442	.450	.456	.452
H	$m=$ .324	.333	.343	.354	.400	.433
I	$m=$ .369	.375	.378	.384	.422	.442

makes little difference whether there is contraction at the ends or not, while if h is less than about one-fifth of the height of weir, p, the quantity k is of little consequence; especially when we consider that results obtained by the use of eq. (5) may sometimes be in error by two, or even three, per cent.

Mr. Rafter's paper containing the account of the experiments just mentioned includes also a useful résumé of the experiments made by Bazin on a great variety of weir forms. See also pp. 222, etc., in Turneaure and Russell's "Public Water-supplies."

138. Submerged or "Drowned" Weirs.—If the height of weir, p, is less than the original depth of the stream, a submerged weir results. But few experiments have been made on this kind of weir. According to Mr. Herschel (Transac. Am. Soc. C. E., 1885, xiv, p. 194) for submerged weirs with sharp crests, up-stream face vertical, and without end-contractions (as in Fig. 106), the following formula may be used, based on the experiments of Francis and also those of Fteley and Stearns:

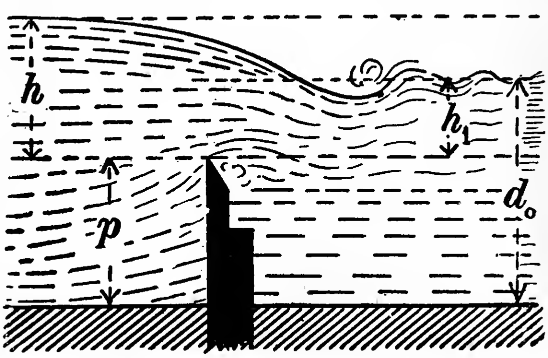


FIG. 106.

$$Q=3.33b(nh)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

for the discharge in cub. ft. per sec.; the length b of the weir and h, the "head on the weir," (see figure,) both being expressed In ft.; while the number or coefficient n depends on the value of the ratio $h_1 \div h$.

Mr. Herschel gives the following table for n:

$h_1 \div h$	n	$h_1 \div h$	n	$h_1 \div h$	n	$h_1 \div h$	n
0.00	1.000	0.20	0.985	0.45	0.912	.70	0.787
.02	1.006	.25	.975	.50	892	.75	.750
.05	1.007	.30	.959	.55	.871	.80	.703
.10	1.005	.35	.944	.60	.846	.90	.574
.15	0.996	.40	.929	.65	.819	1.00	.000

Example.—With the same stream as in the example of § 136, 30 ft. in width, d_0 the original depth $=3$ ft., and with $Q=310$ cub. ft. per sec.; if a sharp-crested weir without end-contractions of 2.6 ft. height, $=p$, be built across the full width of the stream, what increase of depth will be occasioned just above the weir? That is, $h=$?

Solution.—Since, from eq. (6),

$$nh=\left(\frac{Q}{3.33b}\right)^{\frac{2}{3}}=\left(\frac{310}{99.3}\right)^{\frac{2}{3}}=2.126, \quad . \quad . \quad . \quad . \quad (7)$$

we have, putting $n=0.9$ as a first approximation, $h=2.36$ ft.; from which, since $h_1 \div h=(3-2.6) \div 2.36=0.169$, we find a value of 0.992 for n, from the table. For this closer value of n we now derive, from eq. (7), $h=2.14$ ft. as a second approximation.

Again, $h_1 \div h$ would now become $(3-2.6) \div 2.14=0.187$; i.e., from the table, n would be equal to 0.988; and finally, as sufficiently close,

$$h=2.126 \div 0.988=2.15 \text{ ft.};$$

and hence the new depth just above the weir will be $p+h$, or 4.75 ft.

139. Discontinuous, or Incomplete, Weirs. Height of Back-water.—The rise of water in a stream occasioned by the building

of bridge piers, jetties, moles, breakwaters, dikes, or causeways, may be approximately computed by the following methods.

Fig. 107 shows the case of a jetty projecting part way across the width of a stream. The upper part of the figure gives a vertical projection parallel to axis of stream, the lower part a horizontal projection. The width of the stream, originally equal to b', $b+e$, is only b, opposite the end of the jetty; which takes up a portion e of the original width. This causes an increase of depth, $=H$, just above the jetty when a steady flow has again set in. The whole discharge of the stream, Q, must now pass through the narrow width $b=\overline{BC}$.

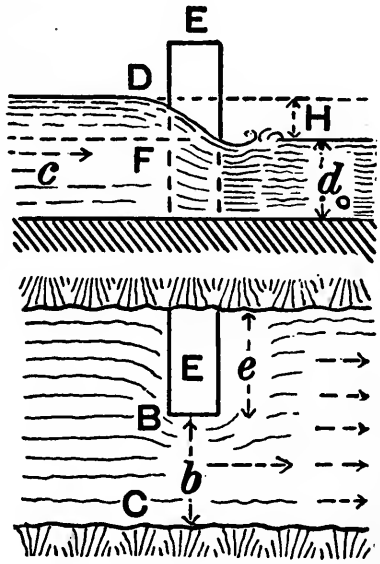


Fig. 107.

The fraction of Q (call it Q_1) passing through the portion DF of the depth may be treated as if flowing through an overfall notch and written

$$Q_1 = \tfrac{2}{3}\mu b H\sqrt{2gH}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

while that, Q_2, passing below the level of point F may be considered as flowing through a vertical rectangular opening (and discharging under water) of a height $=d_0$ (depth of tail-water; original depth) and width $=e$, all filaments having a common velocity $=\sqrt{2g\cdot H}$ (p. 669, M. of E.); that is,

$$Q_2 = \mu b d_0\sqrt{2gH}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

But $Q_1+Q_2=Q$; and hence, finally, considering the two μ's to be about equal, we have

$$Q = \mu b\sqrt{2gH}[\tfrac{2}{3}H+d_0]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If the height H is small or the velocity of approach considerable, with $k=c^2\div 2g$ (c being the velocity of approach, viz.

$$c = Q\div(H+d_0)b', \text{ nearly}) \quad . \quad . \quad . \quad . \quad (11)$$

(see p. 674, M. of E., eq. (3),)

we have, to take the place of eq. (10) above,

$$Q = \tfrac{2}{3}\mu b\sqrt{2gH}[(H+k)^{\frac{3}{2}} - k^{\frac{3}{2}}] + \mu b d_0\sqrt{2g(H+k)}. \quad . \quad (12)$$

The coefficient μ may range from 0.70 or 0.80 to 0.95 according to the degree of rounding of the end of the dike. In the use of eq. (12), which cannot be expected to give more than roughly approximate results, it is best to solve by successive assumptions and trials, to avoid mathematical complications.

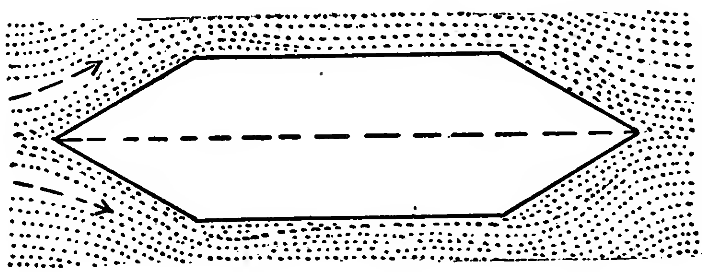


FIG. 108.

In the case of a number of bridge piers, the b of eq. (12) would represent the sum of the widths of the openings between the piers. To prevent the injurious effects of eddies, etc., both ends of the horizontal section of a bridge pier should be rounded or sharpened off, as illustrated in Fig. 108. According to Weisbach and Gauthey, if the ends are rounded or shaped with a very obtuse angle μ may be taken as 0.90; with an acute angle, 0.95; or even 0.97, if two circular arcs meeting at an acute angle are used.

140. Amplitude of Backwater caused by a Weir.—The law by which the depth of the water which has been increased by the weir diminishes with the distance up-stream from the weir must now be investigated. This can be referred to the theory of steady flow with variable motion or "non-uniform motion" in an open channel, as given on pp. 768, etc., of M. of E. Suppose the stream above the weir to be divided into several distinct portions by successive transverse vertical planes. For each of these we may have a separate treatment, the surface of each being considered straight by itself. Fig. 109 shows

a short length of the stream above the weir, the flow being *steady*. Let the depth at A be y_1 ft., the area of cross-section F_1 sq. ft., and the mean velocity v_1 (of stream-lines passing that section); also w_1 = the mean wetted perimeter of portion AC of stream. For section C, y_0, F_0, and v_0 have a similar meaning. The length AC call l_1. Let m and n be points where any stream-line crosses the two sections; z_1 and z_0 their heights above datum through O; and let water-barometer height be b, and the slope of the bed BO be α. Now the fluid pressure at m (since flow is horizontal) is atmospheric plus that due to water height $A..m$, $\therefore \frac{p_m}{\gamma}$, $=$ pressure-head at m, $= b + \overline{Am} = b + \overline{AE} - z_1 = b + y_1 + l_1 \sin\alpha - z_1$; and similarly $\frac{p_n}{\gamma} = b + y_0 - z_0$. For friction-head between m and n we may use the form $\frac{fl_1}{R_1}\cdot\frac{v_m^2}{2g}$ (see eq. (3), p. 757, M. of E.) with R_1 = mean hydraulic radius for AC, $= \frac{1}{2}[F_0 + F_1] \div w_1$; and $v_m^2 = \frac{1}{2}(v_0^2 + v_1^2)$.

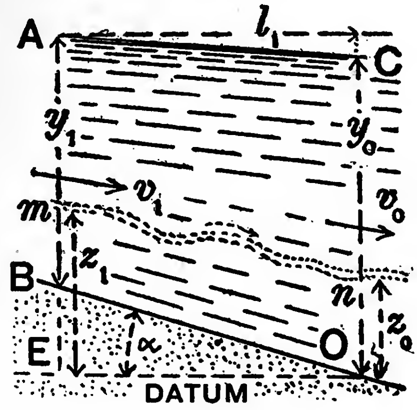


FIG. 109.

Hence Bernoulli's Theorem for the steady flow of the stream-line m to n will give us

$$\frac{v_0^2}{2g} + \frac{p_n}{\gamma} + z_0 = \frac{v_1^2}{2g} + \frac{p_m}{\gamma} + z_1 - \frac{fl_1}{R_1}\cdot\frac{v_m^2}{2g}. \quad . \quad . \quad . \quad (13)$$

With above values of p_m, p_n, R_1, and v_m^2, this reduces to

$$\frac{v_0^2}{2g} + y_0 = \frac{v_1^2}{2g} + y_1 + l_1\left[\sin\alpha - \frac{fw_1}{F_0 + F_1}\cdot\frac{v_0^2 + v_1^2}{2g}\right]. \quad (14)$$

If eq. (14) be applied to the segment of stream next above the weir (see Fig. 110), remembering that the delivery of the stream, Q (cub. ft. per sec.,) is $= F_0 v_0$, etc., so that $v_0 = Q \div F_0$, $v_1 = Q \div F_1$,

etc., we have

$$l_1=\frac{y_0-y_1-\left[\frac{1}{F_1{}^2}-\frac{1}{F_0{}^2}\right]\frac{Q^2}{2g}}{\sin\alpha-\frac{fw_1}{F_0+F_1}\left[\frac{1}{F_1{}^2}+\frac{1}{F_0{}^2}\right]\frac{Q^2}{2g}}; \quad . \quad . \quad . \quad (15)$$

and similarly for the second segment up-stream,

$$l_2=\frac{y_1-y_2-\left[\frac{1}{F_2{}^2}-\frac{1}{F_1{}^2}\right]\frac{Q^2}{2g}}{\sin\alpha-\frac{fw_2}{F_1+F_2}\left[\frac{1}{F_2{}^2}+\frac{1}{F_1{}^2}\right]\frac{Q^2}{2g}} \quad . \quad . \quad . \quad (16)$$

(and the method of further procedure is now evident).

Thus, assuming successive decrements of depth y_0-y_1, y_1-y_2, etc., and computing from these the areas F_1, F_2, etc., we obtain from the above formulæ the distances l_1, l_1+l_2, $l_1+l_2+l_3$, . . . , etc., from the weir, of the sections where the assumed depths will be found.

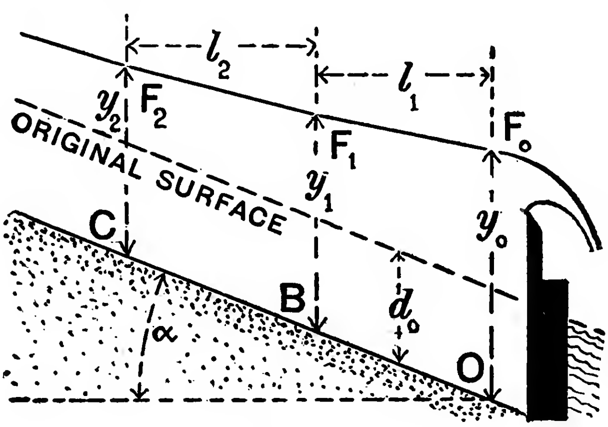


Fig. 110.

141. Numerical Example. Amplitude of Backwater. (The data are from Weisbach's *Mechanics*, but the treatment is more modern.) It is required to determine the amplitude of backwater produced by a weir in a stream 80 ft. wide and originally 4 ft. deep, in which the flow was a *uniform motion* before erection of the weir, if the weir causes the surface (immediately above it) to be raised 3 ft. higher than its original position, and if the discharge of the stream is $Q=1400$ cub. ft. per sec. The bed

is of rock, but fairly smooth, such as would justify the use of a value of $n=0.020$ in Kutter's formula for "uniform motion."

Before the erection of the weir the slope of the surface ($=s$ of § 132) was equal to that of the bed, which is $\sin\alpha$ of our present formulæ. We shall suppose that it is necessary to compute the value of $\sin\alpha$ from a knowledge of the fact that the motion was "uniform" before the erection of the weir. Before erection of weir the mean velocity, v, was $v=1400\div 80\times 4$, $=4.37$ ft. per sec., and the surface of the water was parallel to the bed, so that the relation then holds (p. 757, M. of E.) (see also § 132 of this book)

$$\sin\alpha, =\frac{h}{l}, =\frac{fw}{F}\cdot\frac{v^2}{2g}, =\frac{v^2}{A^2R}. \quad . \quad . \quad . \quad (17)$$

Taking Kutter's coefficient of roughness, n, as 0.020 (see § 132) we find that, with $R=320\div 88=3.6$ ft., the value of Kutter's A (see diagram for $n=0.020$ in Appendix) must lie between 91 and 94. With the value 94 for A, $\sin\alpha$ or s, from eq. (17), would be about .0006; and with this approximation to the slope we find more exactly, from the diagram, $A=93$; the use of which in eq. (17) yields a value of $\sin\alpha=.000615$, which will be used in eq. (15), etc.

We have given, therefore, that $y_0=7$ ft. in Fig. 110 and now inquire, *first*, at what distance, l_1, up-stream from the weir, has the depth diminished to $y_1=6.5$ ft. With $y_0-y_1=7-6.5=0.50$ ft., we have $F_0=80\times 7=560$ sq. ft.; $F_1=80\times 6.5=520$ sq. ft.; $Q=1400$ cub. ft. per sec.; and w_1 may be taken as 93 ft. For this first portion of the stream above the weir we find the mean hydraulic radius R to be $80\times 6.75\div 93=5.8$ ft., for which from the diagram for $n=0.020$ (in the Appendix) Kutter's coefficient A is noted to be about 100; whence the value of the coefficient f, needed in eq. (15), is, $2g\div A^2$, $=0.00644$.

Substituting now in eq. (15) we have (ft. and sec.)

$$l_1=\frac{0.50-(.00000370-.00000319)30380}{.000615-\frac{.00644\times 93}{1080}(.00000370+.00000319)30380}$$

$$=0.484\div 0.000498=973 \text{ ft.}$$

Next assuming a value of $y=6$ ft., whence $F=480$ sq. ft., while R_2, the mean hydraulic radius of this second portion of stream, $=500\div 92, =5.4$ ft. (the mean wetted perimeter being taken as $w_2=92$ ft.). For which R_2 we find, from the diagram, $A=99$, whence $f, =2g\div A^2, =0.00657$. Substitution of these, with other known values, in eq. (16) gives the result $l_2=1029$ ft.

Similarly, with $y_3=5.5$ ft., $y_4=5$ ft., and $y_5=4.5$ ft., we find successively $l_3=1121$, $l_4=1320$, and $l_5=1755$ ft. That is to say, adding the proper lengths,

The height of backwater at the weir is	....	3.0 ft.
" " " " "	973 ft. above the weir is..	2.5 "
" " " " "	2002 " " " " " ..	2.0 "
" " " " "	3123 " " " " " ..	1.5 "
" " " " "	4443 " " " " " ..	1.0 "
" " " " "	6198 " " " " " ..	0.5 "

This method must be looked on as only roughly approximate. Theoretically (see next §) the curve of backwater is asymptotic to the original line of water surface (in ordinary cases), so the height of backwater becomes zero only at an infinite distance from the weir.

142. Amplitude of Backwater for a Shallow Stream of Rectangular Section. Results by Calculus.—Consider a very wide stream of rectangular section, in which the depth was $=d_0$ (same at all sections) and motion *uniform*, velocity $=v$, $=Q\div bd_0$, *before* the erection of a weir; width $=b$ at all sections, before *and after* erection of weir. Fig. 111 shows a profile LMY of water surface *after* erection of weir, also original position TR of surface. Take axes X and Y, as shown. Then, at any point M of profile, $y=MP$ is depth of water and $y'=\overline{MR}$ is the *increase* of depth (or "height of backwater"), due to the weir, at any distance $x=\overline{PO}$ from the weir. The slope of the bed will be denoted by $\sin\alpha$, and the quantity $v^2\div 2g$, or height due to the original mean velocity, by k.

If the foregoing treatment of successive finite reaches along the stream above the weir be applied to vertical slices or laminæ of horizontal thickness dx, the areas of the two faces of

each such lamina being by and $b(y-dy)$, a differential equation may be formed and an integration effected, involving one or two approximations (detail will be found in the works of Weisbach, Bresse, and Grashof, etc.), resulting in the following

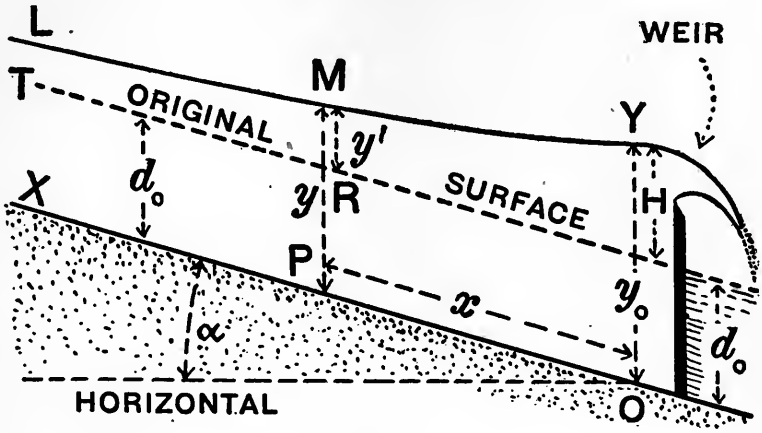


FIG. 111.

equation to the curve of backwater in this case, $L \ldots Y$, or profile of water surface after the erection of the weir, viz.:

$$(\sin \alpha)x = y_0 - y + (d_0 - 2k)(\phi - \phi_0); \quad . \quad . \quad . \quad (18)$$

x and y being the two coordinates of any point M on the curve, and y_0 the depth of the water immediately above the weir; while ϕ is a variable (and abstract number) and a function of the ratio $y \div d_0$. A series of values of ϕ, sufficient for practical purposes, is here given:

TABLE OF VALUES OF THE FUNCTION ϕ.

For $\frac{y}{d_0}$ =	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7
ϕ =	infinite	.680	.480	.376	.304	.255	.218	.189
For $\frac{y}{d_0}$ =	1.8	1.9	2.0	2.2	2.4	2.6	2.8	3.0
ϕ =	.166	.147	.132	.107	.089	.076	.065	.056

By ϕ_0 in (18) is meant the value of ϕ when $y = y_0$.

Hence, for a *shallow* stream of rectangular section, given the data of the weir, height and backwater at the weir, original

depth of stream, d_0, the constant width, b, etc., we can find the "amplitude" x, up-stream from the weir, of the point where any assumed depth y [or height of backwater $y-d_0$] will be found. Of course y must lie between y_0 and d_0 in value.

For $y=d_0$, evidently $x=\infty$ (see table).

To justify the use of this method the *depth must not be greater* than about *one-fifteenth* of the width b.

Example.—Let us apply the foregoing table for ϕ and eq. (18) to the data of the example just treated in § 141.

As already found in that paragraph, $\sin\alpha=0.000615$; while $k=(4.37)^2\div 2g=0.34$ ft. Since $y_0\div d_0=7\div 4=1.75$, we have, from the table, $\phi_0=0.177$. Let us inquire the value for x in order that y may be equal to 6.5 ft.; that is, $y\div d_0=6.5\div 4=1.625$; for which from table we find $\phi=0.211$.

Substitution of these values gives

$$(0.000615)x=7-6.5+(4-0.68)(0.211-0.177);$$

whence $x=997$ ft.

Again, the distance x from the weir at which the new depth will be $y=5.5$ ft., ϕ being now found to be 0.322, is determined by solving the equation

$$(0.000615)x=7-5.5+(4-0.68)(0.322-0.177);$$

i.e., $x=3225$ ft.

It is seen that these results do not differ greatly from those found by the more cumbersome method of § 140.

143. Other Equations for the Backwater Curve.—According to Poirée, the backwater curve may be considered to be a parabola with its axis vertical and having its vertex at Y (see Fig. 111); its actual magnitude in any case being determined by making it tangent to the original surface TR at a point whose abscissa is $x=2H\div\sin\alpha$, with H denoting $y-d_0$ (as marked in figure) or increase of depth at the weir. That is, the equation to the parabola of backwater would be, on this basis,

$$y=H+d_0-(\sin\alpha)x+\frac{(\sin^2\alpha)x^2}{4H} \quad . \quad . \quad . \quad . \quad (19)$$

St. Guilhem's equation to the backwater curve is much more complicated, but was devised to correspond as nearly as possible to actual measurements made of a backwater curve on the river Weser, in Germany. (See p. 772, M. of E.; and Bennett's translation of D'Aubuisson's Hydraulics, pp. 179 and 180). The formula is (for the *English foot*)

$$[y-d_0+x\cdot\sin\alpha]^3=\frac{H^8}{H^5+\dfrac{(x\cdot\sin\alpha)^6}{7.382}}+(x\cdot\sin\alpha)^3. \quad . \quad (20)$$

(Eqs. (19) and (20) are for use with *shallow* streams.)

Other references in this connection are: Engineering Record, July 1892, p. 91; Report of Chief of Engineers of U. S. Army for 1887, p. 1305; DeBauve's Manuel de l'Ingénieur; Annales des Ponts et Chaussées for 1837; Transac. Am. Soc. Civ. Engs., vol. ii, p. 255.

144. Variable Motion, but with Diminishing Depth. Steady Flow.—In the foregoing numerical illustrations of variable motion the cases have been those of depth increasing going downstream, since the backwater due to a weir was under treatment. We now take an example in which the depth diminishes downstream, see Fig. 112. Let the open channel have bottom and

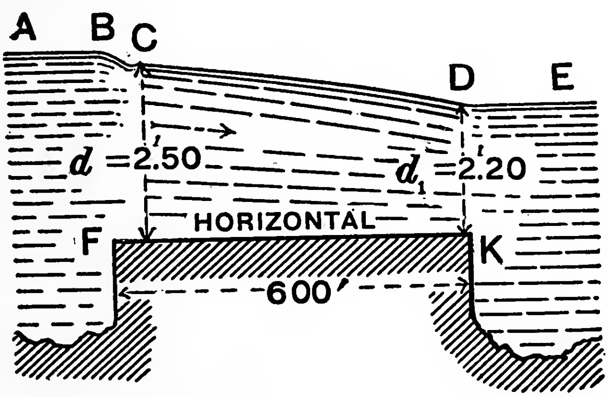


Fig. 112.

sides of rough brickwork (or $n=0.015$ as Kutter's coefficient of roughness), with width $b=10$ ft. and length 600 ft. and a horizontal bed. It connects a large pond A with another pond (or wheel-pit) E. When the positions of the surfaces of the

water in these two ponds are such that depth in the channel is 2.50 ft. at C (an allowance having been made for the small drop BC of the surface) and is 2.20 ft. at D, it is required to compute what the rate of discharge (Q) must be, under these circumstances. The cross-section of channel is rectangular.

Since an approximate value for the mean hydraulic radius is $10 \times 2.4 \div 14.8 = 1.6$ ft. (w_1, the mean wetted perimeter, being $10 + 2 \times 2.4, = 14.8$ ft.); the average slope of the surface also being $0.3 \div 600$ or 0.5 ft. per thousand; we find in the diagram for $n = 0.015$ (in Appendix) the value $A = 107$. From this we have for the coefficient of friction f, $= 2g \div A^2$, $= 0.00562$.

Taking the whole length of 600 ft. as a single reach we may now apply eq. (15) of § 140 with values above obtained for f and w_1 and also $y_0 = 2.2$ and $y_1 = 2.5$ ft.; $\sin \alpha =$ zero; $F_0 = 22$ and $F_1 = 25$ sq. ft.; with $l_1 = 600$ ft. Careful attention being paid to the signs, we finally derive $Q = 66.4$ cub. ft. per second.

This value of Q implies a mean velocity at section C of 2.65 ft. per second. For the water to acquire this velocity at C the surface must fall a vertical distance $BC = (2.65)^2 \div 2g = 0.109$ ft.; so that the whole difference of level between the surfaces of still water in the two ponds must be $0.109 + 0.30$, $= 0.409$ ft., if the above rate of discharge is to take place.

It will be of interest to note that if the bottom (starting at the same point F) were given a downward slope parallel to that which it desired that the surface of water shall have (that is, drop 0.30 ft. in the 600 ft. of length), we have a case of "*uniform motion*" to which Kutter's formula may be applied (see § 132; A being taken from the proper diagram). The result for the discharge is then found to be $Q = 75$ cub. ft. per sec.; which is greater (as of course it should be) than in the first case. The mean velocity, then, in all sections would be 3 ft. per sec., and the drop BC would be 0.15 ft.; necessitating a difference of level from surface A to surface E of $0.15 + 0.30 = 0.45$ ft.

It must be remembered that in all problems of this class there is considerable uncertainty as to the influence of the roughness of the bed which cannot be brought into play with any precision.

145. Open Channel of Horizontal Bed and Shallow Depth. Depth Diminishing Down-stream.—In case the depth diminishes down-stream in steady flow in an open channel of rectangular section with *horizontal bed* and *shallow depth* (the depth not being greater than (say) one-fifteenth of the width), an application of the calculus to the successive vertical laminæ (of horizontal thickness dx) leads finally to the relation (adapted from p. 454 of Ruehlmann's Hydromechanik)

$$2Q^2[fl+2(d-d_1)]=gb^2(d^4-d_1^4); \quad . \quad . \quad . \quad (21)$$

in which b is the (constant) width of the channel (rectangular section), d the depth at any point, and d_1 the depth at any distance l down-stream from the first; while Q is the volume carried per second. The coefficient of fluid friction, f, may be obtained, as previously, from the relation $f=2g \div A^2$ (see § 132, eq. (4)), where A is Kutter's coefficient, to be found from the diagrams in the Appendix.

The quantities d and l may be looked upon as ordinate and abscissa (see Fig. 112), with K as an origin, of the curve formed by the upper longitudinal profile of the water surface. In applying this relation the restriction as to the depth being small should be carefully borne in mind; since otherwise results might be obtained which would be very wide of the truth. (The example worked out in the preceding problem could not be treated by eq. (21), as the relative depth is much too great.)

Example.—Let the width of the channel (with horizontal bed; see Fig. 112, which will serve our present purpose, although the numerals there printed do not now apply) be 100 ft., and the depth at section C be $d=3$ ft. If it is noted that the depth d_1 at section D, 1000 ft. down-stream from C, is 2 ft., at what rate must the water be flowing (volume per sec., $Q=?$) Let the degree of roughness of the bed be such as to correspond to Kutter's $n=0.020$.

The mean slope (with reference to finding f) may be taken as 1 ft. per thousand and the mean hydraulic radius R as 2.25 ft. With these values in the diagram for $n=0.020$ (Appendix) we find $A=85$; and therefore $f, =2g \div A^2, =0.00891$.

Substitution in eq. (21) gives

$$2Q^2[8.91+2(3-2)]=32.2(100)^2(81-16);$$

from which, finally, $Q=980$ cub. ft. per sec.

(**Note.**—If in this case the bed sloped parallel to the surface of the water, the depth being 3 ft. at all sections, we should have a case of "*uniform motion*," to which Kutter's Formula would be applicable; and the result in that case would be found to be $Q=1290$ cub. ft. per second.)

146. Standing Waves.—A "standing wave," or "hydraulic jump," may be formed by the introduction of an obstruction in

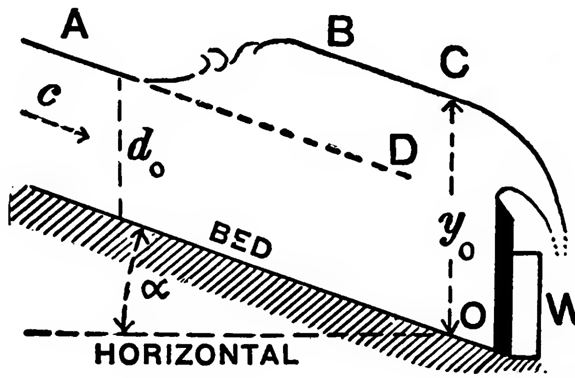


Fig. 113.

a stream whose velocity c is so great (relatively) that k, or $c^2 \div 2g$, is greater than the original depth, d_0, of the stream.

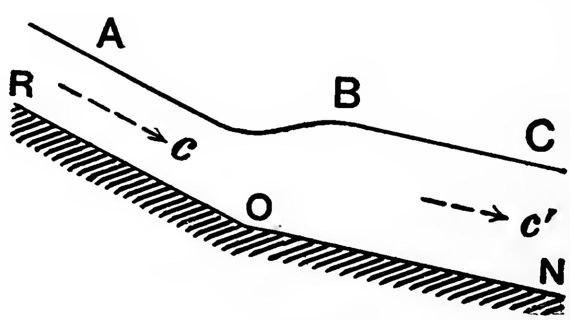


Fig. 114.

Other considerations also enter. For the mathematical treatment involved, see Ency. Britann., article Hydromechanics, p. 501; Weisbach's Hydraulics and Hydraulic Motors, § 154;

Ruehlmann's Hydromechanik, p. 475; and also Merriman's Hydraulics, p. 342. (And Eng. News, July 1895, p. 28.)

A change of slope in the bed (Fig. 114) may also occasion a standing wave.

CHAPTER IX.

Pressure-engines, Accumulators, and Hydraulic Rams.

147. Pressure-engines.—In Fig. 115 we have a previous figure reproduced (see § 6) and now take up more fully the subject of water-pressure machinery. In a water-pressure engine we have in general a piston capable of a reciprocating (to and fro) motion in a cylinder, the edges of the piston fitting accurately, allowing no leakage of water. At the two ends of

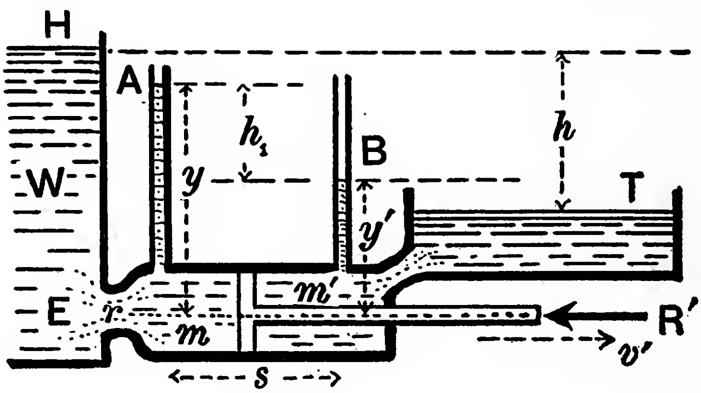


Fig. 115.

the cylinder are ports, or passageways, opened and closed at the proper time by sliding pieces called valves (or if cylindrical in shape, like stoppers, then piston-valves). In this way either side of the end of the cylinder may be put into communication with the supply-pipe r or with the exhaust-pipe leading to the tail-water T (or directly to the outer air). In case the tail-water is below the level of the cylinder the exhaust-pipe is called a *suction-tube* or *draft-tube*. In that case the height of the cylinder above the tail-water is limited to about 20 or 25 ft.

A motor is said to be "*single-acting*" when the "exhaust side," m', is always open to the atmosphere, while the other side, m, communicates alternately with the outer air or with the source of supply. In a single-acting pressure-motor no working force acts in the return stroke, i.e., no useful work is done, the motion being brought about by the inertia of a fly-wheel or by the action of some other working piston if there are more than one piston and cylinder provided.

A "*double-acting*" motor is one in which during the return stroke the two ends of the cylinder change places as regards communicating with the supply E or with the tail-water (or exhaust) T. In such a case about the same amount of useful work is done in the return as in the forward stroke; though account must be taken of the fact that there is a difference between the areas of the two faces of the piston, since on one side the sectional area of the rod must be subtracted from that of the full circle of the piston face.

In the simple case in Fig. 115 no valves are shown and the supply-pipe is very short, so that if a proper resistance R' is provided in the piston-rod the piston will move very slowly and the pressure in m remain nearly equal to that in the still water at E (hydrostatic value); and similarly the pressure at point m' will be but slightly in excess of that due to its depth below the surface of T. When the piston reaches the right-hand end of its stroke (if the engine is "double-acting") the valves are automatically moved in such a way as to admit the "pressure-water" from E to the right-hand face of the piston, while shutting off that end of the cylinder from T; and simultaneously the port leading to the left-hand side of the piston is thrown into communication with T and that with E is shut off. It is also arranged that the resistance R' shall reverse its direction during this return stroke.

Usually the cylinder is not very near to the reservoir W, and a supply-pipe is necessary to conduct the water to the motor. During the motion of the piston the water in this pipe has a certain amount of headway, i.e. velocity, with corresponding energy of motion, so that to prevent "water-hammer"

when the piston stops at the end of the stroke an air-vessel is provided at the down-stream end of the pipe and communicating with it. When the piston stops, some of the water flows into the air-vessel, compressing the air somewhat more than before, and the velocity of the water in the pipe is thus gradually destroyed, or perhaps only partially checked, before the reverse motion of the piston permits new acquirement of velocity. Frequently water-pressure engines are built in pairs, one engine working the valves of the other, in which case, if large air-vessels are provided, the motion of the water in the supply-pipe is almost a "steady flow," instead of intermittent in velocity. Since water is not highly compressible like steam, much larger ports must be provided than for steam-engines, to avoid losses of head.

If the piston-rod is connected to the crank of a shaft and fly-wheel, the motion of the piston is nearly *harmonic*, the maximum velocity occurring at mid-stroke. With a long supply-pipe without air-vessel the velocity of the water in the pipe would be of corresponding character, and the pressure felt by the piston, if there were no fluid friction in the pipe, would be least near the beginning of the stroke, while the water is gaining velocity, attain its average value at mid-stroke, and reach its maximum toward the end, when the previously acquired velocity of the water is checked and finally reduced to zero; so that the final pressure against the piston is greater than the hydrostatic; but on account of fluid friction, which increases nearly as the square of the velocity, and also on account of the use of an air-vessel, the fluctuation of pressure in actual practice is less; the least pressure being at about mid-stroke; the final pressure is, however, still the greatest.

148. Direct-acting Pressure-motor and Pump.—Referring to Fig. 115 (in which A and B are open piezometers), let us suppose that through the use of air-vessels and a long stroke for the piston, with small velocity v', a practically steady flow is realized in the supply- and discharge-pipes, so that Bernoulli's Theorem may be applied; with a constant working-force on the piston. This working-force, if $p_m = (b+y)\gamma$ and $p_m' = (b+y')\gamma$

(b is the height of the water-barometer) are the unit-pressures on the two faces of the piston, will be $F(p_m - p_{m'})$ lbs., and for the equilibrium of the piston in its uniform motion we have

$$F(p_m - p_{m'}) = R' + R_0; \text{ (lbs.)}, \quad . \quad . \quad . \quad . \quad (1)$$

in which F is the area of piston (considered same on both sides), R' the thrust (or pull) in the piston-rod, and R_0 the total friction of edge of piston and sides of rod on the walls and stuffing-box of the motor cylinder.

Let there be a *long* supply-pipe (E to r) of length l and diameter d, in which the loss of head is h_F; and a discharge-pipe leading from m' to reservoir T, for which we have similarly l', d', and h_F'; and let h_E denote the entrance-loss of head, at E, of supply-pipe, and h_r the loss of head due to passage through port at r; and h_r' that due to port leading to discharge-pipe. Also let $\overline{Hm}$ and $\overline{Tm'}$ denote the *vertical* heights between points involved. Then Bernoulli's Theorem applied between points H and m leads to the relation

$$\frac{p_m}{\gamma} = b + \overline{Hm} - h_E - h_F - h_r; \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and similarly, between m' and T,

$$\frac{pm'}{\gamma} = b + \overline{Tm'} + h_r' + h_F'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(The velocity-heads at m and m' are ignored, as very small.) Now the work done per sec. (power) by the force $F(p_m - p_{m'})$ on the piston-rod is [see (1)] $F(p_m - p_{m'})v'$; i.e.,

$$R'v' + R_0v', = Fv'(p_m - p_{m'}), = Q(p_m - p_{m'}); \quad . \quad . \quad (4)$$

in which Q is the rate of discharge (cub. ft. per sec.) through the motor. Substituting from (2) and (3), however, noting that $\overline{Hm} - \overline{Tm'}$ = the whole head h of the "mill-site," we have (ft.-lbs. per sec.)

$$R'v' + R_0v' = Q\gamma[h - (h_E + h_r + h_F + h_r' + h_F')]. \quad . \quad . \quad (5)$$

(*Note.*—The quantity in the bracket $= h_1$, the vertical distance between piezometer summits.)

In passing, we may note that if R' and R_0 were zero, the bracket would be zero; which gives $h = h_E + h_r + h_F + h_r' + h_F'$, or $h = \Sigma$ (friction-heads); and the speed of steady flow then attained (with *very long* pipes) would not necessarily be excessive, since losses of head vary with the *square* of the velocity, nearly. This illustrates the fact that in such a case the motor and pipes contain a hydraulic governing action within themselves, preventing large changes of speed when a change takes place in the value of the resistance R'. For instance, if through a diminution in R' (lbs.) we note that the velocity of the piston has, after adjustment to steady flow, been increased by ten per cent., the vertical drop from H to summit A would be found to have increased by some twenty per cent., thereby diminishing the pressure on the left face of the piston and providing the smaller working force called for by the diminution in R'; and there is no further increase in speed.

As to the special speed which would cause the power $L = F(p_m - p_{m'})v'$ to be a maximum; in such a case (*very long pipe*) it may be proved by the calculus that it is the speed corresponding to the relation that *one-third* of the head h shall equal the aggregate friction-head (nearly); but the consumption of water which such a result would carry with it might be far in excess of the capacity of the "mill-site."

As regards the useful purpose for which the motor is used, let us now suppose that the other end of the piston-rod is attached to, and operates, the piston of a *force-pump* (not shown in figure), this piston moving horizontally in a cylinder with ports and valves enabling each of its extremities to communicate alternately with an inlet-pipe, of length l''' and diameter d''', conducting water from a well or other source of supply, and with a delivery-pipe of length l^{iv} and diameter d^{iv}. We shall assume, also, that by the use of air-vessels, etc., a practically *steady flow* takes place through these two pipes and the pump, by whose operation, maintained by the motor, water is pumped in steady flow, at the rate of Q''' cub. ft. per sec., through a height h''' from source of supply to surface of a receiving-reservoir. Let the loss of head at entrance of inlet-

pipe, and those in the two pipes, and ports of pump, be denoted by h_E''', h_F''', h_r''', h_r^{iv}, and h_F^{iv}, respectively. At least one of the pipes is very long, so that the friction-head in it is much larger than that from all other sources. As before, velocity-heads will be neglected; and by the employment of Bernoulli's Theorem, as previously for the motor itself, we may easily derive an expression for the power exerted in operating the pump (for which R' now acts as a working force), viz.:

$$R'v' = R_0'''v' + Q'''\gamma[h''' + (h_E''' + h_r''' + h_F''' + h_r^{iv} + h_F^{iv}]; \quad (6)$$

in which R_0''' is the total friction on the piston edges and on sides of piston-rod (in pump-cylinder and its stuffing-box).

(N.B. In case the delivery-pipe of the pump terminates in a nozzle to form a fire-jet in the open air, of velocity v ft. per sec., the nozzle being at an elevation h^v ft. above the well; then, in place of the h''' of eq. (6), we should substitute $h^v + \left(1 + \frac{1}{20}\right)\frac{v^2}{2g}$; since in such a case the velocity-head in the jet would be of great importance.)

149. Numerical Example of Foregoing Pump and Motor.— It is required to pump $Q = 0.42$ cub. ft. per sec. through a height of $h''' = 100$ ft., both the delivery-pipe and inlet-pipe (or, it may be, *suction-pipe*) being 4 in. in diameter, and their lengths $l^{iv} = 1200$ ft. and $l''' = 40$ ft. (this means, from the friction-head diagrams in Appendix, friction-head at rate of 28 ft. per thousand feet length).

The available head for the pressure-engine is $h = 40$ ft., the length of its 12-inch supply-pipe 2000 ft., and that of its discharge-pipe 25 ft. (also 12-inch). Determine the necessary rate of discharge Q, of water used by the motor, for the operation of the pump. Consider all pipes to be "clean cast-iron pipe."

From the friction-head diagrams we find $h_F''' = \frac{40}{1000}$ of 28, $= 1.12$ ft. and $h_F^{iv} = \frac{1200}{1000}$ of 28, $= 33.6$ ft.

We have also $Q\gamma = 26.25$ lbs. per sec.

Assuming that the other losses of head in eq. (6) are about 4 ft., and that R_0''' is $\frac{1}{10}$ of R', we have, from (6),

$$0.9R'v' = 26.25\,[100 + 1.12 + 33.6 + 4]; \quad . \quad . \quad . \quad (7)$$

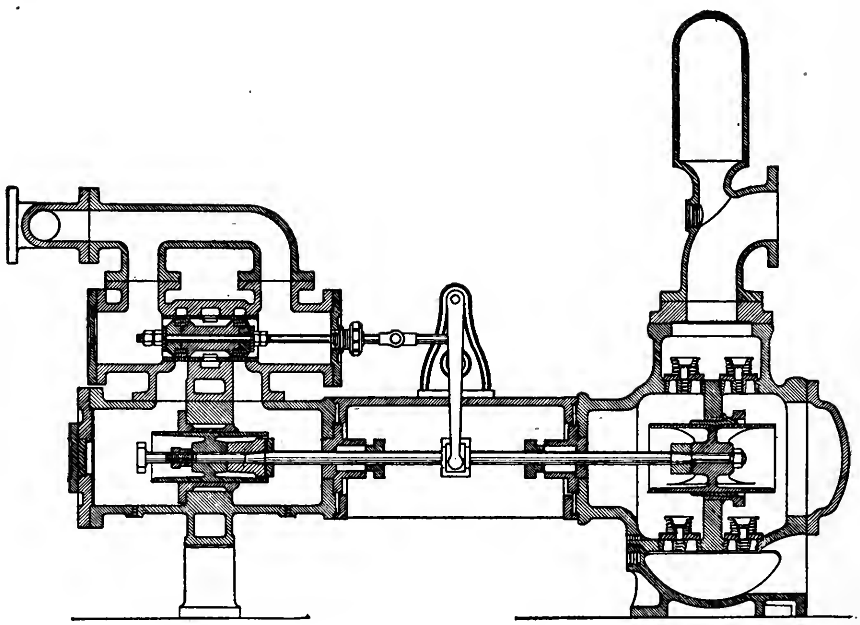


FIG. 115a. Worthington Water-motor Pump.

that is, $R'v'=(26.25\times138.7)\div0.9=4046$ ft.-lbs. per sec. This power, $R'v'$, must be furnished by the motor to operate the pump, and with $R'v'$ known we must now find Q from eq. (5). But here we are met by the difficulty that the friction-heads depend on the velocity of the steady flow in the pipes, that is upon Q itself (diameters being already fixed). It is therefore necessary to solve by trial. Since with no friction of any kind we should have $Q\gamma h=Q'''\gamma h'''$, Q will (very roughly) be three times Q; let us say six times, to allow for friction; i.e., assume for the first trial $Q=6\times0.42=2.52$ cub. ft. per sec.

From the friction-head diagram for 12-inch clean cast-iron pipe we find, for $Q=2.52$, $h_F=\frac{2000}{1000}$ of 3,5, $=7$ ft., while $h_F'=\frac{25}{1000}$ of 3.5, $=0.087$ ft. (neglect); and shall assume the remaining losses of head in eq. (5) as aggregating 2.0 ft. Putting these values into eq. (5) and taking $R_0=\frac{1}{10}$ of R', we have

$$R'v'=Q\times62.5\,[40-(7+2)], \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and hence $Q=(1.1\times4046)\div(62.5\times31)=2.3$ cub. ft. per sec.; which is smaller than the assumed 2.50. A second trial with $Q=2.10$ is practically confirmed by eq. (5), and this result will be adopted.

We next choose a small value for v', say 1 ft. per second, and assume that there is no leakage around the edge of either piston (no "slip," as it is called); whence, writing $\frac{\pi d^2}{4}=2.1\div v'$, we find the proper diameter of the motor cylinder to be $d=1.64$ ft., or close to 20 inches; and similarly with $\frac{\pi d'''^2}{4}=0.42\div v'$, that the diameter of the pump cylinder should be 0.732 ft., or 8.78 inches.

These results can by no means be looked on as exact on account of the uncertainty as to the loss of head in the ports and the frictions R_0 and R_0''' in the cylinders and stuffing-boxes; but will help to give a clear idea of the quantities concerned in the problem and of the method of solution.

150. The Worthington Water-motor Pump.—The Worthington Pumping Engine Co. of New York City, London, etc.,

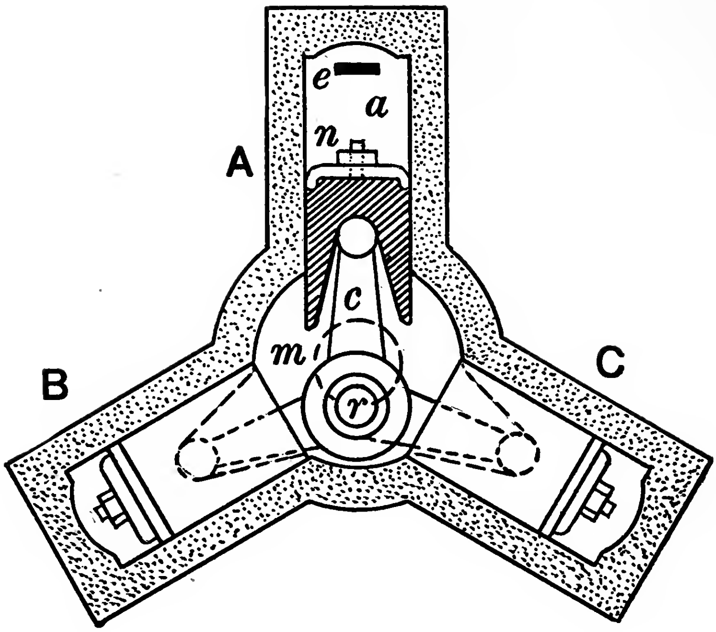


FIG. 116.

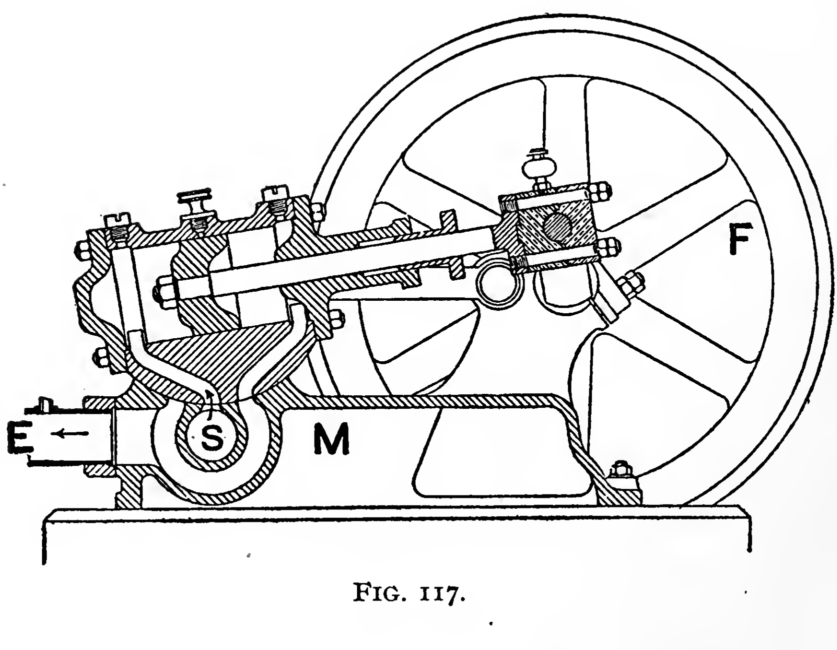


FIG. 117.

manufacture a "water-motor pump" of the type described and illustrated in §§ 148 and 149; i.e., a pair of engines, each direct-connected to a pump. To quote from the award received at the World's Columbian Exposition at Chicago in 1893:

"Two pressure-cylinders, worked with water-pressure at one end, drive two pumps at the other. The valve motion is a tappet motion, one piston-rod giving motion to the valve on the other cylinder. It may be regarded as an hydraulic relay, or pressure intensifier. It is a very useful appliance and can be employed to pump water at a long distance, instead of having an isolated steam-plant, which would require more attention, and, in many cases, an extra boiler and attendant."

In Fig. 115*a* is shown a longitudinal section of one of these Worthington water-motor pumps.

151. **The Brotherhood Pressure-engine.**—A cross-section of this motor at right angles to the shaft is shown in **Fig. 116.** It includes three working cylinders, *A*, *B*, and *C*, set at 120° apart, forming, with their pistons, three distinct motors, each of which is "single-acting," one side of each piston being always open to the atmosphere at *m*. In cylinder *A*, for instance, when the piston *n* moves *out* (down, in the figure) pressure-water is entering the space *a* through the port *e* and the piston, through its connecting-rod *c*, is exerting a thrust against the crank-pin *r*, which revolves continuously in the circle *rmc*, and causes rotation of the main shaft. On the return stroke, the port *e*, by movement of the proper valve, is opened to the atmosphere and the pressure is equalized on the two sides of the piston (except for friction of the piston in the cylinder) and the water expelled. The piston has leather packing around the edge. It is evident that with this arrangement of three pistons and cylinders the engine is always ready to start and cannot be "stalled" on a "dead-center"; since at least one of the pistons is always in a position to exert a thrust against the crank-pin. With a single supply-pipe feeding all three cylinders the flow in the pipe is fairly "steady" although the motion of each piston is variable. This engine is made in

England and runs, when necessary, at high speed; and with very good efficiency. It operates its own valves.

152. The Schmidt Oscillating Engine.—Fig. 117 (taken from Knoke's *Kraftmaschinen des Kleingewerbes*, 1887) shows a cross-section of this engine at right angles to the main shaft. There is no connecting-rod; the piston-rod being attached directly to the crank-pin. To follow the motion of the crank, therefore, the cylinder and piston oscillate on two trunnions projecting opposite the middle of the former, the piston making its strokes within the cylinder correspondingly. F is a fly-wheel on the main shaft. The under portion of the casting containing the cylinder contains ports as shown and terminates in an accurately formed cylindrical surface concentric with the trunnions on which it is mounted. As the cylinder oscillates, this surface moves in water-tight contact with a corresponding surface of the fixed base M, and in such a way that each of the two ports is caused to communicate alternately with the space S supplying the pressure-water, and the exhaust space E through which the water escapes, after use, into the atmosphere. In the position shown in the figure the piston is moving toward the right and pressure-water is acting on its left-hand face; while the water used in the previous stroke is now escaping through the right-hand port into the exhaust-passage E. On the return stroke, the crank-pin is passing below the main shaft, the right-hand port communicates with S, and that on the left with E. The engine is therefore double-acting. It has been considerably used in German.y Two engines of this kind are sometimes used, acting on the same shaft but with cranks 90° apart; so that one engine or the other is always in a position to start.

153. Pressure-engine with Variable Stroke.—When a water-pressure engine is employed to operate a hoist, or to turn a capstan, economy in the use of "pressure-water," which is usually paid for by volume, is favored by proportioning the amount of water to the power actually needed for raising the load, which may be great or small at different times, or perhaps only that of the hoisting-chain; and this is done in the Rigg

and Hastie engines by an automatic change in the length of stroke. In the Rigg engine, three (or four) cylinders radiate from a common fixed shaft turning about it and also oscillating somewhat, their pistons being single-acting and thrusting outwardly against points in the rim of an encircling ring (or "fly-wheel") secured upon the revolving shaft of the capstan or hoist. This revolving shaft is parallel to the first shaft, but eccentrically placed with regard to it. The length of stroke of each piston is equal to twice the distance between the axes of the two shafts. A governor is so connected with a hydraulic "relay" motor that, any slight change of speed due to the power exerted by the pistons being a little in excess or deficiency of what it should be for the proper constant speed in overcoming the resistance offered (whatever its amount), the distance between the two shafts (and consequently the length of stroke) is altered, until the speed returns to the proper value. (See Blaine's Hydraulic Machinery, p. 257.) By this device the amount of pressure-water used is made nearly proportional to the power actually required.

154. Piston with Large Rod. Economy of Water.—In the cylinder of a hydraulic crane using pressure-water the following device is sometimes adopted for economy in the use of the water. When the piston P in the cylinder C (see Fig. 118) makes a full stroke from right to left, the load on the crane is lifted through its full range. The diameter of the piston-rod is so great that the annular area forming the left face of the piston is only about one-half that of the right-hand face (full circle). If a heavy load is to be lifted, pressure-water is admitted on the right, at c, while the other end of the cylinder e (already filled with water) is put into communication with the "tail-water" or exhaust. The resulting working force is then a maximum and the amount of pressure-water used is $\frac{\pi}{4}d^2 \cdot l$ cub. ft.; where l is the

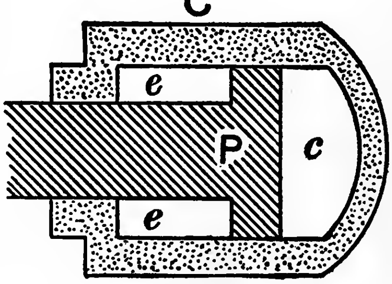


FIG. 118.

length of stroke and d the diameter of cylinder. But if a light load is to be lifted, *both* sides of the piston are thrown into communication with the supply of pressure-water; the resultant working force has now only half its former value and (since the water previously in e is forced back into the pressure-pipes) only *half* the amount of pressure-water is used in the stroke.

155. **Hydraulic Accumulators.**—Many of the smaller machines used in manufacturing plants, dock-yards, etc., such as riveters, presses, punching- and shearing-machines, cranes, etc., etc., require for their operation a store of fluid under high pressure; their action being usually intermittent. Both compressed air and water are employed as fluids for this purpose, the former being extensively used in America, while the latter is given the preference in England and the continent of Europe.

As natural reservoirs rarely provide heads of more than 300 or 400 ft. (or hydrostatic pressures of more than 130 to 170 lbs. per sq. in.), artificial means must in many instances be adopted for creating and maintaining higher pressures, up to 2000 lbs. per sq. in., in a confined body of fluid. When water is used, the storage vessel, etc., is called a *hydraulic accumulator*. Fig. 119 shows (in a diagrammatic way) the vertical section of a simple design of hydraulic accumulator. CD is a strong cylindrical vessel into whose upper end protrudes a ram, or plunger, (i.e., piston and rod in one,) AB. This plunger is loaded (with rings of cast iron, for instance, suspended by rods rr) and water is pumped into the space B through the pipe E until the ram is raised to its highest point. At e is shown an annular space or "gland" into which hemp packing

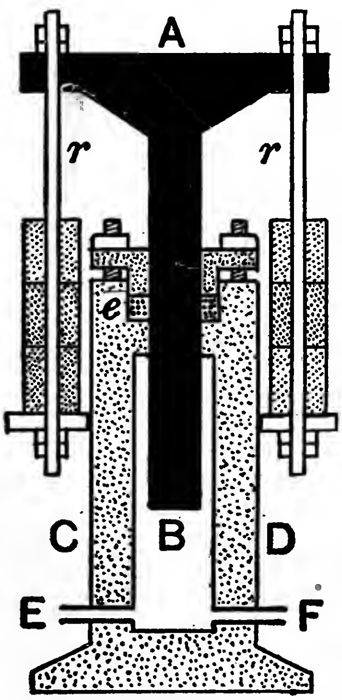


FIG. 119.

is compressed, by the screws holding down the cover of the gland, to prevent leakage of water around the ram (leather packing may also be used; see § 157).

The pressure-water, when needed for occasional operation of machines, passes out through the small pipe F and through other pipes to the particular machine needing to be driven, and the ram and its load gradually sink. When the ram reaches a definite point the pumps are started and it is again raised to its highest position, when the pumps are stopped. Both the starting and stopping of the pumps are automatic. If the demand for power is large and fairly regular, the pumps may be in action continuously; in which case the loaded ram remains nearly stationary, no longer serving as a storehouse of energy, but only as a regulator of the pressure.

If the total load on the ram is G lbs. and it is either stationary or moving with a slow uniform velocity, we have for the unit-pressure in the confined water (neglecting friction at the gland) $p=G\div\left(\frac{\pi}{4}d^2\right)$ (above the atmosphere); in which d is the diameter of the ram *at the gland, e.*

As to the friction occasioned by the hemp packing, which has to be highly compressed to prevent leakage, it is said (Blaine) that if the hemp is well lubricated this friction is about

$$F=\frac{pd}{10}; \quad \ldots \ldots \ldots \quad (8)$$

in which F will be obtained in lbs., if p is expressed in lbs. per sq. in. and d in inches. But in ordinary cases the value of the friction is quite uncertain. From eq. (8) we should have for a 5-inch ram $F=2.5$ per cent. of the load G; which is about three times that of a "U leather" packing (see § 156).

156. The Hydraulic Lift. Hydraulic Jack. Bramah Press.—Considered as a mere diagram, Fig. 119 also serves to illustrate the principle of action of the *direct-acting hydraulic lift,* or elevator; AB being a long ram carrying a car at the upper end. In the actual construction, of course, the car and ram

are made as light as possible, the permanent load being largely counterpoised by weights attached to chains or cables running over pulleys, so that the fluid pressure needed is mainly that required for the temporary load (passengers, freight, etc.).

The *hydraulic jack* is the same in principle, and also the Bramah hydraulic press, the load to be lifted, or pressure to be applied (to a bale of cotton, for instance), playing the part of the loads on the ram in Fig. 119. The hydraulic jack usually contains its own supply of liquid (oil or water) in a special reservoir, a hand-pump, on the side of the apparatus, being used to pump it into the space under the plunger.

157. The Differential Accumulator. Leather-packing. Intensifier.—In Tweddell's differential accumulator (Fig. 120) we have an inversion of position. The ram AB (dense black shading) is *fixed* and placed below, and the cylindrical vessel, CD, which is loaded and movable, is placed above. Also, the ram protrudes through the loaded vessel at A, requiring two packings for the two sliding joints. Here the diameter d' of the upper portion of the ram is a little smaller than that, d, of the lower portion. Consequently the whole weight, G lbs., of the cylinder CD and the loads upon it, is borne by the upward fluid pressure on the area of the ring (difference of the areas of the two circles) $\frac{\pi}{4}d^2-\frac{\pi}{4}d'^2$; i.e., the unit pressure (above the atmosphere) in the space e, when the whole load is sustained, is

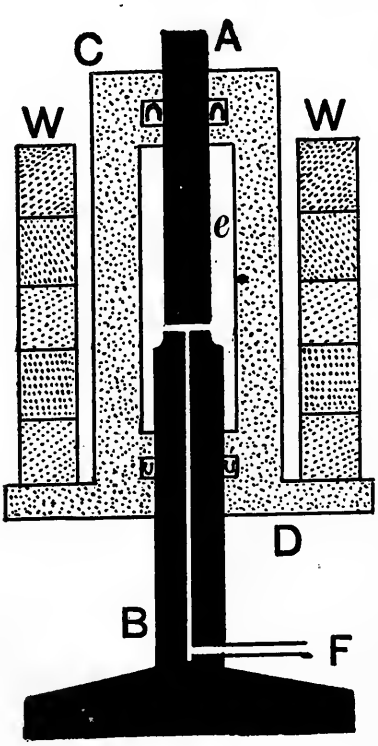


Fig. 120.

$$p=\frac{4G}{\pi(d^2-d'^2)}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

if friction is neglected. By this device, called a "*differential*

accumulator," less load is needed to produce a given fluid pressure, but the amount of pressure-water leaving the vessel for each foot descent of the loaded cylinder is quite small; thus necessitating very frequent working of the pumps, or perhaps their continuous action, to supply the requisite amount.

Water is pumped into the interior space e through the small pipe F (which also serves as an outlet if the flow is the other way) and through passageways in the ram itself, as shown. The two recesses or glands are furnished with water-tight "U-leather" packings like the one shown at A in Fig. 121.

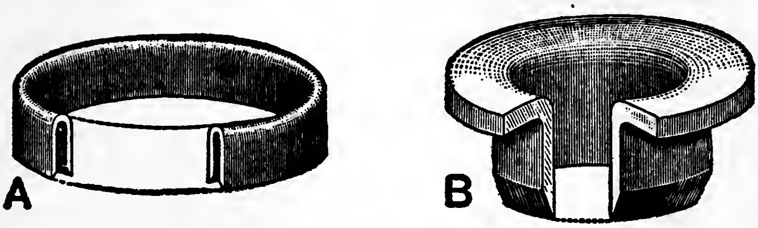


FIG. 121.

This is made of a single piece of leather, pressed into shape (after softening by hot water) in a proper mould. The concave side is turned toward the interior of the cylinder and thus exposes that side to the high internal pressure. This pressure keeps the leather pressed tightly both against the surface of the ram and that of the gland cavity, thus providing a water-tight joint. For the best results the outside of the ram should be of gun-metal or copper, as also the lining of the gland cavity. For small rams or for piston-rods the "hat-leather" packing (see B in Fig. 121) serves a similar purpose. (This cut is from the advertisement of the Detroit Leather Specialty Co. of Detroit, Mich.)

Leather packings are more expensive than those of hemp, but offer much less friction. According to the experiments of Mr. Hick, the friction offered by a well-lubricated U-leather packing is

$$F = 0.032pd; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where p is the internal fluid pressure in *lbs. per sq. in.*, and d the diameter of the ram in *inches;* F being then obtained in

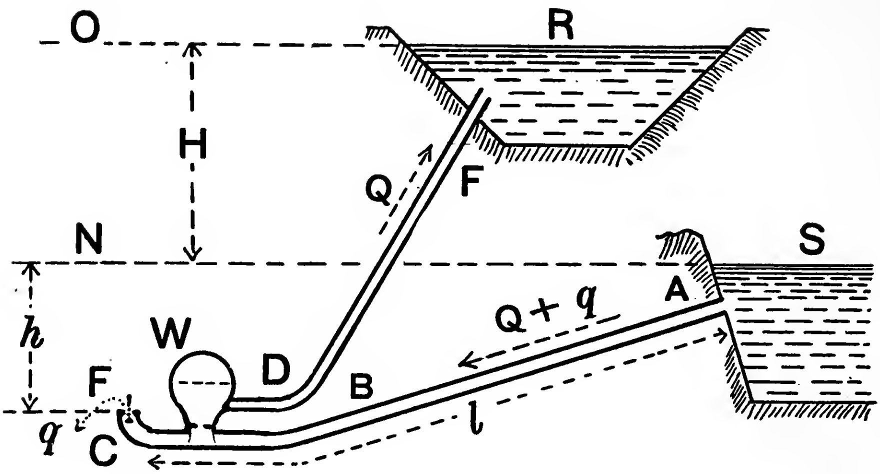


FIG. 122.

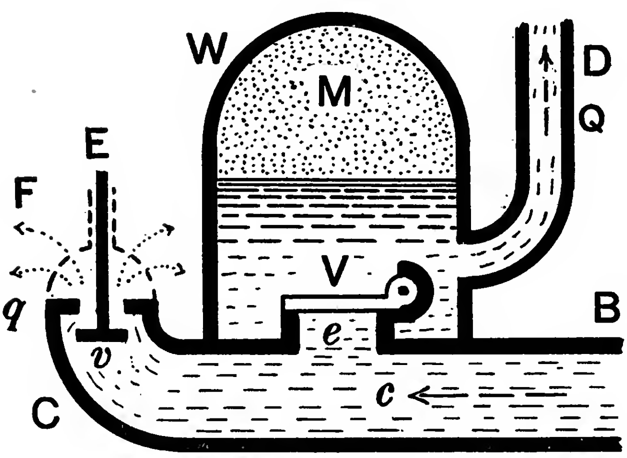


FIG. 123.

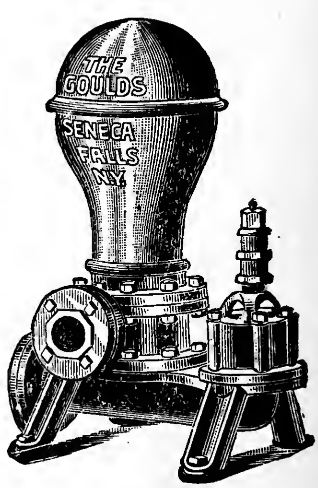


FIG. 124.

lbs. For a new leather, or one badly lubricated, the numeral should be 0.047.

If the load on the ram in Fig. 119 is replaced by the downward hydrostatic pressure on a piston of much larger diameter than that of the ram, moving capable of vertical motion in a fixed cylinder to which water from a comparatively low source is admitted, this piston being attached to the upper end of the ram, the device is called an *intensifier.* In such a case, no water need be withdrawn from, nor pumped into, the upper cylinder.

158. The Hydraulic Ram is a combined water-motor and pump working in successive pulsations and by a kind of mild "water-hammer" action. This action depends on the intermittent starting and stopping of the cylinder of water in the supply-pipe (or "drive-pipe," as it is called in this connection). In the simpler forms the machine consists of an air-vessel, W (see Fig. 122); of two valves, the "waste-valve" and the "check-valve"; and of two pipes, viz., BA, the drive-pipe, supplying water from the supply-pond S, and the delivery-pipe DF, through which a certain amount of the water is pumped into the receiving-tank or reservoir, R, at a higher elevation than the supply-pond. H is the net head through which water is raised (the "lift"), and h the working-head (or "fall") of the "waste-water" (or "motive water"), or that which escapes through the waste-valve at F. If Q (cub. ft. per hour, say) is the rate at which water is pumped through the pipe F, and q the volume per hour of flow through the waste-valve, then the rate of flow through the supply drive-pipe BA is $Q+q$; see figure.

Fig. 123 gives a (diagrammatic) vertical section of the ram proper. At the beginning of a cycle, or pulsation, the waste-valve E at the lower extremity of the drive-pipe is open and the check-valve V at the base of the air-vessel, W, is shut, being held shut for the time being by the high pressure in the air-vessel, which communicates by pipe DF with upper tank R. M is a confined body of compressed air. Under the action of gravity the water in the drive-pipe begins to move

and flow out into the air at F, acquiring greater and greater velocity (unsteady flow). This velocity is finally so great that the pressure of the current on the under side of valve E becomes sufficient to close it abruptly. The moving body of water in pipe CBA is now slightly checked in its velocity and becomes compressed until the pressure rises (very quickly) to a value a little in excess of that on the upper side of valve V, when this latter valve opens and a portion of the "drive-water" is forced into the base of the air-vessel, until its kinetic energy and velocity are entirely exhausted; the immediate effect being a rising of the water-surface in the air-vessel and a further compression of the confined air in M. The water in the drive-pipe having thus been brought to rest but being still slightly compressed, an elastic recoil or rebound takes place and the pressure in the spaces e and v quickly falls to a low value, less than one atmosphere, so that the valve V closes; while E is opened, both on account of its weight and of *the pressure of the outer air*. In other words, the kinetic energy possessed by the drive-water when the valve E first closes is expended in compressing itself (slightly) and then (mainly) in compressing the air in the air-vessel into a smaller compass. Another cycle now begins; and so on, indefinitely. While the water in the drive-pipe is gradually acquiring velocity in the next cycle (and this occupies much the greater part of the time of a cycle) the compressed air in M expands and recovers its former volume, forcing some of the water in the base of the air-vessel through the pipe DF into the tank R; in fact, the flow in this pipe is fairly continuous and "steady," if a large air-vessel is provided.

A small valve not yet described, and not shown in the figure, is the "*snifting-valve,*" in the side wall of the space e, opening inward and closing the entrance of a small pipe leading to the outer air. At the time of the recoil a small quantity of air enters and a little later is carried into the air-vessel; to make good the air lost by being dissolved in the (high-pressure) water in the air-vessel.

In Fig. 124 is shown a "No. 6" ram made by the Goulds

Manufacturing Co. of Seneca Falls, N. Y. The waste-valve is seen on the right, while the opening for attachment of delivery-pipe appears directly in the front, a little to the left of the base of air-vessel. The long horizonta chamber below forms a continuation of the drive-pipe which is attached to it at the extreme left-hand lower corner of the figure. The drive-pipe intended for use with this size of ram is $2\frac{1}{2}$ in. in diameter and of a length, l, equal to that of lift and fall combined; that is, $l=H+h$, while the delivery-pipe has a diameter of $1\frac{1}{4}$ inches.

159. Hydraulic Ram. Efficiency. Experiments.—If we consider the useful power obtained to be the raising of $Q\gamma$ lbs. of water each hour from the level of S to that of R (see Fig. 122); that is, through a height H; and that the whole power expended is that due to a weight $Q\gamma$ of water (from waste-valve) acting each hour through a height h; we obtain the Rankine form for the efficiency, viz.,

$$\eta=\frac{Q\gamma H}{q\gamma h}, =\frac{QH}{qh}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The machine, however, is a complex one; and if we take d'Aubuisson's view that the energy received per unit of time by the ram is $(Q+q)\gamma h$, and that the useful effect obtained therefrom is the raising of $Q\gamma$ lbs. of water per unit of time through a height $H+h$, the form for the efficiency becomes

$$\eta=\frac{Q\gamma(H+h)}{(Q+q)\gamma h}, =\frac{Q(H+h)}{(Q+q)h}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The Rankine form is the one more generally employed and will be adopted here. Under ordinary circumstances (H large compared with h) results based on (2) are not largely in excess of those obtained from (1).

From extensive experiments made by himself in 1804 Eytelwein recommends the following relations to be adopted for the best results:

If Q and q are expressed in *cub. ft. per sec.*, the diameter of the drive-pipe should have a value

$$d=[\sqrt{1.63(Q+q)}] \textit{ feet}. \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The length l of the drive-pipe should be

$$l = H + h + (2\text{ ft.}) \times (H \div h). \quad . \quad . \quad . \quad . \quad (4)$$

The volume of the air-vessel should be equal to that of the delivery-pipe, and the diameter of the latter should be about one half of that of the drive-pipe. The opening of the waste-valve should have the same sectional area as the drive-pipe, and the weight of this valve should be as small as possible. The drive-pipe should be as straight and free from friction as practicable.

For the best results the length of stroke made by the waste-valve in closing should not be too great.

With these proportions adopted, Eytelwein found that the efficiency (Rankine form) diminished with an increase of the ratio of the lift H to fall h; nearly according to the relation (η denoting the efficiency)

$$\eta = 1.12 - 0.2\sqrt{(H \div h)} \quad . \quad . \quad . \quad . \quad (4a)$$

for a range of values of the ratio $H \div h$ from 1 to 20.

This gives the following table:

For $H \div h =$	1	2	4	6	10	15	20
$\eta =$	0.92	0.84	0.72	0.63	0.49	0.34	0.23

A few experiments were made by the present writer on a small ram (No. 2 Goulds) at Cornell University * in March 1899. The data and results are tabulated below. The drive-pipe had a diameter of $\frac{3}{4}$ in. and was 51 ft. long. The delivery-pipe was one inch in diameter and comparatively short, offering but little loss of head. Column A gives pulsations per minute, and B the height of stroke of the waste-valve in closing. $Q\gamma$ and $q\gamma$ are lbs. per minute; H and h are feet.

These experiments were repeated to insure accuracy, each of the three horizontal lines giving the mean results of several in close agreement with each other. In No. 1 the full weight of the waste-valve, which was $6\frac{5}{8}$ ounces, was opera-

* See also Prof. R. C. Carpenter's experiments, mentioned in Kent's *Mechanical Engineer's Pocket-book.*

Experiments on a Hydraulic Ram, Cornell University, March 1899.

No.	Whole Time; Min.	A	B In.	$Q\gamma$	H	$q\gamma$	h	η Effic.
1	12	80	$\frac{7}{8}$	5.50	49.4	26.7	17	0.598
2(s)	15	66	$\frac{7}{8}$	2.30	56.4	21.8	10	0.595
3(s)	16.7	98	$\frac{5}{8}$	4.82	49.4	20.6	17	0.680

tive; there being no provision to counterpoise a portion of it; and the length of its movement, or "stroke," was $\frac{7}{8}$ in. In the other two, Nos. 2(s) and 3(s), a light spring was employed by the use of which the waste-valve was virtually relieved of about one-half of its weight (though, of course, its "mass" was practically unchanged). In No. 2(s), the length of stroke of valve being the same as before, the efficiency is maintained at nearly 60 per cent., notwithstanding the fact that the ratio $H \div h$ is nearly double what it was in No. 1. This is doubtless due to the (relatively) quick closing of the valve. In No. 3(s) the stroke has been made shorter with the effect of increasing the efficiency to 68 per cent.; to which the decrease in the ratio $H \div h$ has also probably contributed somewhat. The pulsation is here very rapid: 98 to the minute.

With a somewhat longer drive-pipe (say 65 ft.) the efficiency would probably have been higher.

160. Hydraulic Ram. Special Designs.—In some designs the check-valve shown as V in Fig. 123 is placed on the side of the chamber e, the top space in which is then used as a dome, or pocket, in which to entrap permanently a small body of air, which serves to lighten the shock and water-hammer effect, preventing the pressure in e from rising much above that in the air-vessel at any time.

The Rife "Hydraulic Engine" (see Engineering News of Dec. 31, 1896) is a large hydraulic ram in which the effective weight of a waste-valve can be varied by means of a weight sliding on a lever. It can also be arranged so that the water

pumped may be taken from a different source (purer water, for instance) from that of the drive-water; the periodic recoil of the drive-water serving to cause the entrance ("by suction") into the space e, of each new instalment of the water to be pumped. A description of some interesting tests of a Rife hydraulic ram may be found in the Stevens Indicator of April 1898 (published at the Stevens Institute, Hoboken, N. J.). The highest efficiency obtained was 75.6 per cent.*

A large ram designed by Prof. D. W. Mead of Chicago is in operation at the village of West Dundee, Illinois, in connection with the local water-works. This machine, with a drive-pipe 2200 ft. long and 10 inches in diameter, under a head of 55 ft., delivers water into a stand-pipe 115 ft. above the ram. On account of the great length of this drive-pipe, the waste-valve, which is circular and 8 inches in diameter, was given a lift, or "stroke," of only $\frac{1}{4}$ inch, thus giving an area of discharge equal to only about one-twelfth of the sectional area of the drive-pipe; and the aggregate area of the (nine) check-valves at the base of the air-vessel is greater than the sectional area of the 10-inch drive-pipe. The duration of one cycle or pulsation of this ram is $4\frac{1}{3}$ seconds, and the pressure in the space e (Fig. 123) never exceeds by more than $2\frac{1}{2}$ lbs. the pressure in the air-vessel, which is 50 lbs. per sq. in. (above atmos.). According to the indicator-cards taken, this highest pressure endures only about one second.

An account of the Pearsall Hydraulic Engine, and of a test of the same, may be found in the Engineering News for Sept. 28, 1889. This is a large hydraulic ram in which the waste-valve is cylindrical in form and is closed, not violently, by the current of flowing water, but *quietly*, by the action of a small motor worked by compressed air taken from the air-vessel. The essential principle involved (referring now to Fig. 123) is practically stated by saying that the chamber e is so furnished with valves (besides V) admitting air from the outside at the proper

* In Engineering News of Aug. 3, 1905, p. 127, is an account of two "Foster Impact Engines," or large hydraulic rams, installed at Bradford, R. I., and working with high efficiency.

times that sufficient air is entrapped and cushioned under the valve V and finally forced into the air-vessel along with the water pumped, not only to make good the ordinary losses of air in the air-vessel, but also to furnish what is needed for the operation of the small motor which opens and closes the waste-valve. In this way the efficiency is increased; 71 per cent. having been attained in the case referred to. From the data given it appears that H was 83 ft.; and h, 17 ft.; while the diameter of drive-pipe was 12 in.

161. **The Phillips Hydraulic Ram** is made in a great variety of sizes (from 3 in. to 48 in. diameter of drive-pipe) by the Columbia Engineering Works of Portland, Oregon, U. S. A. Like the Pearsall ram it has a cylindrical waste-valve, the closing of which is brought about without violence, but the motor which operates its opening and closing is a small, single-acting, water-pressure engine, receiving its pressure-water from the supply-pond. This takes but little power; as the waste-valve from its cylindrical form is always "balanced." Sufficient air is cushioned and entrapped to supply the small losses of the air-vessel and also, by a small piston, to operate the exhaust-valve of the pressure-engine.

The diagram shown as Fig. 125 has been kindly furnished by the makers of the ram, and the following clear description of its operation is taken from their pamphlet:

"The drive-pipe is connected at A. This pipe, which is of a size suitable for handling the desired quantity of water, determines the normal size of the ram. The waste-valve C being open, the water flowing down the drive-pipe escapes. At the same time water from the main supply is flowing through the operating-pipe G and the upper part of operating-valve D into the waste-valve cylinder H. This water raises the waste-valve piston E and with it the waste-valve C, thus closing the opening and causing a stoppage in the main flow of the water. The energy stored up in the flowing water is now liberated and forces part of the latter through discharge-valves LL into the main air-chamber. This discharge is continued until an equilibrium between the respective pressures

above and below the discharge-valve is established. At the time the waste-valve closes, air is lodged in the chamber *I*. This air is compressed by the moving water column and part of it is delivered through pipe *M* into the air-chamber, thus giving the latter a constant supply at each impulse. The pressure in chamber *I* will also be imparted through pipe *K* to the lower part of the operating-valve *D*, where it raises the piston *N*, thereby closing the smaller beveled valve above and cutting off communication between the operating-pipe *G* and the under side of waste-valve piston *E*. At the same time, an exhaust opening is made at the top of operating-valve *D* for the operating water, which now escapes and allows piston *E* and with its waste-valve *C* to drop by gravity, thus making the beginning of a new cycle."

162. Hydraulic Air-compression.—Intermittent.—The action of the hydraulic ram is easily arranged to bring about the compression of a confined body of air, as also its delivery into a storage-tank, without the pumping of any water. The apparatus used at the Mt. Cenis tunnel is described in Davey's "Pumping Machinery" (London, 1900), p. 285. At the bottom of a closed vertical cylindrical vessel containing air at one atmosphere pressure, water from a pipe communicating with an elevated reservoir is permitted to enter by the opening of a valve; its initial velocity being zero. The resistance offered by the ai to the advance of the water into the vessel being small at the beginning, the velocity of the water increases at first and reaches a maximum, after which its energy of motion is gradually given out in compressing the air still further, and, later, when the air-pressure is sufficient to open the valve leading to the storage-tank, to perform the work of delivery; that is, to force the air at this final constant pressure into the tank. Dimensions are so designed that the water is brought to rest just before it reaches the upper end of the compressing vessel. At that instant, by the automatic operation of the proper valves, further entrance of water is prevented and that already in the compressing vessel allowed to flow out into the atmosphere and the vessel to fill up with a new charge of air

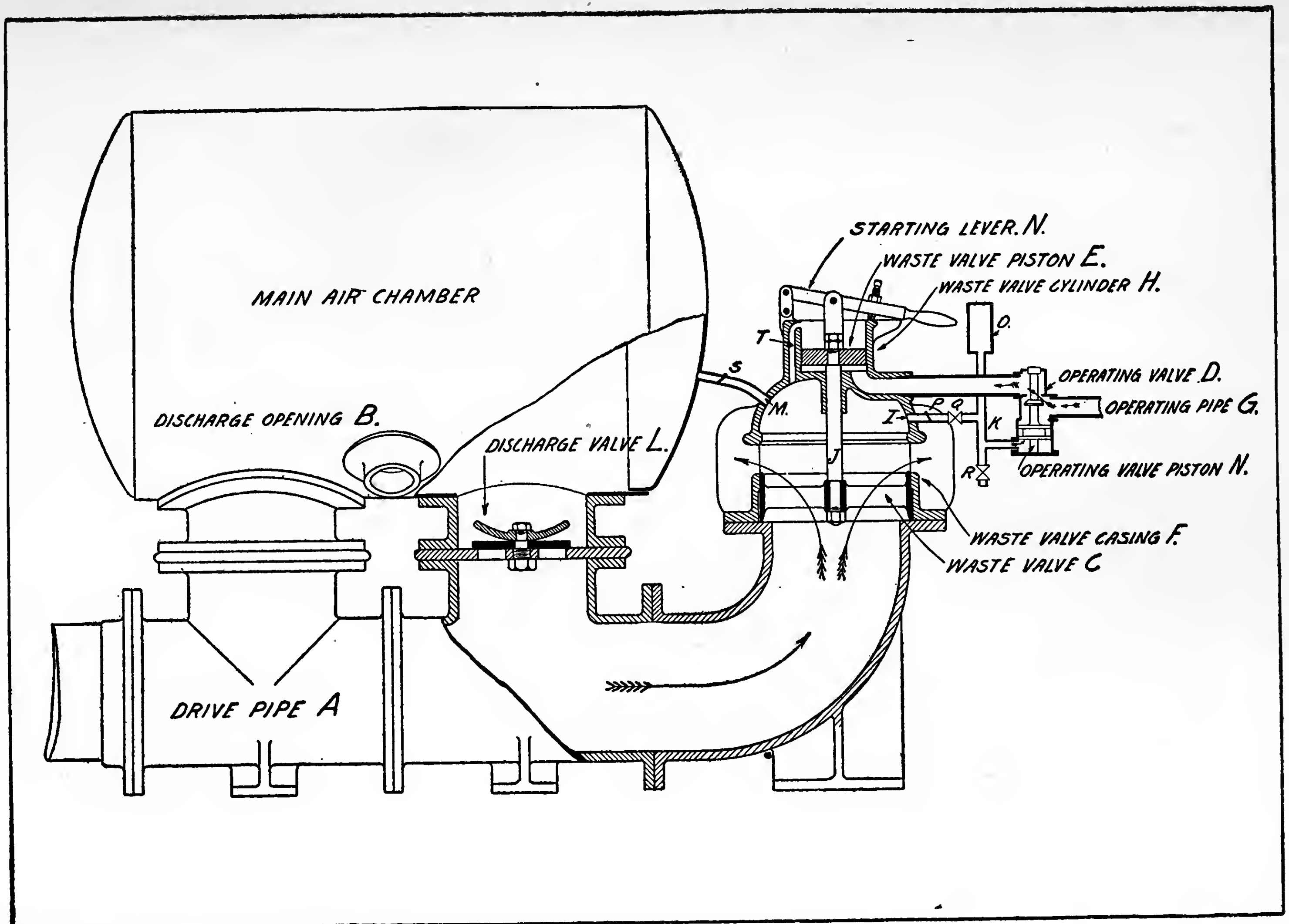


FIG. 125. Diagram of Phillips' Hydraulic Ram.

from the outside; and the cycle is repeated indefinitely. In this way air may be compressed to a pressure very much greater than the hydrostatic pressure corresponding to the head of water used. At the Mt. Cenis tunnel the head, h, of the supply-pond was 85 ft., and the final pressure of the air 75 lbs. per sq. in. (above atmos.). The theory of this operation is as follows:

Let the sectional area of the vertical compressor cylinder be F' and its length l', and let the design be such that the water which has entered it during the compressing of the air completely fills it, and has just come to rest at the end of the stroke. The weight of this water is then $F'l'\gamma$. Also, let h' denote the vertical depth of the center of gravity below the surface of the supply-pond. Let p_m indicate the (unit) pressure of the compressed air in the storage-tank, and p_a that of the outer atmosphere.

We are now to note that W', the work of overcoming the air-resistance at the front face of the advancing body of water during the stroke, will be the same (if we assume the compression to be adiabatic) as that on the front face of the piston of the air-compressor treated on p. 636, M. of E. If, in the analysis of pp. 631 to 637, M. of E., the more accurate value 1.41, of Poisson's exponent had been used, instead of 1.50 (see p. 623, M. of E.), we should have obtained for the work done in one stroke by the thrust in the piston-*rod* of the air-compressor of pp. 636 and 637 (after a little transformation and using present notation) the expression

$$W = 3.44 F'l'p_a\left[\left(\frac{p_m}{p_a}\right)^{0.290} - 1\right]; \text{ ft.-lbs.}, \quad . \quad . \quad . \quad (5)$$

in place of eq. (2) of p. 637, M. of E.

Now consider the collection of rigid bodies comprising all the particles of water in the pond and supply-pipe (long or short), and the fact that all these particles are at rest both at beginning and end of the stroke, so that both initial and final amounts of kinetic energy are zero. During the stroke the center of gravity of this whole body of water, whose weight

is G lbs., sinks through some small vertical distance Δh, so that the working force G does the work $G \cdot \Delta h$; while the surface of the pond sinks slightly, through a distance Δs, so that the atmospheric pressure on that surface does the work $F'' \cdot p_a \cdot \Delta s$ (where F'' is the area of pond surface). This last item of work corresponds (and is equal) to the work done by the atmosphere on the hinder face of the piston in the ordinary air-compressor; that is, we may write $F''p_a \cdot \Delta s = F'p_a \cdot l'$ (since the volumes $F''\Delta s$ and $F'l'$ are equal); and are also to note that $W' = W + F'p_a l'$, i.e., that $W' = W + F''p_a \cdot \Delta s$. Hence the term $F''p_a \cdot \Delta s$ will cancel out in the final summation of work (see below).

By the theorem of work and energy, then, (p. 149, M. of E.,) applied for one stroke to the collection of rigid bodies in question, remembering that, by § 32, $G \cdot \Delta h = F'l'\gamma \cdot h'$, and neglecting all friction for the time being, we have

$$(F'l'\gamma) \cdot h' = 3.44F'l'p_a\left[\left(\frac{p_m}{p_a}\right)^{0.290} - 1\right]; \quad . \quad . \quad (6)$$

that is, finally,

$$\frac{p_m}{p_a} = \left[1 + \frac{h'}{3.44b}\right]^{3.44} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Here it is to be noted that h' is not the full head, h, of the "mill-site," but smaller, since the compressor cylinder is vertical; and that b denotes the height of the water-barometer (i.e., about 34 ft. at sea-level and less at higher altitudes).

Applying (7) to the case of the Mt. Cenis apparatus assuming that h' was 85 ft. and that b was 29.5 ft., we obtain the result $p_m = 8.12p_a$; and if p was 12.8 lbs. per sq. in., p_m is 91 lbs. per sq. in. above the atmosphere.

If in this apparatus the supply-pipe is quite long and of length l, with diameter d and sectional area F, and if by allowing at first a portion of the water to escape into the outer air until that in the pipe has a velocity $= c$ when permitted to enter the compressing cylinder, eq. (6) becomes

$$(F'l'\gamma) \cdot h' + \frac{Fl\gamma}{g} \cdot \frac{c^2}{2} - \left[f(\pi d \cdot l)\gamma \frac{v_m{}^2}{2g}\right] \cdot \frac{F'l'}{F}$$
$$= 3.44F'l'p_a\left[\left(\frac{p_m}{p_a}\right)^{0.290} - 1\right], \quad (8)$$

since we must now introduce the initial kinetic energy of the water in the supply-pipe and the work spent on skin friction. (See eq. (1) on p. 695, M. of E.) γ is the weight of unit volume of water, and v_m is an average velocity of the water in pipe during the stroke (a value rather difficult to estimate).

163. Hydraulic Air-compression. Continuous.—When water is agitated in contact with the atmosphere it becomes charged with small air-bubbles. In still water these bubbles would ascend with about 1 ft. per second velocity; so that in a descending current of (say) 4 ft. per second they would be carried along by the current, and with an absolute velocity of about 3 ft. per second.

A hydraulic method for the continuous production of compressed air is founded on these facts and was invented about 1878 by Mr. J. P. Frizell. It has the simplicity of involving no moving parts whatever.

It requires, of course, a "mill-site" with some fall h and water-supply Q cub. ft. per sec. A vertical "descending shaft," or pipe, conducts water from the head-water to a horizontal shaft, which, again, leads into a vertical "ascending shaft" terminating under the surface of the tail-water. Water can enter the "descending shaft" only by flowing over the horizontal edge of a funnel; the radial converging streams meet each other at the bottom where the funnel empties into the "descending shaft" and break into foam by mutual collision and agitation at that point; then enter the top of the descending shaft. With proper regulation of dimensions, the descending current, which occupies the full section of the shaft, has sufficient velocity to entrain the air-bubbles; which, after the horizontal shaft is entered, gradually work their way to the upper part of this shaft, where, before the junction with the ascending shaft, they rise and collect in a bell or air-chamber and form a body of compressed air whose pressure is practically equal to that (hydrostatic) corresponding to the depth of the horizontal shaft below the surface of the tail-water; this supply is drawn upon through proper air-pipes. The water, having

left the air behind it, rises through the ascending shaft and joins the tail-water (see Frizell's "Water-power," p. 426).

The *Taylor* method of hydraulic compression employs practically the three shafts mentioned above or their equivalent, but the mode of charging the water with the air-bubbles is different. The water, in steady flow, enters the upper end of the descending shaft or pipe through a *constricted* sectional area; the internal fluid pressure at that part of the flow being thus given a value less than one atmosphere, by proper design of the parts. Small openings in the walls of this constricted part of the passageway communicate by pipes with the outer atmosphere, and through them air is forced in by the outside air-pressure and joins the current of water in the pipe. In short, the principle of Sprengel's air-pump is employed for introducing the air (see p. 656, M. of E.).

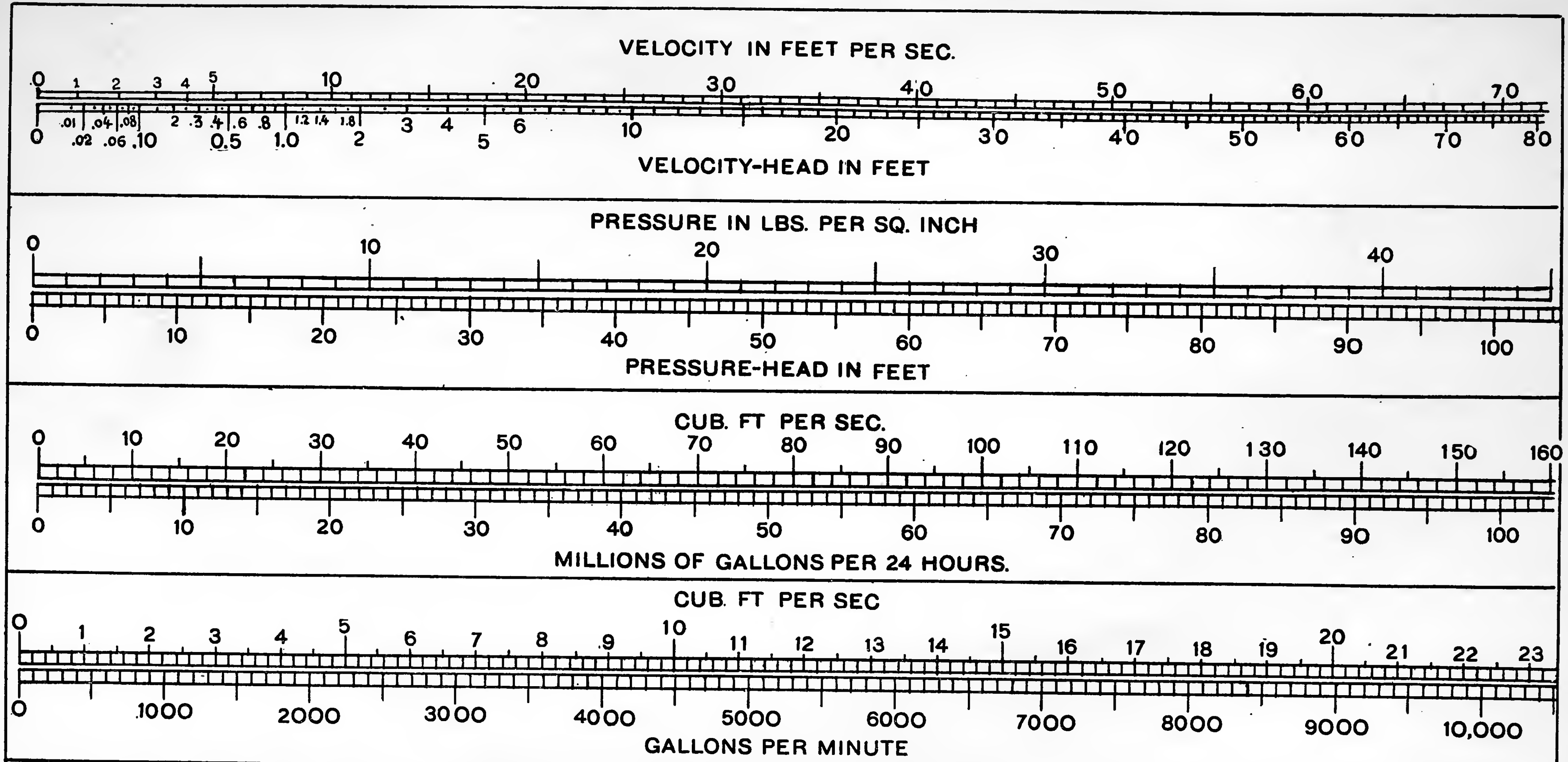
As before, the air-bubbles become disengaged from the water at the lowest point of the apparatus where the current is slow and horizontal, and is collected in a suitable chamber. Of course, the higher the pressure desired for the compressed air the greater the necessary depth of the lowest point of the shafts below the surface of the tail-water.

Two installations nvolving the Taylor method have been built: one at Magog, Province of Quebec, Canada, where a final pressure of 52 lbs. per sq. in. is obtained, the efficiency of the plant being about 62 per cent.; and the other at Taftville, Conn. (1900). The compressed air is used to operate compressed-air engines either near by or at a distance. (See London Engineering, June 1898, p. 562; and Frizell's "Water-power," p. 470. Also see Journ. Assoc. Engin. Societies, Jan. 1901, p. 35; Engineering News, May 1901, p. 406. For transmission of compressed air in pipes, see pp. 786–795, M. of E.)

APPENDIX

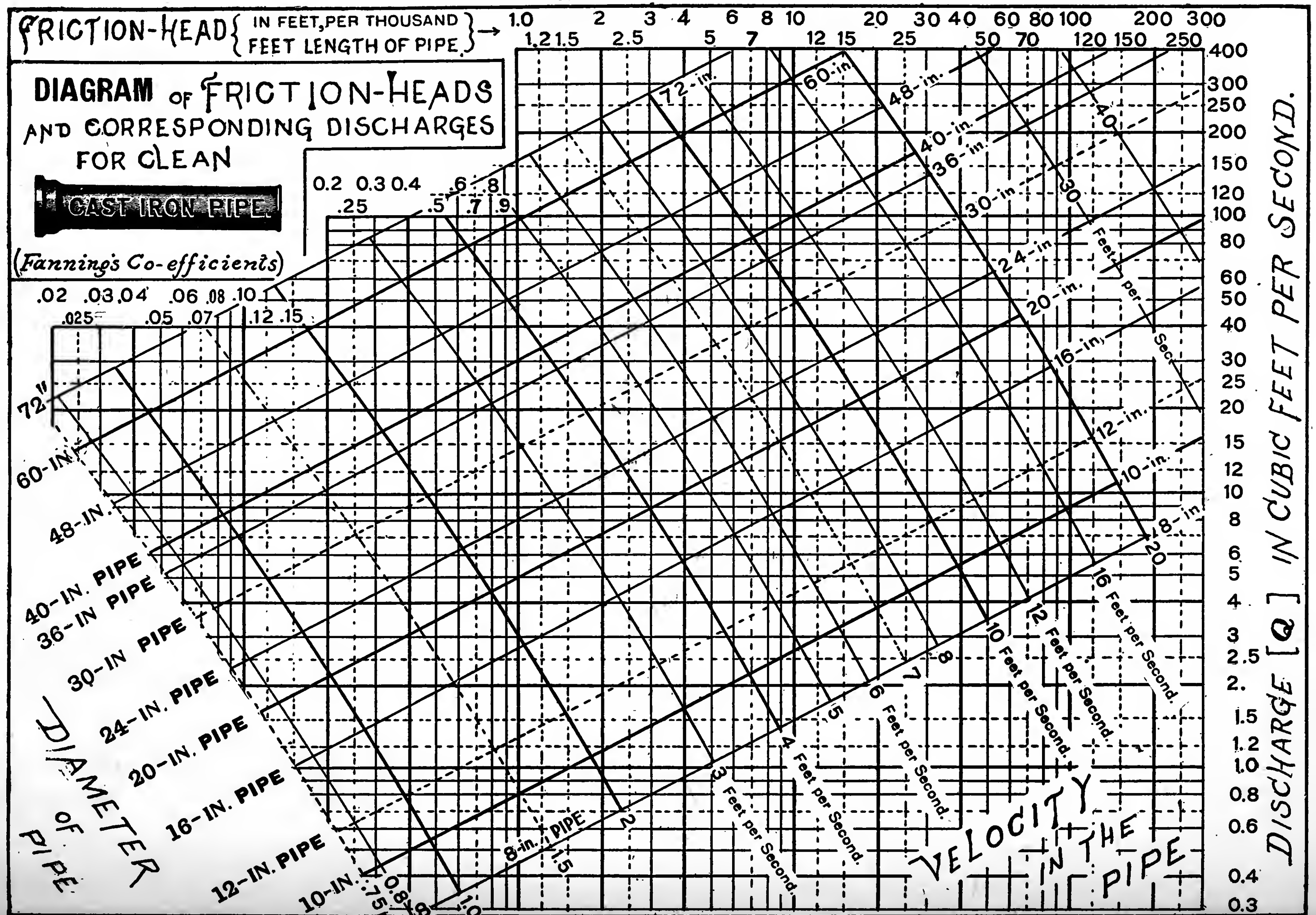
OF

DIAGRAMS AND TABLES.



CONVERSION SCALES

See p. 190.


FRICTION-HEAD { IN FEET, PER THOUSAND FEET LENGTH OF PIPE } →
DIAGRAM OF FRICTION-HEADS AND CORRESPONDING DISCHARGES FOR CLEAN
CAST IRON PIPE.
(Fanning's Co-efficients)
DISCHARGE [Q] IN CUBIC FEET PER SECOND.
VELOCITY IN THE PIPE
DIAMETER OF PIPE
72"
60-IN.
48-IN.
40-IN. PIPE
36-IN PIPE
30-IN PIPE
24-IN. PIPE
20-IN. PIPE
16-IN. PIPE
12-IN. PIPE
10-IN.
8-in. PIPE
3 Feet per Second.
4 Feet per Second.
5 Feet per Second.
6 Feet per Second.
10 Feet per Second.
12 Feet per Second.
16 Feet per Second.
30 Feet per Sec.

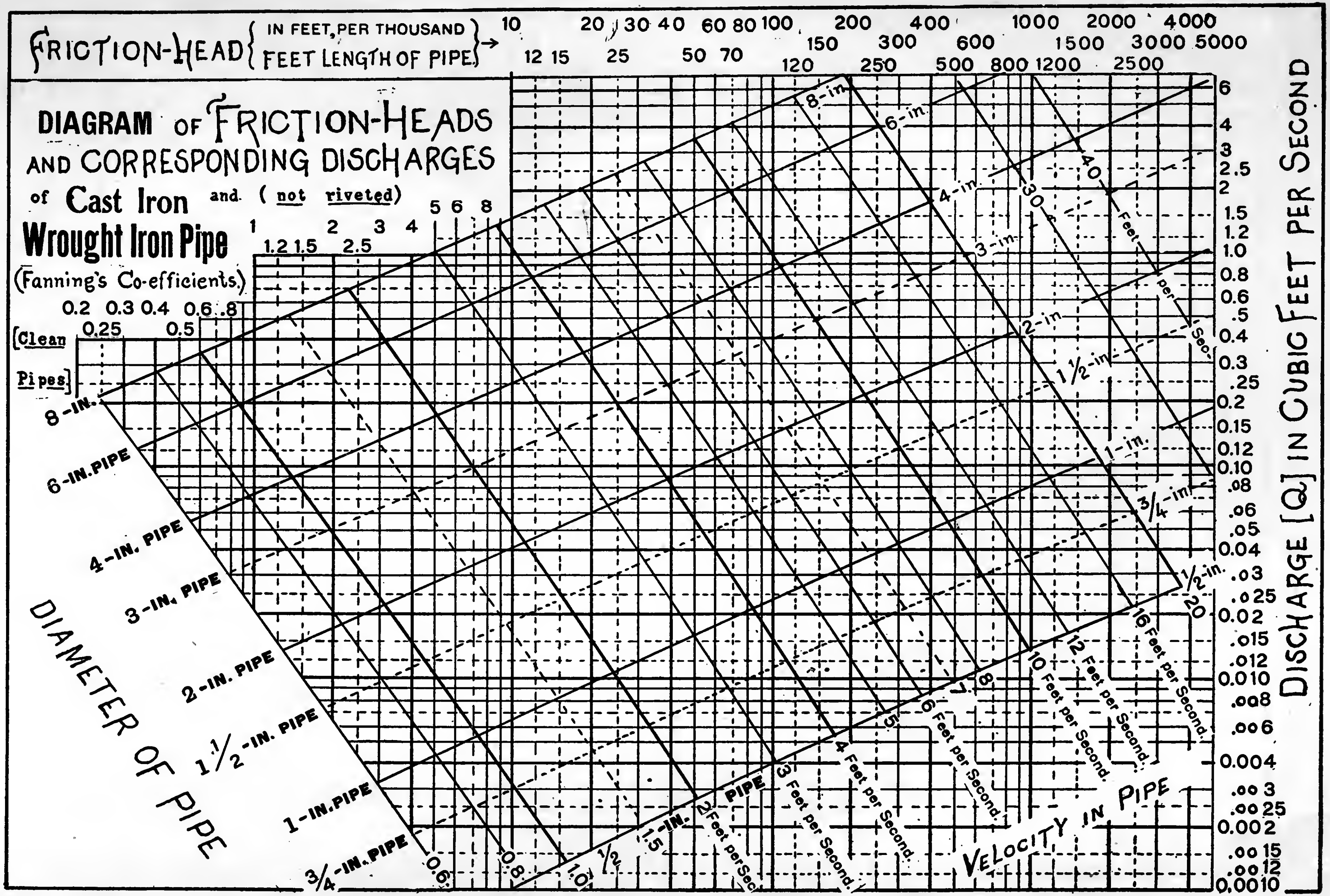
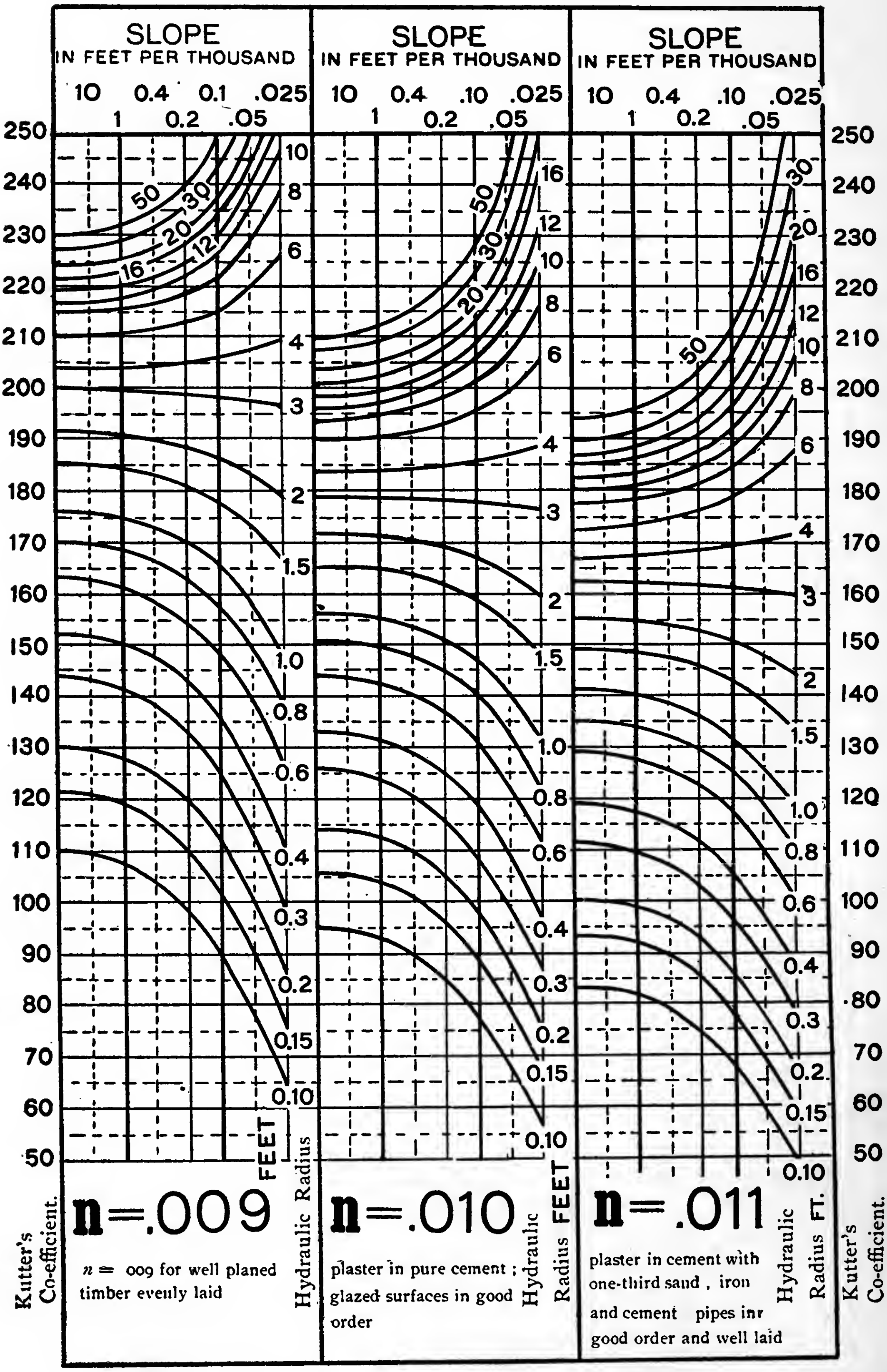
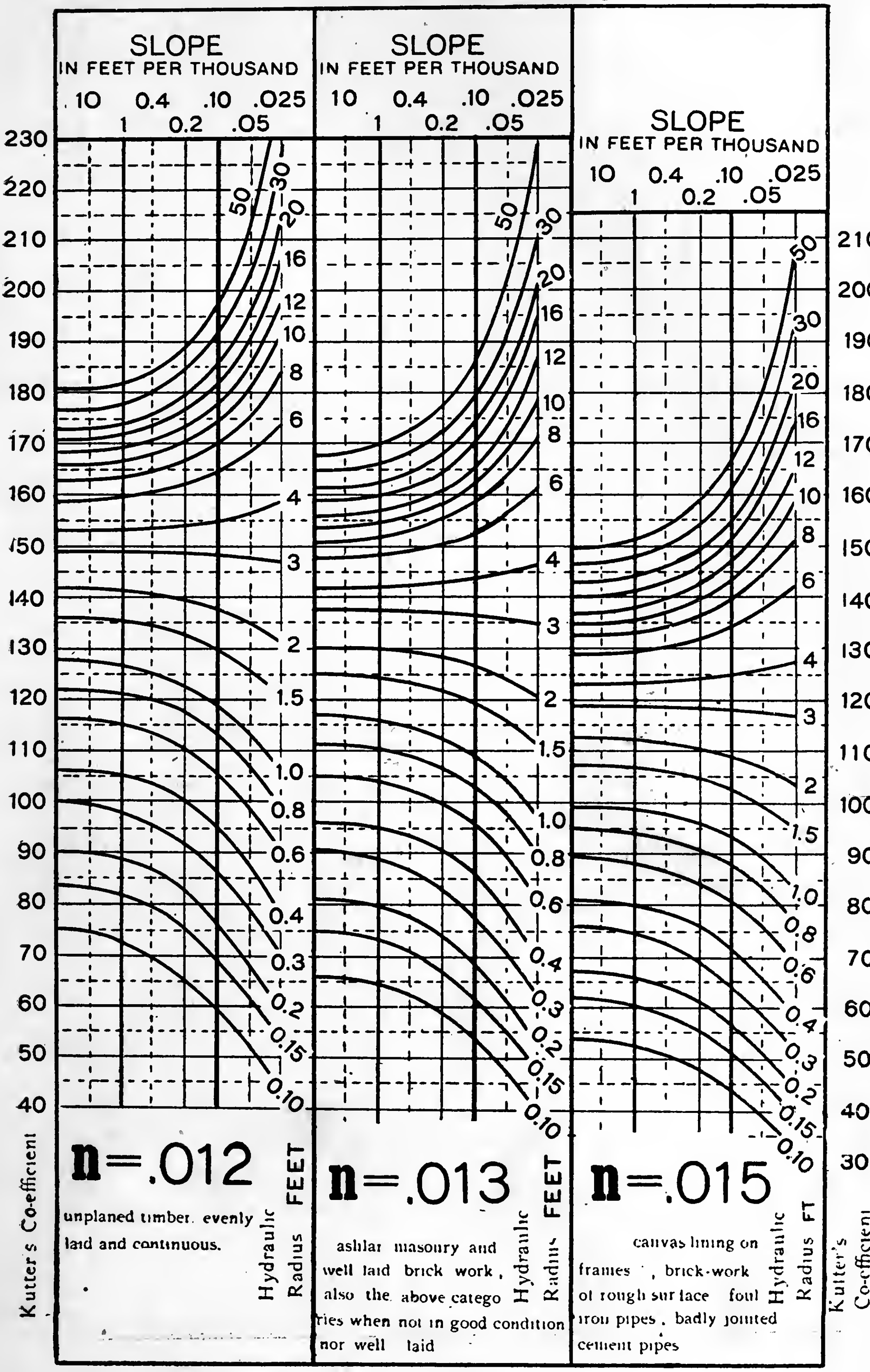


DIAGRAM OF FRICTION-HEADS AND CORRESPONDING DISCHARGES of Cast Iron and (not riveted) Wrought Iron Pipe
(Fanning's Co-efficients)
[Clean Pipes]
FRICTION-HEAD { IN FEET, PER THOUSAND FEET LENGTH OF PIPE }→
10 12 15 20 25 30 40 50 60 70 80 100 120 150 200 250 300 400 500 600 800 1000 1200 1500 2000 2500 3000 4000 5000
0.2 0.25 0.3 0.4 0.5 0.6 .8 1 1.2 1.5 2 2.5 3 4 5 6 8
DISCHARGE [Q] IN CUBIC FEET PER SECOND
6 4 3 2.5 2 1.5 1.2 1.0 0.8 0.6 .5 0.4 0.3 .25 0.2 0.15 0.12 0.10 .08 .06 .05 0.04 .03 .025 0.02 .015 .012 0.010 .008 .006 0.004 .003 .0025 0.002 .0015 .0012 0.0010
DIAMETER OF PIPE
8-IN. 6-IN. PIPE 4-IN. PIPE 3-IN. PIPE 2-IN. PIPE 1 1/2-IN. PIPE 1-IN. PIPE 3/4-IN. PIPE 1/2-IN. PIPE
8-in. 6-in. 4-in. 3-in. 2-in. 1 1/2-in. 1-in. 3/4-in. 1/2-in.
VELOCITY IN PIPE
0.6 0.8 1.0 1/2 1.5 2 Feet per Sec. 3 Feet per Second. 4 Feet per Second. 5 6 Feet per Second. 7 8 10 Feet per Second. 12 Feet per Second. 16 Feet per Second. 20 30 40 Feet per Sec.

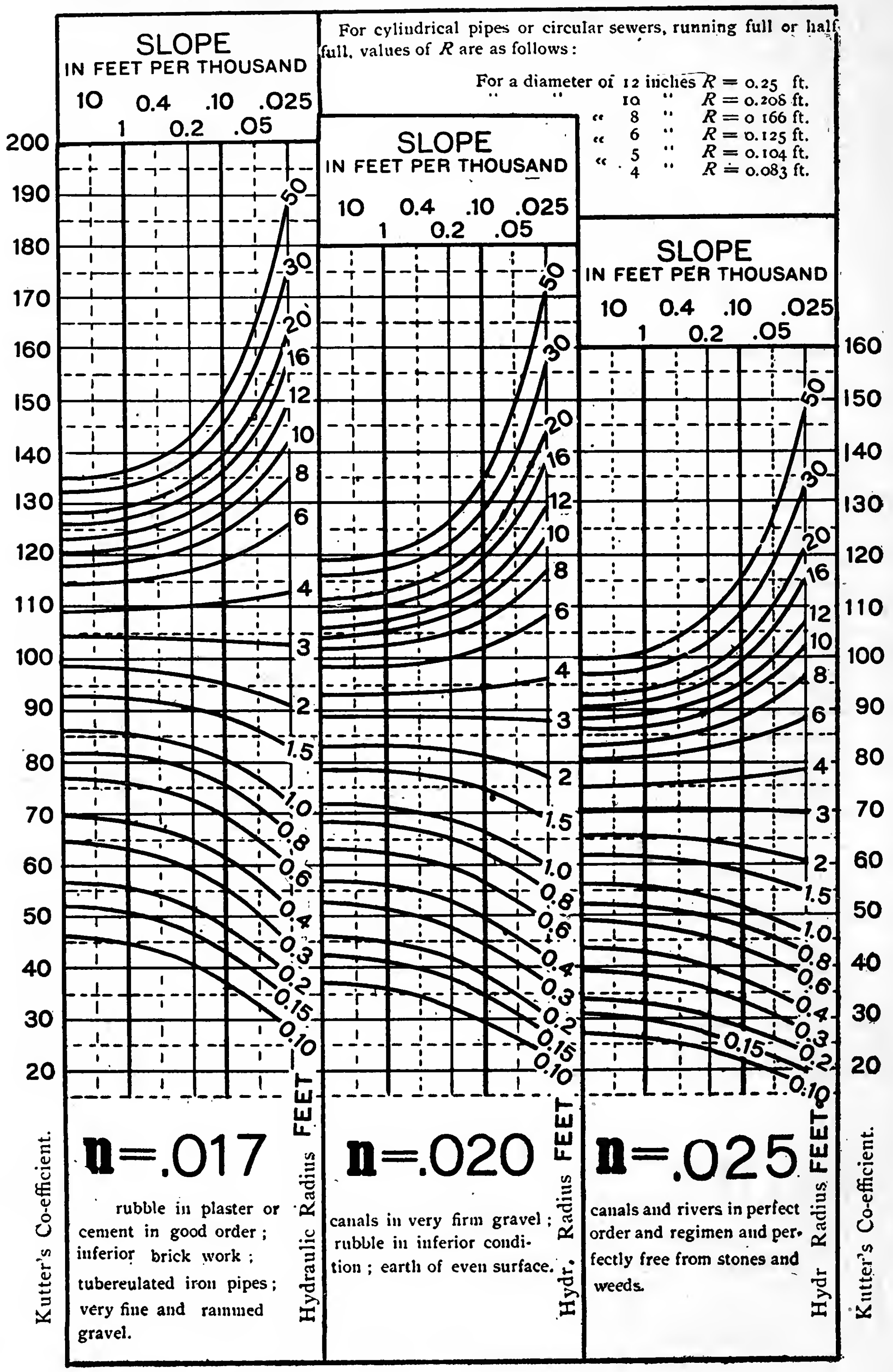
DIAGRAMS. Based on the Formula of Ganguillet and Kutter.



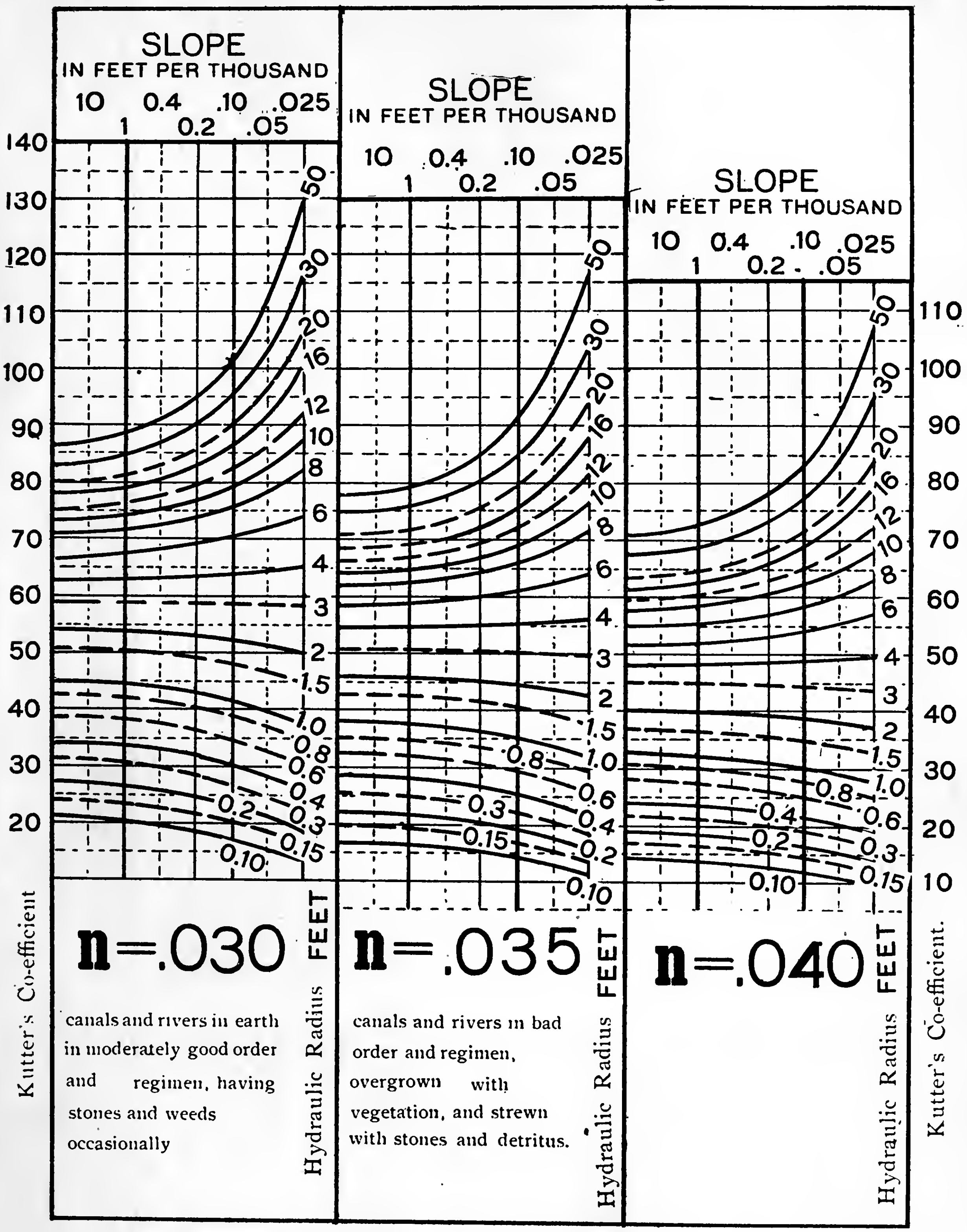
DIAGRAMS. Based on the Formula of Ganguillet and Kutter.



DIAGRAMS. Based on the Formula of Ganguillet and Kutter.



DIAGRAMS. Based on the Formula of Ganguillet and Kutter.



LOGARITHMS (BRIGGS').

N	0	1	2	3	4	5	6	7	8	9	Dif.
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	42
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	38
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	35
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	32
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	30
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	28
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	26
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	25
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	24
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	22
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	21
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	20
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	19
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	19
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	18
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	16
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	16
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	15
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	15
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	14
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	14
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	13
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	13
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	13
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	12
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	12
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	12
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	11
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	11
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	11
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	10
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	10
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	10
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	10
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	10
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	9
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	9
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	9
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	9
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	8
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	8
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	8

N. B.—Naperian log = Briggs' log × 2.302.
Base of Naperian system = e = 2.71828.

LOGARITHMS (BRIGGS').

N	0	1	2	3	4	5	6	7	8	9	Dif.
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	8
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	8
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	8
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	7
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	7
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	7
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	7
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	7
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	7
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	6
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	6
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	6
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	6
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	6
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	6
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	6
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	6
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	5
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	5
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	5
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	5
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	5
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	5
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	5
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	5
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	5
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	4

N. B.—Naperian log = Briggs' log × 2.302.
Base of Naperian System = *e* = 2.71828.

TRIGONOMETRIC RATIOS (Natural); including "*arc*," by which is meant the "π-measure" or "circular measure" of the angle; e.g., arc 100° = 1.7453293, = $\frac{100}{180}$ of π.

arc	degr.	sin	csc	tan	cot	sec	cos		
0.000	0	0.000	inf.	0.000	inf.	1.000	1.000	90	1.571
0.017	1	0.017	57.3	0.017	57.3	1.000	1.000	89	1.553
0.035	2	0.035	28.7	0.035	28.6	1.001	0.999	88	1.536
0.052	3	0.052	19.1	0.052	19.1	1.001	0.999	87	1.518
0.070	4	0.070	14.3	0.070	14.3	1.002	0.998	86	1.501
0.087	5	0.087	11.5	0.087	11.4	1.004	0.996	85	1.484
0.105	6	0.105	9.6	0.105	9.5	1.006	0.995	84	1.466
0.122	7	0.122	8.2	0.123	8.1	1.008	0.993	83	1.449
0.139	8	0.139	7.2	0.141	7.1	1.010	0.990	82	1.432
0.157	9	0.156	6.4	0.158	6.3	1.012	0.988	81	1.414
0.174	10	0.174	5.8	0.176	5.7	1.015	0.985	80	1.396
0.192	11	0.191	5.24	0.194	5.14	1.019	0.982	79	1.379
0.209	12	0.208	4.81	0.213	4.70	1.022	0.978	78	1.361
0.227	13	0.225	4.45	0.231	4.33	1.026	0.974	77	1.344
0.244	14	0.242	4.13	0.249	4.01	1.031	0.970	76	1.326
0.262	15	0.259	3.86	0.268	3.73	1.035	0.966	75	1.309
0.279	16	0.276	3.63	0.287	3.49	1.040	0.961	74	1.291
0.297	17	0.292	3.42	0.306	3.27	1.046	0.956	73	1.274
0.314	18	0.309	3.24	0.325	3.08	1.051	0.951	72	1.257
0.332	19	0.326	3.07	0.344	2.90	1.058	0.946	71	1.239
0.349	20	0.342	2.92	0.364	2.75	1.064	0.940	70	1.222
0.366	21	0.358	2.790	0.384	2.605	1.071	0.934	69	1.204
0.384	22	0.375	2.669	0.404	2.475	1.079	0.927	68	1.187
0.401	23	0.391	2.559	0.424	2.356	1.086	0.921	67	1.169
0.419	24	0.407	2.459	0.445	2.246	1.095	0.914	66	1.152
0.436	25	0.423	2.366	0.466	2.145	1.103	0.906	65	1.134
0.454	26	0.438	2.281	0.488	2.050	1.113	0.899	64	1.117
0.471	27	0.454	2.203	0.510	1.963	1.122	0.891	63	1.099
0.489	28	0.469	2.130	0.532	1.881	1.133	0.883	62	1.082
0.506	29	0.485	2.063	0.554	1.804	1.143	0.875	61	1.064
0.523	30	0.500	2.000	0.577	1.732	1.155	0.866	60	1.047
0.541	31	0.515	1.942	0.601	1.664	1.167	0.857	59	1.030
0.558	32	0.530	1.887	0.625	1.600	1.179	0.848	58	1.012
0.576	33	0.545	1.836	0.649	1.540	1.192	0.839	57	0.995
0.593	34	0.559	1.788	0.675	1.483	1.206	0.829	56	0.977
0.611	35	0.574	1.743	0.700	1.428	1.221	0.819	55	0.960
0.628	36	0.588	1.701	0.727	1.376	1.236	0.809	54	0.942
0.646	37	0.602	1.662	0.754	1.327	1.252	0.799	53	0.925
0.663	38	0.616	1.624	0.781	1.280	1.269	0.788	52	0.908
0.681	39	0.629	1.589	0.810	1.235	1.287	0.777	51	0.890
0.698	40	0.643	1.556	0.839	1.192	1.305	0.766	50	0.873
0.716	41	0.656	1.524	0.869	1.150	1.325	0.755	49	0.855
0.733	42	0.669	1.494	0.900	1.111	1.346	0.743	48	0.838
0.750	43	0.682	1.466	0.933	1.072	1.367	0.731	47	0.820
0.768	44	0.695	1.440	0.966	1.036	1.390	0.719	46	0.803
0.785	45	0.707	1.414	1.000	1.000	1.414	0.707	45	0.785
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